A RESIDUAL STRESS CRACK OPENING APPROACH AFTER AN OVERLOAD DURING FATIGUE

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ABSTRACT
The compressive residual stresses following overloads during fatigue loading is the major parameter influencing the retardation effect observed in the fatigue life duration [1]. In this paper, we propose a new residual stress opening approach based on a specific criterion controlling the debonding at the crack tip. This approach is expressed as a crack opening displacement or as a crack opening stress criterion; it is then compared to experimental results obtained by Lang and Marci [2]. This comparison shows that the stress opening criterion including the cyclic material behaviour leads to a good prediction of the crack opening. A systematic parametrical study will be carried out in order to validate this major conclusion.

1 INTRODUCTION
The crack propagation under fatigue is highly influenced by the residual stress field at the crack tip. This compressive stress field near the crack tip controls the crack retardation phenomenon observed by several authors since Schijve [1] in 1962. This phenomenon depends on the shape of the loading process and particularly depends on the overloads or underloads during the process. In the technical literature, this delay was analysed as a consequence of various physical mechanisms such as crack tip blunting, crack tip strain hardening, branching, crack closure (induced by plasticity, oxidation or roughness), residual compressive stresses ahead of the crack tip, etc…

The concept of $\Delta K_{eff}$, proposed by Elber in 1970 [3], has been generally accepted by the scientific community in order to describe the overload effect. This concept consists in introducing a stress intensity factor (SIF) range into the Paris law, namely:

$$\frac{da}{dN} = C(\Delta K_{eff})^m \quad \text{with} \quad \Delta K_{eff} = K_{max} - K_R$$

where $C$ and $m$ are the Paris law constants and $K_R$ ($R$ for Restriction) is a critical value of the stress intensity factor from which $\Delta K$ contributes to the crack growth.

Historically, during 20 years, Elber model, dealing with the crack closure concept, was generally admitted. It is only in the 1990’s, that some researchers, on the basis of former work of Schijve [1] and Marci [4], reintroduced other phenomena such as the residual compressive stresses ahead of the crack tip in the restriction of the stress intensity factor range. Consequently, the $\Delta K_{eff}$ concept remains valid; nevertheless the capacity of the crack closure concept seemed not to be sufficient to explain the delay phenomena [5]. After this statement, and in order to better model the delay phenomenon with $\Delta K_{eff}$, we propose, in this paper, a new approach based on the use of the residual compressive stresses. This approach leads to determine the crack opening behaviour after an overload.

2 POSITION OF THE PROBLEM
It is well known that during the fatigue service under constant amplitude loading, an overload involves a delay of the crack growth rate, which increases with the overload
amplitude [5]. The problem remains the understanding of the crack growth evolution during this period. In fact, the retardation effect depends on the calculation of the minimum crack growth rate, which depends on $K_R$ (eqn (1)). $K_R$ may have several definitions such as being the limit between the curved part and the linear part of the compliance curve or related to compressive residual stresses ahead of the crack tip [2].

In this paper, according to several works [1, 2, 4], we decide to take into account only the residual stresses as responsible of the retardation effect. In order to distinguish the principal mechanism of the crack growth propagation, Lang and Marci [2] have developed a test procedure, named CLPM (Crack Propagation Load Measurement), which allows to determine the effective part of the propagation. According to these test results, we have developed a numerical methodology allowing to determine the restriction of the effective stress intensity factor range $\Delta K_{eff}$.

For the material and the specimen used in [2], test procedure of CPLM enables to obtain the $K_R$ value, as a function of the unloading cycle following the overload:

$$K_R = \left(0.322 + 0.57 R_U + 0.23 R_U^2 - 0.145 R_U^3\right) K_{\max} \quad [-0.7 < R_U < 1] \quad (2)$$

where $R_U$ is the load ratio following an overload, thus $R_U = K_{ul}/K_{\max}$ (where $K_{ul}$ corresponds to the unloading cycle following the overload). Equation (2) is displayed on Figure 1 hereafter. The calculation of $K_R$ is performed by means of finite element analysis.

