MATHEMATICAL MODELLING OF DYNAMIC PROCESSES OF IRREVERSIBLE DEFORMING, MICRO- AND MACROFRACTURE OF DAMAGEABLE SOLIDS AND STRUCTURES

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ABSTRACT

Thermomechanical processes, which proceed in deformable solids under intensive dynamic loading, consist of mechanical, thermal and structural ones, which correlate themselves. The structural processes involve the formation, motion and interaction of defects in metallic crystals, phase transitions, the breaking of bonds between molecules in polymers, the accumulation of microstructural damages (pores, cracks), etc. Irreversible deformations, zones of adiabatic shear and microfractures are caused by these processes. Dynamic fracture is a complicated multistage process including an appearance, evolution and confluence of microdefects and a formation of embryonic microcracks, pores their grow up to the break-up of a bodies with division into separate parts.

The present paper include new results in the next scopes:

1) development the thermodynamically correct mathematical models of damageable thermoelastoviscoplastic medium (microfracture);
2) development the methods for determination of “nonstandart” constants of medium models, connected with microfracture of material;
3) numerical simulation of destruction (fragmentation) of constructions (macrofracture);
4) numerical investigation of some problems for damageable solids and structures (dynamical deforming
and fracture of thick-walled cylindrical and spherical shells under explosion; dynamical deforming and fracture of thick-walled two-layer shell, filled with liquid, under impact and high velocity penetration).

1 INTRODUCTION

It is common practice to recognize the following three basic types of dynamic fracture: viscous, brittle, and with formation of zones of adiabatic shear. The viscous fracture (observed in metals like aluminum and copper as well as in solid fuels and explosive) is characterized by formation and evolution of near-spherical pores during a process of plastic deformation. It is typical for the brittle fracture of a body that arbitrary oriented coin-shaped microcracks (capable to grow during deformation) are formed in great number of steels. If the rate of deformation is high, then process of plastic flow is adiabatic. In a number cases, the liberated heat is concentrated in thin domains whose thickness ranges up to several tens of microns. These domains are located along the surfaces of maximal tangential stresses; this leads to a considerable increase of characteristics of the plastic flow along these surfaces. In particular, such fractures with formation of zones of deformation shear are observed in still cylinders loaded by an explosion and in cases of punching the barriers by percussion mechanisms with flat front acts (forcing a “plus” out of a barrier).

Since a great number of the above-mentioned are formed in the process of dynamical deformation, it is difficult to consider each of such microfracture individually. In this connection, in recent years some approaches have been developed, whereby certain internal variables characterizing the evolution of microdamages are introduced into determining equations. This line of investigation is called mechanics of continual (or scattered) fracture.

The development of mechanics of continual fracture was originated in the papers (Kachanov [1]; Rabotnov [2]), dealing with theory of creeping of materials, where one scalar damage parameter was introduced. A short time later, tenser measures of damage were proposed (Il'yushin [3]). Attempts to introduce these tensors are undertaken up to now (Astaf'ev, etc. [4]).

The introduction of the damage parameters into the system of internal variables and the usage of thermodynamic principles of continuum mechanics make possible the construction of thermodynamically correct coupled models of damageable solids (Coleman, etc. [5]; Kiselev, etc. 6—10, 12, 16, 17]; Kiselev [11]).

2 MODELS OF MECHANICS OF CONTINUAL FRACTURE

In particular, was constructed the thermoelastoviscoplastic model with two damage parameters, permitting to description microfracture in domains of intensive tension and by formation of zones of adiabatic shear [10, 11]. Since we describe the viscous fracture with formation of spherical micropores and the fractures with formation of shear zones, we choose in the capacity of the damage parameters the following two invariants of the tensor w_{ij}: the scalars $w = w_{kk}/3$ (the volume damage) and $a = \sqrt{w'_{ij}w'_{ij}}$ (intensity of the damage tensor deviator $w'_{ij} = w_{ij} - wd_{ij}$). We shall assume that in domains of intensive tension the parameter w describes the accumulation of micropores type damages (which make disappear under compression) and the parameter a describes the shear fracture. As is done is classical theories (Kachanov [1]; Rabotnov [2]), we shall interpret the parameter w as relative decrease of the effective load-bearing elementary area due to formation of micropores inside the specimen. The parameter w may be considered as a volume content of micropores in the material. In the damage-free material we have $w = a = 0$; if damages are accumulated, then w and a increase in such a manner that they remain less than 1.

