VALIDATION OF MASSIVELY PARALLEL SIMULATIONS OF DYNAMIC FRACTURE AND FRAGMENTATION OF BRITTLE SOLIDS

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ABSTRACT

Massively parallel finite element simulations of dynamic fracture and fragmentation of brittle solids are presented. Fracture is introduced by the adaptive insertion of cohesive elements. The model is validated against specially designed experiments in a collaborative effort between the experimental and the computational groups. Mesh sensitivity issues are addressed through the renormalization of the cohesive law.

1 INTRODUCTION

We present the results of massively parallel numerical simulations of dynamic fracture and fragmentation in brittle solids. Our approach is based on the use of cohesive models to describe processes of separation leading to the formation of new free surface. Within the framework of the conventional finite element analysis, the cohesive fracture models are introduced through *cohesive elements* embedded in the bulk discretizations. These cohesive elements bridge nascent surfaces and govern their separation in accordance with a cohesive law [1]. The cohesive elements are introduced adaptively in the simulation, driven by the fracture criterion naturally introduced by the cohesive law.

Cohesive theories of fracture intrinsically define a length-scale and a time-scale [1]. The intrinsic length-scale is related to the extent of the fracture process zone. To avoid spurious mesh effects, the finite element size must resolve the process zone. Furthermore, cohesive element approaches force the fracture surface to conform to the element faces, but otherwise the crack paths are arbitrary. Consequently, a sufficiently fine mesh is needed to avoid nonphysically constraining the crack path. Nevertheless, with a resolved mesh, the present method affords accurate solutions, and in particular branching and fragmentation are easily handled.

In this work we assess the validity and the predictive ability of the cohesive models and the computational algorithms. We present careful quantitative validation against experiments designed specifically for this purpose by A. J. Rosakis *et al.* In relation to the mesh dependency observed for under-resolved meshes, we explore the concept of renormalization of cohesive laws.

2 VALIDATION

The ability of cohesive theories of fracture and their numerical implementation through cohesive elements to reproduce the essential features of the dynamic brittle fracture process observed in experiment has already been established. In particular, our simulations are able to reproduce qualitatively the hierarchy of the branching phenomenon (micro and macro branching). An illustration is provided in Fig. 1. Features present in our simulations such as the characteristic spacing of crack branches, their trajectory, self-organized crack patterns, and the eventual fragmentation (c.f. Fig. 2), are reminiscent of experimental observations, for instance these of Fineberg and Sharon [2]. A detailed study of these issues is currently being pursued [3].

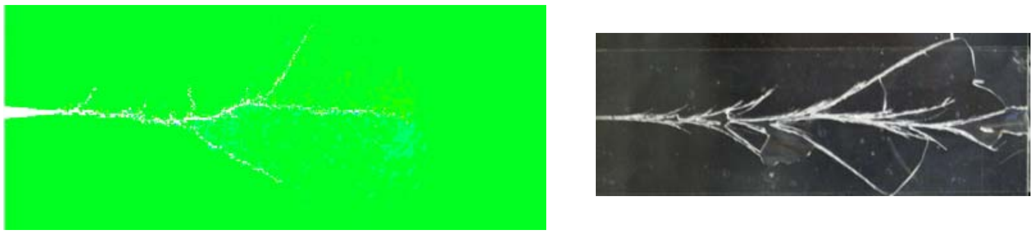


Figure 1. Dynamic instability under impact of projectile (hierarchy of micro- and macro-branching)