![Figure 1: Evolution of $K_R$ for an overload (eqn (2)).](image)

**3 NUMERICAL MODELLING**

Finite element analysis has been conducted with Abaqus Code [6], in order to derive the effective stress intensity factor range namely residual stress crack opening phenomenon ($K_{op}$ equivalent to $K_R$ in eqns (1) and (2)).

**3.1 Modelling aspects**

Nowadays, the crack opening modelling becomes the main subject of investigation for many researchers. In particular, recent works from Ellyin and Wu [7], Pommier [8] and Solanki et al. [9]
have been released. Indeed, computational means have greatly been improved, allowing large element meshes in order to evaluate precisely the stress gradient at the crack tip in elastic-plastic materials. But many assumptions have to be made to model the complexity of fatigue crack growth. So various modellings developed in the literature have covered these following purposes:

- Plane stress or plane strain conditions, depending on the geometry and on the $K_{op}$ measurements location.
- Two-dimensional or three-dimensional finite element modelling; the latter is much more CPU time consuming, but allows to consider the plastic zone size at the surface and in the deep of specimens.
- Specimen geometry: CT, MT or others.
- Element type and element size at the crack tip. The necessity to dispose of enough number of elements in the plastic zones is often enlightened.
- Crack surface contact modelling.
- Crack advance scheme: generally, the methods are based on the following principle: releasing nodes, assuming a growth of one element size, and then applying cyclic loadings. But, one can debond the crack surfaces at the maximum load, the minimum load, after one or two cycles. The main difficulty is to define a sequence of crack advances and loadings that refers to the crack growth phenomenon. But it is worth noting that this decoupling has no physical considerations, and no stress or strain criteria drive the crack tip advance in this model.
- the $K_{op}$ crack opening SIF assessment. Either displacement or stress criteria at the crack tip can be considered.

Our modelling choices will be introduced in the next part of this paper.

3.1.1 Finite element modelling choices

Refined meshes are needed at the crack tip to model the crack growth by FEM in order to precisely calculate the stresses and strains in the monotonic and in the reverse plastic zones. Furthermore, the loading cycles have to be finely discretized to determine $K_{op}$. Calculations are non-linear, on the one hand by geometrical aspects, as we take into account the contact between the crack surfaces, on the other hand by material aspects, i.e. the material response is elastic-plastic. Only two-dimensional calculations have been performed on a common CT specimen in which plane strain conditions are fulfilled.

**Element type**: 8 nodes, biquadratic quadrilateral elements have been chosen, with reduced integration. Plane strain locking is avoided thanks to hybrid elements from Abaqus, which allow to calculate the hydrostatic stresses only at the centre of the element and so prevent from numerical singularities when the Poisson’s ratio tends to 0.5 (especially at the crack tip).

**Material behaviour law**: Cyclic plasticity occurs at the crack tip and our model is based on the assumption that this behaviour governs the crack opening level. To better represent the material, we use the Chaboche constitutive law which combines a non-linear kinematic hardening and an isotropic hardening.

**Element size**: Size of the elements is determined thanks to the Irwin’s plastic radius $r_p$.

$$r_p = \frac{1}{6\pi} \left( \frac{K_{\text{max}}}{S_p} \right)^2$$  \hspace{1cm} (3)
where \( S_y \) is the initial yield strength. Solanki et al. [9] suggests at least 3-4 elements in the reverse plastic zone and at least 10 elements in the monotonic plastic zone. In our model the element size is about 10 microns, which corresponds to about 5 elements in the reverse plastic zone after an overload (see Figure 2).

![Figure 2: Monotonic and cyclic plastic zones.](image)

**Crack tip debonding:** The Abaqus procedure requires to define two distinct initially bonded surfaces between which the crack will propagate. The first one is based on the faces of the crack elements and the other one is created with a rigid surface, as only half of the specimen is modeled. Debonding is linked with a reduction of the transmitted force between both surfaces at specified nodes during computations. Convergence difficulties can occur for large-strain problems, typically at the crack tip and a specific debonding curve is required. Furthermore, to model fatigue crack growth, elastic-plastic stress (associated to cyclic behaviour law) have to be stabilized before debonding the nodes.