The system of constitutive equation for a model of damageable thermoelastoviscoplastic medium is as follows:

$$e_{kk} = \frac{s}{K} + a_v\,(T - T_0) + \Lambda\int_0^w \frac{\partial j}{\partial s}dw\,,\; e_{ij}^{e} = \frac{S_{ij}}{2m} + A\int_0^a \frac{\partial j}{\partial s_{ij}}\,, \tag{1}$$

$$e_{ij}^{p} = \frac{S_{ij}}{2h}\,\frac{S_? - \sqrt{\frac{2}{3}}\,Y}{S_?}\,H(S_? - \sqrt{\frac{2}{3}}\,Y)\,,\; \dot{w} = j\,(w\,,s\,) = B\left(\frac{s}{1-w} - s_*\right)H\left(\frac{s}{1-w} - s_*\right) +$$

$$+\,w\,\frac{s - s^+}{4h_0}\,H(s - s^+) + w\,\frac{s - s^-}{4h_0}\,H(s^- - s)\,,$$

$$s^+ = -\frac{2}{3}Y_0\ln w\,,\, s^- = -s^+\,,\; \dot{a} = y\,(w\,,a\,,S_?) = C\left(\frac{S_?}{(1-w)(1-a)} - S_?^*\right)H\left(\frac{S_?}{(1-w)(1-a)} - S_?^*\right),$$

$$h = h_0\,(1-w)(1-a)\,,\;\; Y = Y_0\,(1-w)(1-a)\,,\;\; r\;?_s\,\dot{T} + a_v\,s\,\dot{}\,T = S_{ij}\,\dot{e}_{ij}^{p} + \Lambda\dot{w}^2 + A\dot{a}^2 - div\,\vec{q}\,,$$

$$\vec{q} = -k\;grad\,T\,,\;\; S_? = \sqrt{S_{ij}S_{ij}}\,.$$

Here are $s_{ij}, e_{ij}^{e}, e_{ij}^{p}$ — the components of the stress tensor, elastic and no elastic (viscoplastic) deformation tensors, respectively ($e_{ij} = e_{ij}^{e} + e_{ij}^{p}$; $e_{kk}^{p} = 0$); T is the absolute temperature; $\vec{q}$ — is a heat flux; r is density; A, D, C, Λ, s_*, $S_?^*$ — constants of materials, connected with damage parameters w and a; K_0, m_0, h_0, Y_0 — volume module, shear module, dynamic viscosity and static yield of elasticity for an undamaged material; $?_s$ is the heat conductivity at constant stress; a_v is the coefficient of cubic expansion; k is the coefficient of heat conduction; $H(x)$ is Haviside function; the dot over symbols indicates the material derivative with respect to tine.

The kinetic equation for the volume damage w consists of three terms. The first one has the form of the Tuler – Bucher equation and describes the stage of formation and initial growth of the volume damage w . Then, as w is accumulated, the second term describing the viscous growth in domains of tension of the material comes into play (Kiselev, etc.[12]). The third term describes the viscoplastic flowing in pores when the material is compressed.

Note that the equation for w taken without the dynamical problem on a single spherical pore of inner radius a and other radius b in a viscoplastic incompressible material.

