NONLOCAL DAMAGE-PLASTICITY MODEL FOR FAILURE OF PLAIN CONCRETE

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ABSTRACT

Failure of concrete in tension and compression is simulated by means of a local (with fracture energy adjustement) and a nonlocal damage-plasticity constitutive model. A three-point bending test, representing tensile failure, and a prism subjected to eccentric compression, representing compressive failure, are analysed. The mesh size is varied to study, whether the local and nonlocal model are capable to describe the failure process independently of the mesh size.

The results for tensile failure are independent of the mesh size for both models. Compressive failure, on the other hand, cannot be described by the local model. The results of this model are polluted by pathological mesh size dependence, since the size of the region of localized strains does not depend on the element size. The nonlocal model, however, is capable to describe compressive failure independently from the mesh size.

1 INTRODUCTION

Failure of plain concrete in tension and compression is characterized by softening, which is accompanied by the development of regions of highly localized strains. In the present paper a local and nonlocal damage-plasticity constitutive model is used to simulate tensile and compressive failure of plain concrete. It is investigated whether these models are capable to describe the failure process in an objective way.

The constitutive model is based on three-dimensional elasto-plasticity with the yield condition written in terns of the effective stress, combined with a scalar damage model with the damage evolution driven by the plastic strain. This model was developed in an earlier study by the authors [1]. The plasticity model is characterized by a pressure-sensitive yield condition, non-associated flow and pressure-dependent hardening. Softening is described by the damage model, using an equivalent strain measure based on the volumetric plastic strain, which links, for instance, the plastic volumetric expansion in compression to the evolution of damage. The damage history variable defined as the maximum equivalent strain reached in the history of the material, is related to the damage variable by an exponential softening law.

In the context of the standard continuum theory, a constitutive law with softening leads to localization of inelastic strains into a layer of width h_{t}. For tensile failure, h_{t} is related to the size of the finite elements. There are two main approaches to overcome the resulting pathological sensitivity of the numerical results to the discretization:

- Adjustment of the softening law with respect to h_{t}, so that the dissipated energy is independent of the size of the finite elements, see [2].
- Application of a localization limiter, which leads to a size of the localization zone independent of the discretization.

The first approach, denoted as local, assumes a numerically represented fracture process zone of width h_{t}, which is mesh size dependent. In the present study, the value h_{t} was determined as the projection of the element size normal to the direction of the maximum principal strain.

The localization limiters enforce a specific size of the region of inelastic strains independent of the discretization. We use the integral-type nonlocal model, which achieves this by weighted spatial averaging of a suitable state variable. In the present study the equivalent strain ε_{f} was averaged, as proposed by Pijaudier-Cabot and Bažant [3] for nonlocal damage:

$$\bar{\varepsilon}\left(\boldsymbol{x}\right) = \int_{\mathrm{V}} \alpha\left(\boldsymbol{x}, \boldsymbol{\xi}\right) \tilde{\varepsilon}\left(\boldsymbol{\xi}\right) \mathrm{d}\boldsymbol{\xi} \tag{1}$$

where $\bar{\varepsilon}_{\mathrm{f}}$ is the nonlocal equivalent strain, α is the weight function and V is the spatial domain representing the body. The weight function is defined as

$$\alpha\left(\boldsymbol{x}, \boldsymbol{\xi}\right) = \frac{\alpha_{\infty}\left(||\boldsymbol{x} - \boldsymbol{\xi}||\right)}{\int_{\mathrm{V}} \alpha_{\infty}\left(||\boldsymbol{x} - \boldsymbol{\zeta}||\right) \mathrm{d}\boldsymbol{\zeta}} \tag{2}$$

The function α_{∞} is the bell-shaped truncated polynomial function

$$\alpha_{\infty}\left(r\right) = \begin{cases} \left(1 - \frac{r^2}{R^2}\right)^2 & \text{if } |r| \leq R \\ 0 & \text{if } |r| \geq R \end{cases} \tag{3}$$

The energy per unit area dissipated by the nonlocal model in a uniaxial tension test is $G_{\mathrm{F}} = kRg_{\mathrm{F}}$ where R is the nonlocal interaction radius, g_{F} is the area under the local stress-strain curve and k is a proportionality factor, which ranges between 1.6 and 2 depending on the shape of the softening curve, see [4].

