In this contribution the well established approach to fatigue failure by means of load controlled Wöhler tests is compared with a fracture mechanical approach, which considers the softening behaviour of concrete as observed in deformation controlled tensile tests. Therefore, a series of load as well as deformation controlled uniaxial tensile tests on notched and on unnotched concrete prisms was carried out, respectively. The main parameters in the experiments were the number of cycles to failure, the curing conditions, the concrete grade and the frequency and the deformation rate, respectively. The load controlled tests provided Wöhler lines which were nearly parallel to each other for all investigated parameters, except for the tests on high strength concrete showing a line more inclined. The experimental results from the deformation controlled tests showed in particular that for an increasing number of load cycles the corresponding envelope curves differed significantly from the monotonic curve. This clearly showed that the previous conventional assumption of a unique envelope curve for the fatigue behaviour of concrete is admissible only for a fatigue loading with a low number of deformation controlled cycles, but cannot be maintained if the number of such loading cycles is approx. 1000 and higher. The comparison of the two different approaches to fatigue yielded in the conclusion that both approaches are useful for different applications and they can not be converted reciprocally by just a simple transformation formula.

1 INTRODUCTION
The establishment of a European high-speed railway transportation system was and still is one of the challenging tasks for railway construction engineers. For this railway net often superstructures with ballastless slab tracks made of concrete or asphalt are used instead of the usual crushed stone subbase. The advantages of ballastless slab tracks are a stable and durable rail position, the reduced requirement of intensive maintenance, a long-term durability and an individual adjustment to various velocity and load profiles. Furthermore, ballastless slab track constructions are lower in height which may be useful in tunnels to require less ground excavation and allow higher banking resulting in a reduced length of artificial building structures. On the other hand – compared to crushed stone subbase – there is a restricted possibility for rail position correction, a higher investment cost, a limited facility for recycling and a higher sound emission. In a calculation the reduced stiffness of the load bearing parts of ballastless slab track structures has to be taken into account by replacing the bedding modulus with a specific support point stiffness, see also [1].

Since traffic loads are occurring periodically the concrete structures have to be investigated concerning their behaviour under cyclic loading conditions. So far the experiments to characterise the fatigue behaviour of concrete structures subjected to a high number of loading cycles were load-controlled tests, so called Wöhler tests. Due to the predetermined upper and lower load level these tests are not capable to detect the softening behaviour of concrete, which might be essential to know with regard to durability of the concrete members. In this contribution the conventional approach to fatigue failure by means of Wöhler tests is compared with a fracture mechanical approach based on the results of deformation-controlled cyclic tensile tests, which provide also the softening behaviour of concrete.
2 EXPERIMENTAL PROGRAM OVERVIEW

Both, the load controlled and the deformation controlled tensile tests with non-rotatable boundaries were performed on notched concrete prisms schematically shown in fig. 1, left. The tests were carried out on specimens of two concrete grades, namely a normal strength concrete (NSC) with a water-cement ratio w/c of 0.55 and a high strength concrete (HSC, w/c = 0.30). For both concrete grades a maximum aggregate size of 16 mm was applied. The prisms were casted horizontally in metal forms. A special kind of curing to simulate the moisture condition of mass concrete was achieved by applying a three layer system of protection against desiccation (further denoted as sealed specimen). The specimens termed as unsealed were simply stored in a climatic chamber at a relative humidity of 65 % and a temperature of 20 °C immediately after demoulding representing the moisture condition of slender concrete members without proper curing. The compressive strength determined at the concrete age of 28 days showed a medium value of \( f_c = 50.5 \) MPa in the case of NSC and \( f_c = 109.9 \) MPa for HSC. The age of the concrete at testing was 280 days. To include the effect of the loading rate two different frequencies were used (1 Hz and 10 Hz) in the Wöhler tests. In the deformation controlled tests the deformation rates were 5 \( \mu \)m/s and 0.5 \( \mu \)m/s, respectively. The details may be found in [2].

3 LOAD CONTROLLED WÖHLER TESTS

A typical load-deformation relation obtained from the load controlled tests is shown in fig. 1, right.