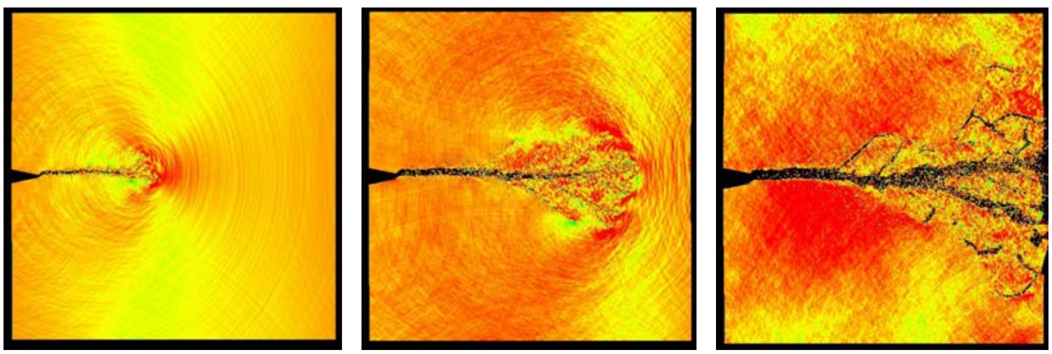


Figure 2. Snapshots (increasing times to the right) of the fracture process of a square pre-notched PMMA plate subjected to an initial uniform strain rate in the vertical direction.

Despite the encouraging qualitative agreement, a comprehensive quantitative validation of the models and algorithms of current use has been lacking. Such a validation effort is essential to establish the predictive ability of our simulation tools. A first computational difficulty stems from the disparity of scales between the typical sample sizes and the cohesive length scale associated with brittle materials. In order to obtain mesh-insensitive results, this cohesive length needs to be resolved, which in turn leads to extremely intensive calculations, only attainable by massively parallel implementations. A more fundamental challenge faced in this effort is the complexity of the phenomenon. Figure 3 shows an illustrative experiment, and sketches a succession of complex and sometimes competing events. From this image it is apparent that it is unreasonable to expect a detailed quantitative agreement between simulation and experiment, since the observed behavior is too complex and not even repeatable from an experimental point of view. Moreover, it is unclear on what grounds a meaningful comparison should be based. For this reason, it is essential to design experiments targeted at probing individually the mechanisms identified in the complex picture previously presented.

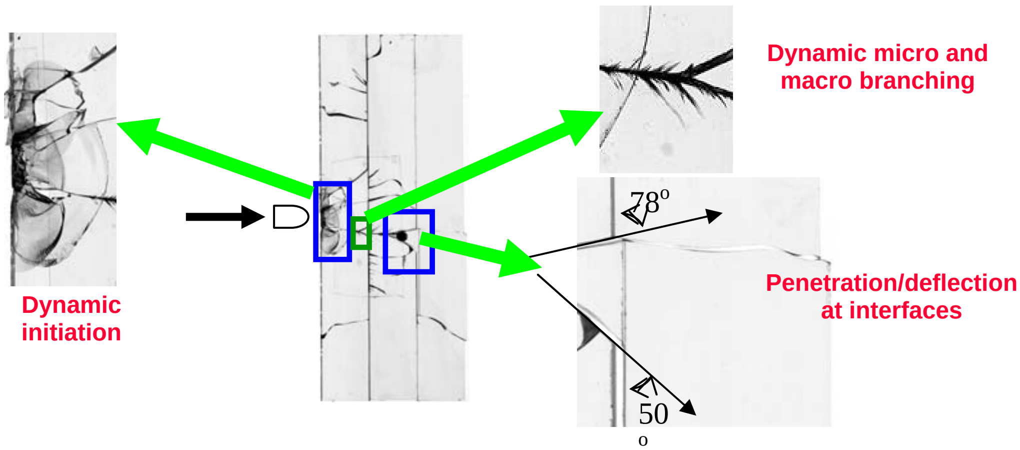


Figure 3. Phenomenon of catastrophic fragmentation: (1) dynamic crack propagation, curving and kinking, (2) triggering of branching instability, and (3) interaction of opening and shear dynamic cracks with interfaces (deflection/penetration).