We assume that the yield limit Y and shear modulus m depend on temperature, pressure, density, accumulated plastic strain, as in model of Steinberg - Guinan (Wilkins [13]):

$$Y_0 = Y_{00}\,(1 + b\, e_?^p)(1 - b s\, (\frac{r_0}{r})^{1/3} - h(T - T_0)), \quad Y_{00}\,(1 + b\, e_?^p) \le Y_{\max}, \quad Y_{00} = 0 \tag{2}$$

for $T > T_m$, $T_m = T_{m0}\,(\frac{r_0}{r})^{2/3} \exp(2g_0\,(1 - \frac{r_0}{r}))$, $m_0 = m_{00}\,(1 - b\, s\, (\frac{r_0}{r})^{1/3} - h(T - T_0))$,

where $e_?^p = \sqrt{\frac{2}{3} e_{ij}^p e_{ij}^p}$ is intensity of tensor of plastic deformation; T_m is temperature of melting of material; Y_{00}, m_{00}, T_{m0}, b, h, b, g_0 are materials constants. It is accepted that

$$s_* = s_*^0 \frac{Y_0}{Y_{00}}, \quad h_0 = h_{00} \frac{m_0}{m_{00}}, \quad S_?^* = S_{?0}^* \frac{Y_0}{Y_{00}} \tag{3}$$

In formulas (2) - (3) we denote by two zeros the parameters of the undeformed material: $r = r_0$, $s = 0$, $T = T_0$.

This model develops the model for elastoviscoplastic medium Socolovsky – Malvern type and takes into account the formation and accumulation of damages in domains of intensive tensor, their disappearance under compression as well as the heating effects and the accumulation of damages in domains of intensive tension, their disappearance under compression as well as the heating effects and the accumulation of damages under shear. The mechanical, structural, and heat processing are mutually dependent.

The evolution of the intensive plastic flow and accumulation of microstructural damages may be considered as a process of prefracture of the material. The entropy criterion of limiting specific dissipation (Kiselev, etc. [6]):

$$D = \int_0^{t_*} \frac{1}{r}(d_M + d_F + d_T)dt = D_*; \qquad d_F = \Lambda \dot{w}^2 + A\dot{a}^2, \quad d_T = k\,\frac{(gradT)^2}{T}, \tag{4}$$

is proposed as the criterion of the beginning of macrofracture (i.e., the beginning of formation of cracks (new free surfaces) in material). Here t_* is the time of the beginning of fracture; D_* is a constant of the material (the limiting specific dissipation); d_M, d_F and d_T are mechanical dissipation, dissipation of continuum fracture and thermal dissipation.

If zones of large stresses appear in the body (as, for example, in the problem on plane collision of plates with spallation fracture (Kanel, [14]), then the major contribution to dissipation (4) is made by d_M and by term $\Lambda \dot{w}^2$ from d_F . As for the developed shearing plastic flow with formation of zones of adiabatic shear, the major contribution to dissipation D is made by d_M , d_T and by the term $A\dot{a}^2$ from d_F (like in the problem of forcing a "plug" out of a barrier by a percussion mechanism with a flat front cut) (Fomin, etc. [15]).

When criterion (4) is fulfilled at some point of material, a microcrack should be formed there, i.e., a new free surface that will spread over the body. Thus, the problem on calculation of the further deformation becomes self-depend in the framework of computational mechanics of deformable solids.