![Load-deformation curve](image)

As can be seen in fig. 1, right the deformation in the last few cycles before the final failure increases over proportionally, which indicates a decreasing stiffness of concrete at this stage. By regarding the mean deformation \( \bar{\delta}_m = \frac{1}{2}(\delta(F_{upper}) + \delta(F_{lower})) \) in Wöhler tests the relations shown in fig. 2, left could be obtained. The increase of the deformation with increasing actual number of load cycles – referred to the number of load cycles at final failure – provide a so-called cyclic creep curve, which can be divided into three parts predominantly representing three phases of cracking, but also creep deformations. The phase of micro crack initiation and primal creep up to a number of cycles of approximately 0.2·N (in fig. 2 denoted as phase I) is characterised by an extensive increase of deformation \( d\delta/dN > 0; d^2\delta/dN^2 < 0 \). Phase II shows an approximately linear ascent meaning a constant value of the \( d\delta/dN \) up to about 0.8·N. Above a number of load cycles of about 0.8·N the deformations increase dramatically up to final failure (phase III). In general, the described shape of the \( \delta-N \) curve could be observed for all investigated parameters (degree of load, curing conditions and frequency). The use of high strength concrete instead of normal strength concrete resulted in a shift of the cyclic creep curve to the higher
values of deformations. This effect can be drawn back mainly to higher absolute values of the upper load level for the same loading degree in the case of high strength concrete, which may lead to larger elastic deformations and a more pronounced primal creep.

![Figure 2: Effect of the increasing number of load cycles on mean deformation (left) and mean stiffness (right) as observed in load-controlled cyclic tensile tests on notched concrete prisms](image)

The stiffness decrease with an increasing number of load cycles is shown in fig. 2, right. The method used for the calculation of the inclination of mean straight lines within hysteretic loops is leaned on [3]. Again, there is only a minor effect of the investigated parameters degree of the loads, concrete grade, curing conditions and frequency with regard to the curve shape. Similar to the curves obtained for the deformations three phases can be identified, however, with the first phase ending already at a number of load cycles of about \(0.05 \cdot N\). The observed stiffness in the last load cycles is up to 60% less than the initial stiffness, which is proportional to the modulus of elasticity of the undamaged concrete at the first load cycle.

![Figure 3: Effect of the upper load level on the attained number of load cycles to failure in load-controlled cyclic tensile tests (Wöhler lines)](image)
If the above discussed mean straight lines of the two load cycles following each other are linked at the points of the upper and the lower load, they build an area characterising the dissipated energy in this state of cracking. The so obtained curves have nearly the same shape as the curve for the deformations shown in fig. 2, left. Again we have the above described three phases from a strong increase in the early beginning and a continuously increase in the main part of the tests resulting in the unstable crack growth immediately before the final failure occurs.

If the results of the load controlled tensile tests are plotted in a stress-number of load cycles at failure diagram the Wöhler lines shown in fig. 3 can be obtained. First, it is worth to be mentioned that for two combinations of parameters – normal strength and high strength concrete prisms, both sealed and loaded with a frequency of 10 Hz – could bear more than 10 million load cycles without a final failure at a degree of load \( S = \sigma_{\text{upper}} / \sigma_{\text{lower}} = 0.6 \). These specimens were undertaken a monotonic deformation controlled uniaxial tensile test afterwards. The load-deformation relations obtained from these monotonic tests showed similar characteristic mechanical and fracture mechanical values as obtained in deformation controlled fatigue tests with a high number of load cycles (see following section). This clearly supports the assumption of a constant fatigue limit for concrete as it is true e.g. for steel. With a glance at the Wöhler lines in fig. 3, which show similar slopes for the different parameters, it can be stated, that the frequency and the curing conditions have no significant effect on the fatigue behaviour of concrete in tension. In contrast to this observation the effect of the concrete grade is reflected by the Wöhler line being more inclined in the case of high strength concrete, which indicates its higher sensibility to fatigue.

### 4 DEFORMATION CONTROLLED UNIAXIAL TENSILE TESTS

In contrast to the Wöhler tests described in the previous section the deformation controlled uniaxial tensile tests have been performed by a constant increment of the total deformation from cycle to cycle (i.e. \( \Delta \delta = d \delta / dn = \text{const.} \), where \( n = \text{number of load cycles} \), see fig. 4.

When the preset value for the deformation \( \Delta \delta \) in the following cycle was reached the specimen was unloaded until the lower reversal point \( \delta_{\text{min}} \) was attained. The lower reversal point \( \delta_{\text{min}} \) was defined as a function of the lower load level \( F_{\text{min}} = \text{const.} = 0 \, \text{N} \). The deformation increment \( \Delta \delta \) was determined by dividing the critical crack opening (i.e. the crack opening at which no tensile stresses can be transmitted any more across the crack) defined from the monotonic tests by the desired number of load cycles to failure. As maximum crack opening in the monotonic case a constant value of \( w_{\text{cr}} = 160 \, \mu \text{m} \) was chosen from the literature, see [4].