In the spirit of Ref. [4], and with the validation effort in view, Rosakis and co-workers have designed a set of well controlled experiments to investigate the penetration/deflection of dynamic cracks by interfaces of different inclination angles and strengths. These experiments, on pre-notched Homalite-100 plates, have simple boundary conditions and well defined loading conditions, which ensure repeatability and facilitate the setup of the simulations. Full-field diagnostics provide a wealth of metrics for the quantitative comparison. Figure 4 illustrates the experimental configuration and the loading setup.

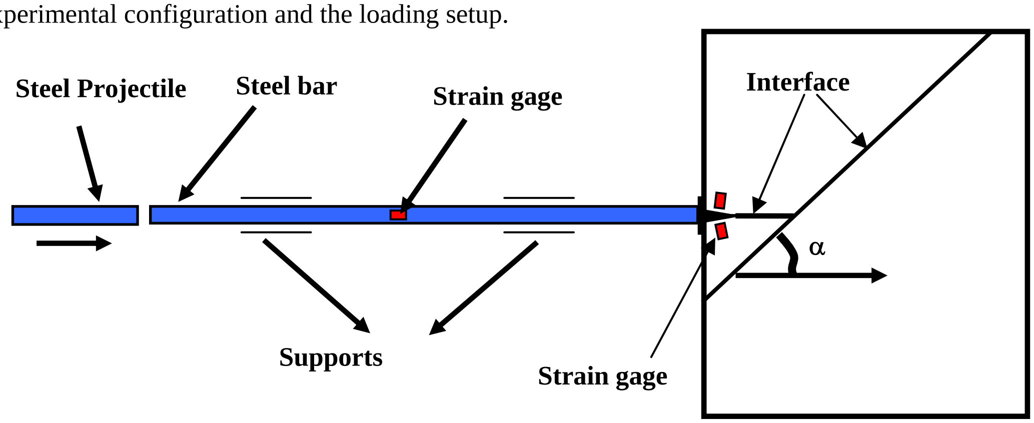
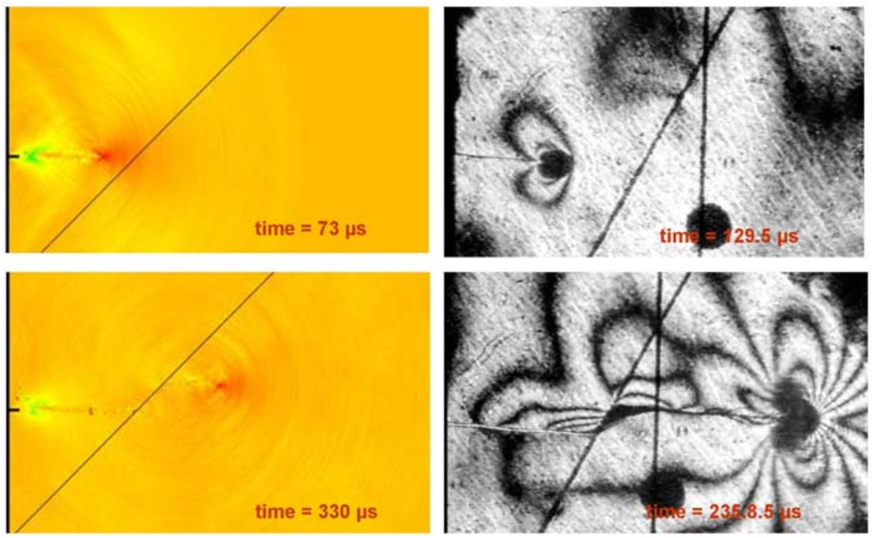


Figure 4: Modified Hopkinson bar loading setup.

Preliminary simulations of these experiments have been performed, and the penetration/deflection behavior has been reproduced to some extent, without any parameter fitting (c.f. Fig. 5). A detailed quantitative agreement is still object of ongoing work. In particular, we are addressing an accurate measurement of the cohesive law from independent experiments by an inverse method [5], as well as the dynamic friction coefficient relevant to precisely model the loading. On the other hand, fully resolved multi-million elements simulations are being pursued. Methods to alleviate the intense computational cost are currently under investigation, as described in the next section.



