# APPROPRIATE EXTENDED FUNCTIONS FOR X-FEM SIMULATION OF PLASTIC FATIGUE CRACK GROWTH

T. Elguedj[1], A. Gravouil[1] and A. Combescure[1]
[1] LaMCoS, Laboratoire de Mécanique des Contacts et des Solides
UMR 5514, INSA Lyon
Bat. Jean d'Alembert, 18,20 rue des Sciences 69621 Villeurbanne France

ABSTRACT

The eXtended Finite Element Method (X-FEM) was used with success in the past few years for Linear Elastic Fracture Mechanics. In this paper we propose to extend this method to fatigue crack growth analysis in the case of confined plasticity. A new plastic enrichement basis is therefore extracted from HRR non-linear fields and introduced in X-FEM coupled with a Newton like iteration scheme and a radial return method for plastic flow. Comparisons are made for mode I loading with a finite element code and show good agreements.

## 1. INTRODUCTION

This paper presents, in the context of the eXtended Finite Element Method, an elastic-plastic fatigue crack growth without remeshing analysis in homogeneous, isotropic, two-dimensional solids subject to mixed mode loading conditions. This method is based on asymptotic crack-tip fields under elastic-plastic conditions also called HHR fields (see Hutchinson [1] and Rice and Rosengren [2]) and involves an X-FEM formulation with elastic-plastic enrichment functions similar to those proposed in Rao and Rahman [3] (coupled with a meshless method). A Fourrier analysis of these functions shows that they are capable of capturing the HHR singularities. A numerical example is presented to illustrate the proposed method and compare it with finite element results.

## 2. ELASTIC-PLASTIC ENRICHMENT BASIS

### 2.1. Space discretization - Elastic case

In the presented method, we use the eXtended Finite Element Method, first introduced in Black *et al.* [4] and Moës *et al.* [5], in which an enrichment basis is added to the classical finite element basis approximation. This is done using the partition of unity method developped in Babuska and Melenk [6]. The enriched basis shape functions are associated to new degrees of freedom and the displacement field can be written (see Moës *et al.* [5]):

$$U = \sum_{i \in \mathcal{N}} N_i(x) U_i + \sum_{i \in \mathcal{N}_{cut}} N_i(x) H(x) a_i + \sum_{i \in \mathcal{N}_{branch}} \sum_{\alpha} N_i(x) B_\alpha(x) b_{i,\alpha} \tag{1}$$

$\mathcal{N}$is the set of the standard finite element nodes, $\mathcal{N}_{cut}$ the set of nodes which belong to elements completely cut by the crack and $\mathcal{N}_{branch}$ the set of nodes containing a crack front. $N_i$ are the standard finite element shape functions, $H$ is a Heaviside function which value is 1 if $x$ is above the crack surface and -1 if $x$ is under. $[B_\alpha]$ is derived from the LEFM asymptotic displacement field (see Fleming *et al.* [7])

$$[B_\alpha] = \left[ \sqrt{r} sin(\frac{\theta}{2}), \sqrt{r} cos(\frac{\theta}{2}), \sqrt{r} sin(\frac{\theta}{2}) sin(\theta), \sqrt{r} cos(\frac{\theta}{2}) sin(\theta) \right] \tag{2}$$

### 2.2. Elastic-plastic case

In the case of non-linear materials, the asymptotic displacement field is different and the basis presented in Eq.2 is no more valid. For power-law hardening elastic-plastic material, asymptotic displacement fields (also called Hutchinson-Rice-Rosengren singularity fields) can be calculated (see Hutchinson [1] and Rice and Rosengren [2] for mode I conditions and Pan [8] and Pan and Shih [9] for mode II, III and combined conditions). The uniaxial stress-strain relation is given by the Ramberg-Osgood law:

$$\frac{\varepsilon}{\varepsilon_0} = \frac{\sigma}{\sigma_0} + \alpha \left(\frac{\sigma}{\sigma_0}\right)^n \tag{3}$$