![Fig. 4](attachment://fig4.png)

**Fig. 4**: Typical load-deformation relation (left) and deformation control procedure (right) in the deformation controlled tension tests
The main finding in the experiments is that for an increasing number of load cycles the envelope curves of the $\sigma$-$\delta$ relations differ significantly from the corresponding monotonic curve, see fig. 5, left. Because of lower values of $f_{tn}$ for the fatigue tests with a high number of load cycles these curves are below the curves for the monotonic tests and the tests with a low number of cycles in the first, steeper part of the stress-deformation relation. In the second, shallow part of the softening curve the average curves for the cyclic tests are nearly congruent when a deformation of about 150 $\mu$m is reached. The curve for the monotonic loading is slightly higher at this deformation region and coincides with the curves for the cyclic loading at a deformation of about 350 $\mu$m. This means a decrease of the fracture energy $G_f$ with an increasing total number of load cycles, if this value is defined as the area under the envelope curve. The mentioned observations clearly show that the conventional assumption of a unique envelope curve for the fatigue behaviour of concrete cannot be maintained especially for the case of high number of load cycles, see also [2, 5].

5 COMPARISON OF THE TWO FATIGUE APPROACHES
The following analysis is an attempt to compare the Wöhler tests in their characteristics with the deformation-controlled uniaxial tensile tests as described in the previous sections. Fig. 5, left presents the stress-deformation relations of monotonic and cyclic tests. In the case of cyclic tests only the envelope curves are plotted for the sake of clearness. Additionally, the crack opening at final failure is marked by filled squares and triangles for different upper stresses, i.e. the measured deformation at the upper load in the last hysteretic loop sustained by the specimen in the Wöhler tests. As can be seen from the figure the values of the crack opening at final failure in the Wöhler tests are quite near the corresponding deformation measured in the deformation-controlled monotonic tests independent of the frequency, the concrete grade and the degree of load $S$.

A more detailed glance at the deformations in fig. 5, right shows clearly that a failure criterion on the basis of deformations would be a rough and inaccurate approach with regard to the failure induced by a steady increase of deformations. The characteristic points from the cyclic creep curve in the Wöhler tests are transferred to the deformation controlled curve at the stress level of the Wöhler test (in fig. 5, right marked as dotted line). From this no agreement between the two approaches can be found which may be drawn back mainly to a different loading history prior to the achievement of the considered deformations.

![Fig. 5: Stress-deformation relations of monotonic and cyclic tests and crack openings at final failure in Wöhler tests (left), and accessory detailed consideration (right)](image-url)
The number of load cycles reached at the point B is different for both approaches as well – 210 in the Wöhler tests and nearly 5000 in the case of deformation-controlled fatigue tests: Characteristically the stress level of the Wöhler test is never reached in the case of a deformation controlled tests with higher numbers of load cycles. In contrast to the own observations Subramaniam et al. [6] related both fatigue approaches at the point B on the basis of the well-known Two Parameter Model. Following [6] the marked points A, B and C are exactly on the monotonic stress-deformation curve. However, the relation by Subramaniam et al. is developed for the assumption of a constant toughness of the material, independent on the crack length, which is inaccurate for concrete.

Since a failure criterion on the basis of deformations is not suitable it was investigated to prove the validity of the energy based failure criterion. However the comparison of the consumed energy during the two fatigue tests adduced no correlation between the two approaches. This is mainly due the fact that in the Wöhler tests an over proportional increase of the consumed energy is observed in the last few cycles before final failure whereas in the deformation-controlled tests the maximum of the fracture energy is almost reached in this area of deformations. Furthermore, the absolute values of the energy in the Wöhler tests are merely around 10% of the fracture energy obtained in the deformation-controlled fatigue tests. Additionally comparisons carried out with regard to the energy consumed up to the crack state reached in the Wöhler tests as well as the consideration of the energy within the hysteretic loops produced no further findings, see [2].

In contrast to the observed stiffness in the two approaches is in the same amount. Nevertheless the stiffness is not suitable for a correlation as well since it covers just a very small range of crack opening in the case of Wöhler tests.

6 SUMMARY AND CONCLUSIONS

Comprising we can postulate that the fracture mechanical approach is more suitable to investigate the crack formation and propagation in concrete structures subjected to cycle tensile loads. By means of deformation-controlled tests complete stress-deformation curves can be obtained. Furthermore, the results of these tests show a smaller scatter compared to the conventionally used Wöhler tests, mainly due to the constant deformation increment. However, if it is intended to investigate the life cycle of real concrete structures bearing a fatigue loading the Wöhler tests are more convenient due to a simpler test set-up and the time saving. Moreover, in the most practical cases the failure of concrete structures is “load controlled” as to be observed in ballastless slab tracks.

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