In the following, both the local and the nonlocal approaches are used to model the tensile and compressive failure of plain concrete specimens.

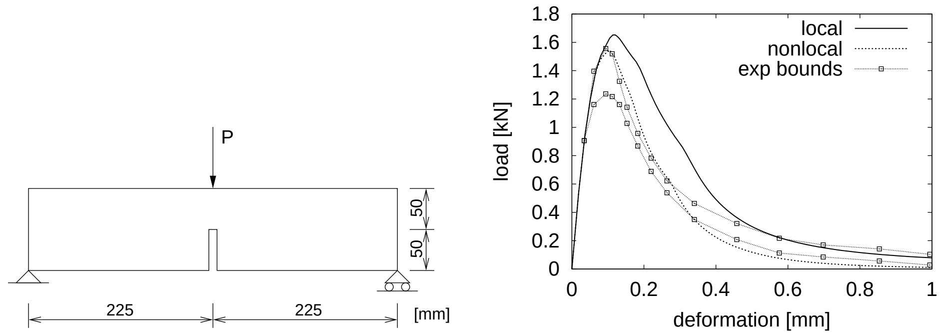
2 STRUCTURAL EXAMPLES OF TENSILE AND COMPRESSIVE FAILURE

Two structural examples of tensile and compressive failure are used to study the performance of the local and nonlocal damage-plasticity model. Of special interest are the regions of localized strains, in which either the damage part or the plasticity part or both are active. Therefore, we call them active regions. Triangular plane-strain finite elements with linear displacement interpolation were used to discretize the specimens. The mesh size was varied.

The first example is a three point bending test of a single-edge notched beam tested by Kormeling and Reinhardt [5], see Fig. 1a. The material parameters are the Young's modulus $E = 30$ GPa, the Poisson's ratio $\nu = 0.2$, the tensile strength $f_{\mathrm{t}} = 2.4$ MPa and the fracture energy $G_{\mathrm{F}} = 113$ J/m^2. The nonlocal interaction radius was set to $R = 25$ mm and the proportionality factor to $k = 1.6$.

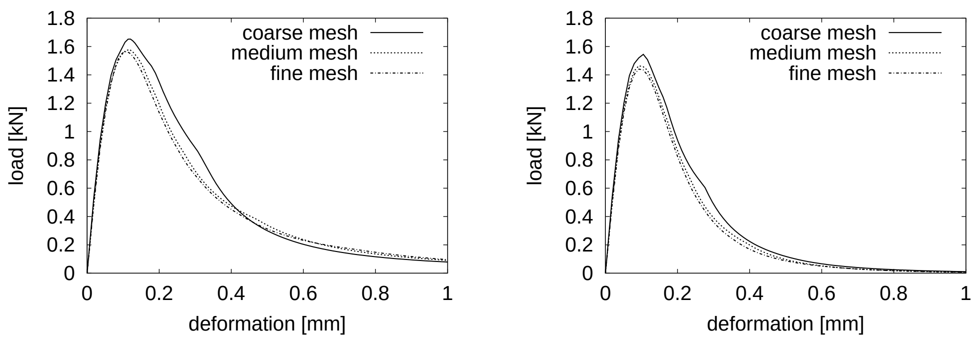
Three meshes of different element size were used. The load-displacement curves of the local and nonlocal analyses with the coarse mesh are compared to the experimental bounds in Fig. 1b. The load-displacement curves for the analyses with the three different mesh sizes are shown in Fig. 2a and b for the local and nonlocal model, resp. The corresponding active regions are shown in Fig. 3 and Fig. 4.