with $\sigma_0$ the reference stress, $\varepsilon_0 = \sigma_0/E$ the reference strain, $E$ Young's modulus, $\alpha$ a material constant and $n$ the hardening exponent. The displacement is proportional to a power of $r$ but depends on the hardening exponent $n$ for pure in-plane loading condition

$$u_i \sim r^{\frac{1}{n+1}} \tilde{u}_i(\theta, n) \tag{4}$$

After the calculation of the $\tilde{u}_i(\theta, n)$ functions for various $n$ a Fourrier analysis is performed. In order to verify the hypothesis of the Fourrier analysis, the *tilde* functions are extended from the interval $[0; \pi]$ to the interval $[0; 4\pi]$, by preserving symmetry and anti-symmetry of the elastic asymptotic field, and the variable is taken to be $\theta/2$ instead of $\theta$. It appears that using the eight first non-zero harmonics ( $cos(k\theta/2)$ and $sin(k\theta/2)$ for k in $\{1, 3, 5, 7\}$) is sufficient to describe the HRR fields as shown in Figure 1. The elastic enriched basis is then replaced by the one presented in Eq.5, in which the Fourrier terms have been combined in order to have only one function which is discontinuous between $\theta = \pi$ and $\theta = -\pi$.

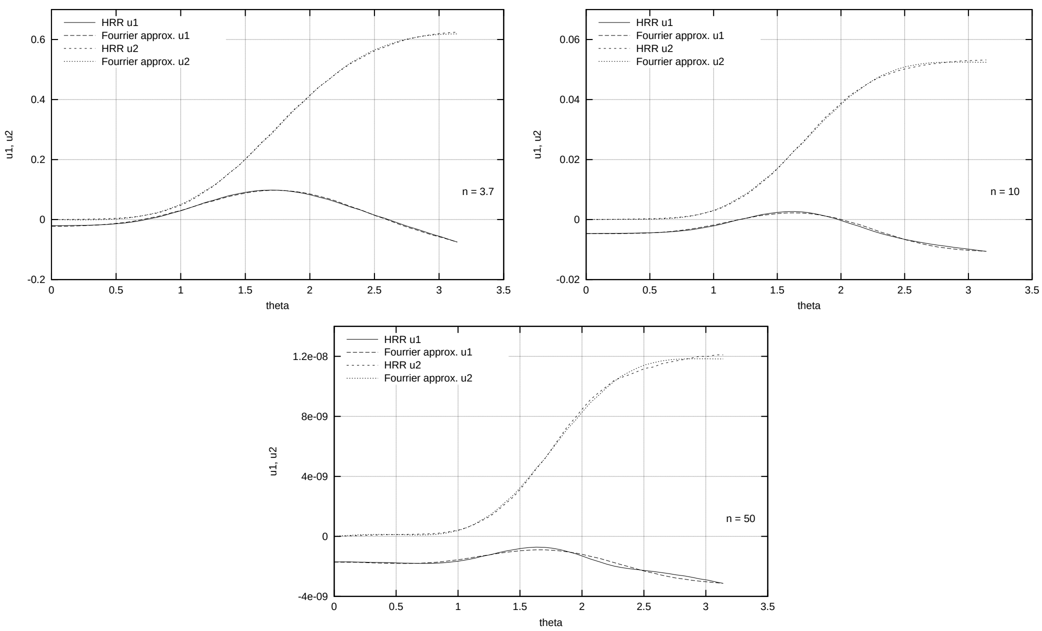


Figure 1. Approximation of HRR displacement field for mode I plane strain

$$[B_\alpha] = \left[ r^{\frac{1}{n+1}} sin\left(\frac{\theta}{2}\right), r^{\frac{1}{n+1}} cos\left(\frac{\theta}{2}\right), r^{\frac{1}{n+1}} sin\left(\frac{\theta}{2}\right) sin(\theta), \right.$$
$$\left. r^{\frac{1}{n+1}} cos\left(\frac{\theta}{2}\right) sin(\theta), r^{\frac{1}{n+1}} sin\left(\frac{\theta}{2}\right) sin(3\theta), r^{\frac{1}{n+1}} cos\left(\frac{\theta}{2}\right) sin(3\theta) \right] \tag{5}$$