### 3.1.2 Crack opening criteria

At least two types of criteria can be defined:

- **Displacement criterion:** based on the displacement of crack surface nodes near to the crack tip. Pommier [8] determines the displacement variation ratio of the second node behind the crack tip (CTOD) with the maximum displacement of this node. This criterion can be expressed as: 
  \[
  \frac{U_{\text{op}} - U_{\text{min}}}{U_{\text{max}} - U_{\text{min}}} = 1.5 \%
  \]
  where \( U_{\text{max}} \) and \( U_{\text{min}} \) are respectively the maximum and minimum displacements of that node, and \( U_{\text{op}} \) is the calculated crack opening parameter.

- **Stress criterion:** based on the transition between compressive and tensile stresses either at the crack tip [7], behind the crack tip, or in the elements surrounding it. This criterion is defined in accordance with the hypothesis that the crack can grow only when the compressive stress field at the crack tip is released.

### 3.2 Application

#### 3.2.1 Specimen used

A CT specimen has been chosen for our studies (width: 50 mm, thickness: 10 mm). This specimen was used in Lang and Marci tests. Modelling, meshing and boundary conditions are presented in Figure 3.
To prevent from any short crack phenomenon or failure conditions, our calculations have been carried out with a 6 mm crack. The material used is an aluminium alloy Al7475-T7351 with the following characteristics: $S_y = 350$ MPa, $E = 71500$ MPa and $v = 0.3$. The constitutive behaviour of the material is well described by using a Chaboche plasticity model (with kinematic hardening parameters: $C = 290$ MPa, $C_A = 50000$ MPa, and isotropic cyclic hardening parameters: $Q = 0$, $b = 15$) [10].

3.2.2 Loading sequence
Considering a kinematic cyclic hardening behaviour of the material, it has been checked that only two loading cycles are needed to stabilize the crack tip plastic zone. As a consequence, these two cycles are applied before each debonding, in order to simulate a fatigue crack growth.

The following sequence has been considered for our simulation: 2 cycles at Constant loading ($K_{\text{max}} = 8.1$ MPa.m$^{1/2}$, $K_{\text{min}} = 0$), 1 element (2 nodes) debonding ($\leftrightarrow$), 2 cycles at constant loading, 1 Overload of 3x8.1 i.e. ($K_{\text{max}} = 24.3$ MPa.m$^{1/2}$, $K_{\text{min}} = 0$), 1 element debonding, a Decrease at loading to $K_{ul}$ and then $N$ cycles at constant loading. The sequence could be noted as $2C_0^8 \leftrightarrow 2C_0^8 \leftrightarrow O^{24.3}_0 \leftrightarrow D_{Kul} \leftrightarrow C_{Kul}^8$ (see Figure 3 with $K_{ul} = 0$).

3.2.3 Comparison of displacement and stress criteria
The stress criterion has been split into two criteria (see Figure 3).

![Figure 3: Meshing of CT specimen – loading cycles and $K_{op}$ assessment location.](image)

![Figure 4: Comparison between FEM calculations and experiments from Lang and Marci.](image)
• Stress criterion n°1: crack opening assessment at the cycle following the overload.
• Stress criterion n°2: crack opening assessment after 2 cycles following the overload. This criterion has been considered with reference to Lang and Marci tests, in which the crack opening assessment procedure could have lead to stabilize the plastic zone around.

Results obtained with these different criteria are reported in Figure 4. Stress criterion n°2 best fits the experimental results, whereas the displacement criterion seems to underestimate $K_{op}$.

4 CONCLUSIONS
This study presents a complete methodology of crack opening phenomenon modelling by finite elements. Many assumptions have been made to take into account the complexity of fatigue crack growth: fine 2D quadratic model, Chaboche constitutive law, crack tip debonding procedure and several crack opening criteria (displacement and stress).

Finally, the simulation results are in good agreement with existing experimental results and validate the suggested methodology related to numerical modelling of the residual stresses ahead of the crack tip. The crack opening phenomenon seems to be better represented by the stress criterion than by the displacement criterion and it will be adopted to determine the loading level from which one can observe the crack debonding.

Further works will be focused on the generalisation of the present methodology for constant and variable amplitude loadings and for validation by using experimental test results dealing with the crack opening phenomenon.

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