Crack deflection and subsequent penetration for an interface angle of 45°

Crack deflection and subsequent penetration for an interface angle of 60°

Figure 5. Qualitative comparison of deflection behavior in simulation (left) and experiment (right).

3 RENORMALIZATION

As previously noted, simulations concerned with materials possessing as small characteristic length scale may require a large number of elements in the finite element discretization. By way of example, simulations of the Rosakis *et al*. experiment may require as many as 75 million elements. This level of resolution is often prohibitively expensive even when advanced distributed computing environments are available. An alternative approach is to compensate for the lack of resolution by a suitable renormalization of the cohesive law. Specifically, we apply the scaling proposed by Nguyen and Ortiz [6], which relates the effective cohesive energy of a material layer to the interplanar potential under the assumption of nearest-neighbor interactions. A generalization of this renormalization which accounts for arbitrary interactions and surface relaxation has been proposed by Hayes *et al*. [7]. A rigorous mathematical proof of the universality of the renormalized cohesive law has been given by Braides *et al*. [8].

We assess the feasibility of this approach numerically in a double-cantilever-beam test configuration [9] and present preliminary results. Specifically, we endow the cohesive elements with a cohesive law scaled according to the mesh size. Thus, the cohesive strength scales as $1/\sqrt{h}$ and the critical opening displacement scales as $\sqrt{h}$, where *h* is the characteristic mesh size. The resulting renormalized cohesive law represents the effective behavior of a layer of material of thickness *h*, and has the property that the corresponding cohesive length is automatically resolved by the mesh. In addition, the fracture energy remains invariant under the renormalization. Fig. 6(a) shows the position of the crack tip as a function of time for several under-resolved meshes. The strong mesh sensitivity is apparent in this figure. However, the solutions for these meshes nearly collapse into one curve when renormalized cohesive laws are used, as shown in Fig. 6(b).

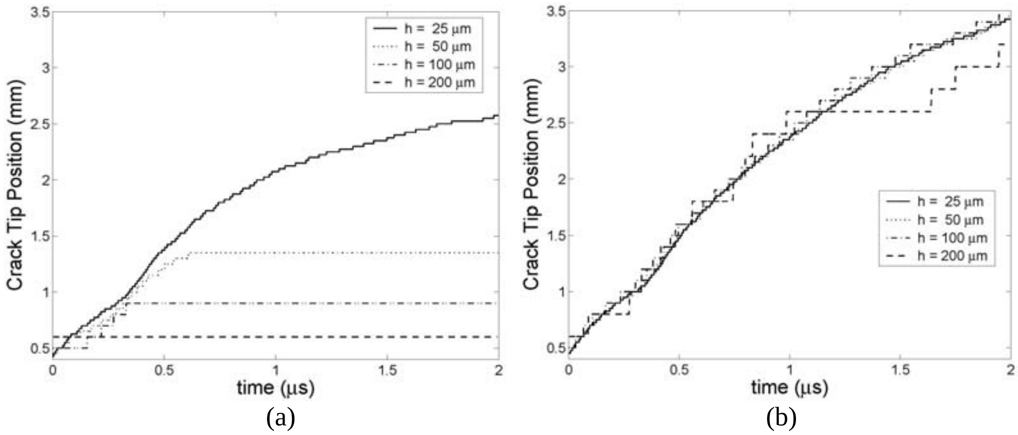


Figure 6. Double-cantilever beam test: solutions for under-resolved meshes with actual (a) and renormalized (b) cohesive parameters.

ACKNOWLEDGEMENTS

The authors gratefully acknowledge the support of the DOE, ASCI-ASAP, through a grant (#DIM.ASC 1.1.7S LLNL.ASCP) to Caltech.

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