3 THE METHODS FOR DETERMINATION MATERIAL CONSTANTS

Models for damageable media contain some "nonstandard" constants, connected with damage parameters and subjected to determining. As, the models with single damage parameter (Kiselev, etc. [6, 7]) contain tree such constants. In addition, these models contain fourth unknown constant – a constant of limiting specific dissipation. For determining these constants we use a method, based on comparison the results of physical and numerical experiments of the problem of flat collision of two plates with spallation destruction in a plate-target (Kiselev, etc. [6, 7]). Note, that experiments with spallation destruction are today the most informative and detailed for constructing dynamic constitutive equations for materials under high parameters (Kanel, etc. [14]). For model of porous medium [7] intended for describing behavior of solid fuels and explosive, we used the problem of compression of a spherical microscopical pore filled of gas medium (Kiselev, etc. [12]). The model with two damage parameters (see (1), (4)) contained seven such constants: A, B, C, Λ , s_* , $S^*_?$, D_* . For determining these constants under quasidynamical deforming the modelling the method, based on numerical and physical modelling of processes of quasidynamical twisting and tension of thin-walled tubular samples with destruction and with following mathematical data handing, was proposed (Kiselev, etc. [9]). However, we don't know published results of experiments, which make possible to determine constants for real materials.

For dynamical loading of materials with destruction there are many experiments of flat collision of the plates with spallation destruction (Kanel, etc. [14]), which make possible to determine the constants of damageable media (1), (4) (Kiselev, etc. [16]).

4 DYNAMICAL DEFORMING AND FRACTURE OF THICK-WALLED CYLINDRICAL AND SPHERICAL SHELLS UNDER EXPLOSION

For the first time were carry out numerical investigation of problem of irreversible dynamic deforming and fracture of thick-walled spherical and cylindrical shells as taking account of microfracture with formation and development of defects micropores type and zones of adiabatic shear, as macrofracture up to full destruction of constructions. Determined main regularities of dynamical irreversible deforming and fracture of shells in wide range of loading which bring as to spallation and shear fracture as total destruction of shells.

5 DYNAMICAL DEFORMING AND FRACTURE OF THICK-WALLED TWO-LAYER SPHERICAL SHELL, FILLED WITH LIQUID, UNDER IMPACT AND HIGH VELOCITY PENETRATION

We consider two-layers container filled by water. The outer layer is of thermoprotecting composition material, which is modeled by Maxwell-type thermoviscoelastic medium. Second,

thinner layer is aluminum. Irreversible deformation and microfracture dynamics of aluminum layer is modeled as dynamics of damageable type thermoelasticviscoplastic medium (eqns (1) – (4)).
Liquid, which filled the container, is described by wide-range equation of state by Kuznetsov. This equation of state is extended into low pressure range by special approximation based on experimental data (Kiselev, [18]).
We consider irreversible dynamic deformation and destruction of filled container numerically in two dimensional axially symmetric geometry. This task is solved by numerical modeling on Lagrangian mesh by method similar to Wilkins one (Wilkins, [13]; Fomin, etc. [15]).
Situation of container impact on rigid wall with initial velocity $V_0 = 100 m/s$ at time $t = 54.2 ms$ present on figure 1.
Situation of high velocity penetration of container by steel ball ($V_0 = 5 km/s$) present on figure 2. Irreversible deformation and microfracture dynamics of steel ball is modeled as dynamics of damageable type thermoelasticviscoplastic medium (eqns (1) – (4)). Pressures in the water reach about 1500 bar and temperatures are about $2000\ C$. Container is disrupted.

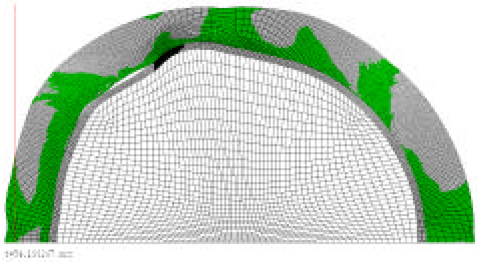

Figure 1: Impact of container on rigid wall.

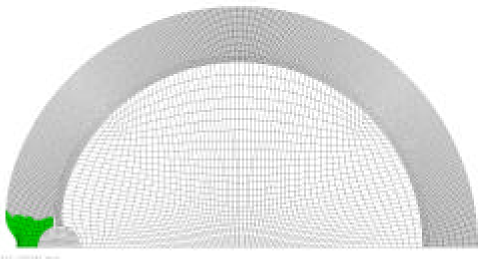

Figure 2: High velocity penetration of container.

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