The local model overestimates the load capacity and ductility observed in the experiments, see Fig. 1b. This might be explained by the shape of the active region, which is not a straight layer, but is forced to follow the unstructured mesh lines. The results of the nonlocal model is in better agreement with the experimental bounds, since here the active zone is not influenced by the mesh



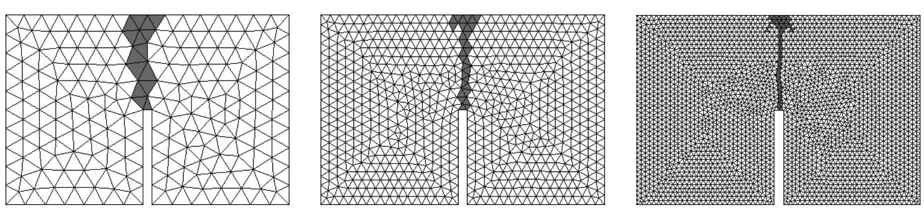
(a) (b)

Figure 1: a) Geometry and loading setup of the three point bending test. b) Comparison of the local and nonlocal FE analyses with the experimental bounds.



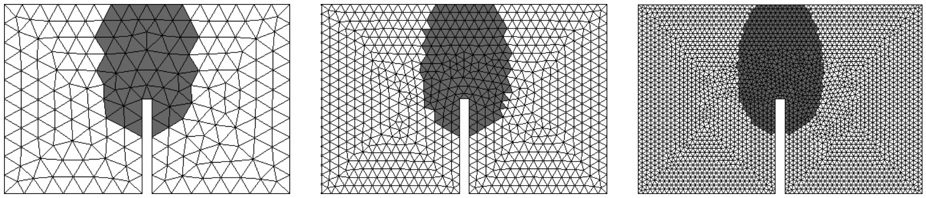
(a) (b)

Figure 2: The load displacement curves for different mesh sizes for a) the local model and b) the nonlocal model



coarse mesh medium mesh fine mesh

Figure 3: The active regions of the 3 point bending test for the local model at 1/10 of the peak load in the post-peak regime.

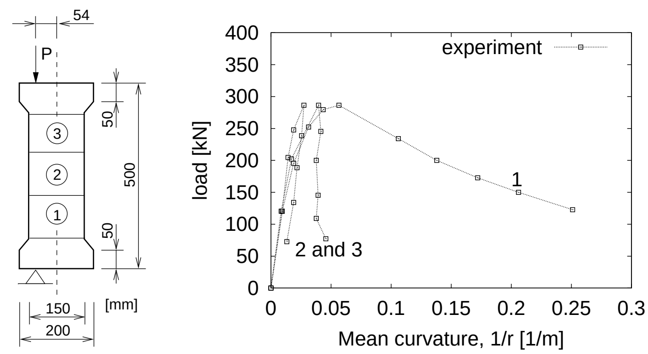


coarse mesh medium mesh fine mesh

Figure 4: The active regions of the 3 point bending test for the nonlocal model at 1/3 of the peak load in the post peak regime.

lines. Nevertheless, both models give an almost mesh size independent description of tensile failure (Fig. 2), where the load displacement curves converge to a lower bound with decreasing mesh size.

The second example is a concrete prism loaded in eccentric compression tested by Debernardi and Taliano [6]. The geometry and loading setup is shown in Fig. 5a.



(a) (b)

Figure 5: a) Geometry and loading setup of the eccentric compression test. b) Comparison of the load mean curvature curves of the local and nonlocal model with the experimental ones.

The material properties were chosen as the Young's modulus $E = 30$ GPa, the Poisson's ration $\nu = 0.2$, the compressive strength $f_{\mathrm{c}} = 56$ MPa, the tensile strength $f_{\mathrm{t}} = 4$ MPa and the fracture energy in uniaxial tension $G_{\mathrm{F}} = 100$ J/m^2. The nonlocal interaction radius was set to $R = 0.025$ and the proportionality factor to $k = 1.6$.

The results are presented by means of load-mean curvature relations in three different regions of the specimens; see Fig. 5a,. The mean curvature is defined as the average strain measured on the left and right boundary of the region divided by the distance between the two edges. The experimental results are presented in Fig. 5b. The results of the finite elements are not directly compared to the experimental results, since plane strain finite elements were used in the analyses. The compressive response of concrete in plain strain is characterized by a significantly greater ductility than in plane stress, which would better correspond to the experimental setup. However, a plane stress version of the constitutive model is not yet available. Two meshes of different element size were used. The load-mean curvature curves with the corresponding active region plots are presented in Fig. 6 and Fig. 7 for the local and nonlocal model, resp.