### 2.3. Implementation in X-FEM

The elastic tip enrichment is replaced by the plastic tip enrichment given in Eq. 5. A Newton iterative procedure is used to compute equilibrium equation and a radial return scheme for the plastic flow. Gauss quadrature points are used to compute plastic flow. For elements cut by the crack high order terms in Eq5 require a new integration scheme with high number of Gauss quadrature points quadrangles.

## 3. NUMERICAL EXAMPLE

A mode I SE(T) specimen was chosen, with $n = 3.7$. Comparison is made between a standard finite element code with a mesh composed of 892 six nodes triangle elements and a 380 four nodes quadrangles in X-FEM with discontinuous enrichment and a four terms plastic enriched basis for crack tip. Results are presented on Figure 2 for the Crack Opening Displacement and show good agreements between FE and X-FEM computations.

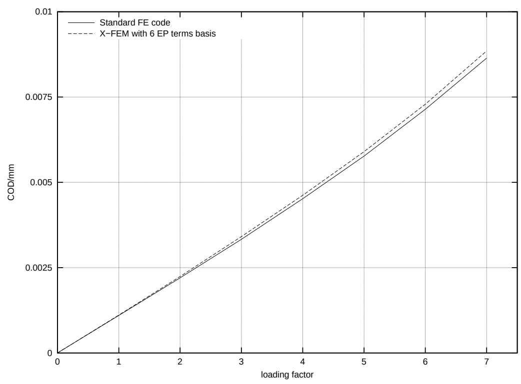


Figure 2. Comparison of the COD

## 4. CONCLUSION

An X-FEM elastic-plastic fatigue crack growth without remeshing analysis in homogeneous, isotropic, two-dimensional solids is presented. The method involves a new enriched basis function to capture the Hutchinson-Rice-Resengren singularity fields in elastic-plastic fracture mechanics. A numerical example is presented to illustrate the proposed method. Fracture parameters, such as crack tip opening displacement, evaluated by the proposed method is in good agreement with elastic-plastic finite element results.

## REFERENCES

1. Hutchinson JW. Singular behavior at the end of a tensile crack in a hardening material. *Journal of the Mechanics and Physics of Solids* 1968; **16**:13–31.
2. Rice JR, Rosengren GF. Plane strain deformation near a crack tip in a power-law hardening material *Journal of the Mechanics and Physics of Solids* 1968; **16**:1–12.

3. Rao BN, Rahman S. An enriched meshless method for non-linear fracture mechanics *International Journal for Numerical Methods in Engineering* 2004; **59**:197–223.
4. Black T, Belytschko T. Elastic crack growth in finite elements with minimal remeshing *International Journal for Numerical Methods in Engineering* 1999; **45**:601–620.
5. Moës N, Dolbow J, Belytschko T. A finite element method for crack growth without remeshing *International Journal for Numerical Methods in Engineering* 1999; **46**(1):133–150.
6. Babuska I, Melenk JM. The Partition of unity method *International Journal for Numerical Methods in Engineering* 1997; **40**:727–758.
7. Fleming M, Chu YU, Moran B, Belytschko T. Enriched Element-free Galerkin methods for crack tip fields. *International Journal for Numerical Methods in Engineering* 1997; **40**:1483–1504.
8. Pan J. Asymptotic analysis of a crack in a power-law material under combined in-plane and out-of-plane shear loading conditions *Journal of the Mechanics and Physics of Solids* 1990; **38**(2):133–159.
9. Pan J, Shih CF. Elastic-plastic analysis of combined mode I, II and III crach-tip fields under small-scale yielding conditions *Internationl Journal of Solids and Structures* 1992; **29**(22):2795–2814.