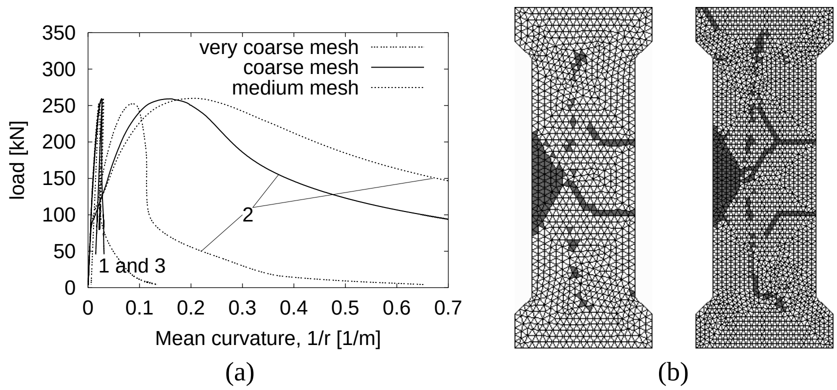


Figure 6: a) The load-mean curvature curves of the local model for three different mesh sizes. b) The active region plots of the coarse and medium mesh at 170 kN in the post peak regime.

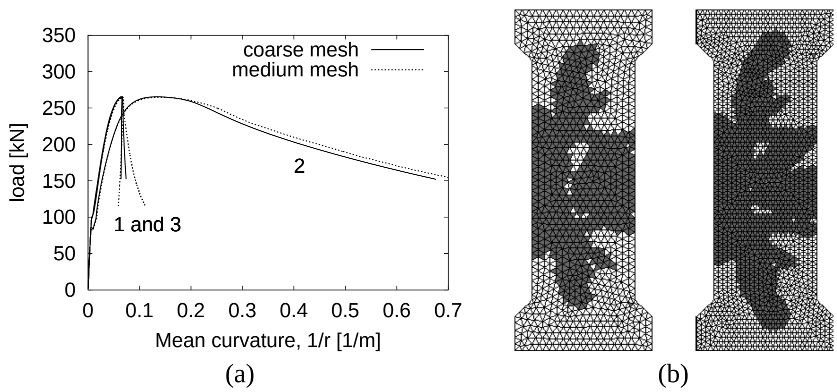


Figure 7: a) The load-mean curvature curves of the nonlocal model for two different mesh sizes. b) The active region plots of the coarse and medium mesh at 170 kN in the post peak regime.

The general behavior is well captured by the analyses. The strains in the compressive zone localize in region 2, which is manifested by a reduction of the load accompanied by increasing curvature. Regions 1 and 3, on the contrary, show an unloading response in form of decreasing load accompanied by decreasing curvature. A similar response was observed in the experiments.

The results of the analyses with the local model for the different mesh sizes (see Fig. 6a) show a clear mesh dependence: the finer the mesh, the more ductile the response. This trend was confirmed by an additional analysis with a very coarse mesh. The results of the analyses with the nonlocal model, on the contrary, show almost no mesh dependence. The two meshes result in almost the same response in form of the load-mean curvature relations, depicted in Fig. 7a. The mesh dependence of the local model can be explained by the active regions plotted in Fig. 6b. The size and shape of the active regions in the compressive zone is almost mesh-size independent. Thus, the fundamental assumption of a mesh-dependent active region is violated and the local model exhibits pathological mesh dependence. The nonlocal model, on the other hand, relies on an mesh-independent active region. Thus, this model gives mesh-independent results, when modeling compressive failure of plain concrete.

3 CONCLUSIONS

The evaluation of the performance of a local and nonlocal damage-plasticity model in describing the tensile and compressive failure of plain concrete leads to the following conclusions:

- Both the local and nonlocal models are able to describe tensile failure, exemplified by a three point bending test, independently of the mesh size.
- Compressive failure, exemplified by an eccentric compression test, can only be modeled by the nonlocal approach. The results of the local model are polluted by pathological mesh dependence. This mesh dependence is caused by a diffuse region of inelastic strains, which violates the assumption of localization made for the adjustment of the softening modulus.

These conclusions need to be confirmed by further investigations. In particular, additional example of compressive failure should be studied.

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