A PROBABILISTIC MODEL FOR THE FORMATION AND PROPAGATION OF CRACK NETWORKS IN HIGH CYCLE FATIGUE

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ABSTRACT
Thermal striping is observed in various situations. A probabilistic method based on an initial distribution of crack initiation sites is proposed to describe different aspects of the formation and propagation of crack networks in thermomechanical fatigue. The interaction between cracks is modeled by considering shielding effects created by each activated crack. The model can be used to predict the crack density under two different hypotheses.

1 INTRODUCTION

Various components in nuclear power plants are subjected to thermomechanical loadings during their lifetime. Thermal striping networks are observed in some areas of the residual heat removal system (RHR) of nuclear power plants. They consist of a dense and multidirectional crack network on the inner surface of pipes (Fig. 1) [1]. This is not troublesome as long as a fracture analysis indicates that the cracks will not grow notably during the remaining service life. Consequently, the evaluation of crack initiation, their subsequent propagation and coalescence in structures subjected to thermomechanical loadings is very important to determine investigation periods and maintenance programs. The inception of thermal cracking is due to a biaxial stress state: namely, axial or hoop cyclic thermal stresses and mean tensile stresses. It is proposed to analyze crack nucleation and growth by using a unified probabilistic framework.

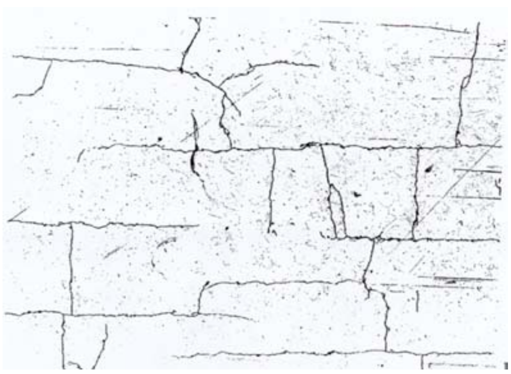


Figure 1: Striping network on the inner surface of a RHR system.

For a crack network, let us distinguish three scales [2]. First the microscale, which is related to the stage I of the fatigue process, depends upon the details of the microstructure. The cracks are considered as microstructurally short (they will be referred to as microcracks). This scale ends when mesocracks are initiated. The second scale corresponds to the propagation of mesocracks (i.e., stage II fatigue) that form the network. The cracks are considered as physically small. Last, the third scale is concerned with coalesced mesocracks that form a macrocrack (i.e., a long crack). Since stage II fatigue is concerned, the cracking directions are assumed to be aligned along the principal stress directions $\mathbf{d}_i$ (here constant during the whole load history). Each direction will be considered independently. The stress σ will denote any of the principal stresses $\sigma_1 \geq \sigma_2 \geq \sigma_3$.

Two hypotheses can be considered for the crack network formation. The first one assumes that all mesocracks initiate on existing defects during the first quarter of the first cycle of loading [3]. In this case, crack initiation is a function of stress level and not the number of cycles. The initial distribution of defects is defined by a Poisson point process. The material microstructure is therefore approximated by point defects, potential crack propagation sites, with *random* locations. The second hypothesis consists of continuous initiation of mesocracks during loading. Mesocrack initiation is defined as a function of cyclic stress amplitude and number of cycles. The microstructure is modeled in terms of sites where crack initiation can occur after some incubation time. The sites are also described by a Poisson point process.

2. INSTANTANEOUS INITIATION

Let us assume that fatigue cracks initiate on initial flaws considered as mesocracks of size *a*. Their propagation is driven by the stress intensity factors (SIF) $K = Y\sigma\sqrt{a}$ where Y is a dimensionless geometry parameter. The propagation occurs when $K_{\max} g(R)$ is greater than the threshold stress intensity factor K_{th}, where K_{max} is the maximum value of stress intensity factor during the loading cycle, R the load ratio and g a function describing the effect of the load ratio on mesocrack opening. By considering the above-mentioned definition of the stress intensity factor, the initiation stress σ_{th} of each mesocrack can be related to its initial size a by

$$\sigma_{th} = \frac{K_{th}}{Yg(R)\sqrt{a}} \tag{1}$$

The active sites where the propagation condition is satisfied (i.e., $\sigma > \sigma_{th}$) are assumed to follow a Poisson point process. The probability of finding $N_s = n$ active sites within a zone Ω (i.e., volume, surface or length) of size Z is described by a Poisson distribution

$$P(N_s = n, \Omega) = \frac{[\Lambda(\Omega)]^n}{n!} \exp[-\Lambda(\Omega)] \tag{2}$$

where Λ(Ω) is the average number of active sites in Ω, which is related to the intensity λ_t of the Poisson point process by

$$\Lambda(\Omega) = Z\lambda_t(\sigma) \tag{3}$$

that can be modeled by a power law of the applied stress

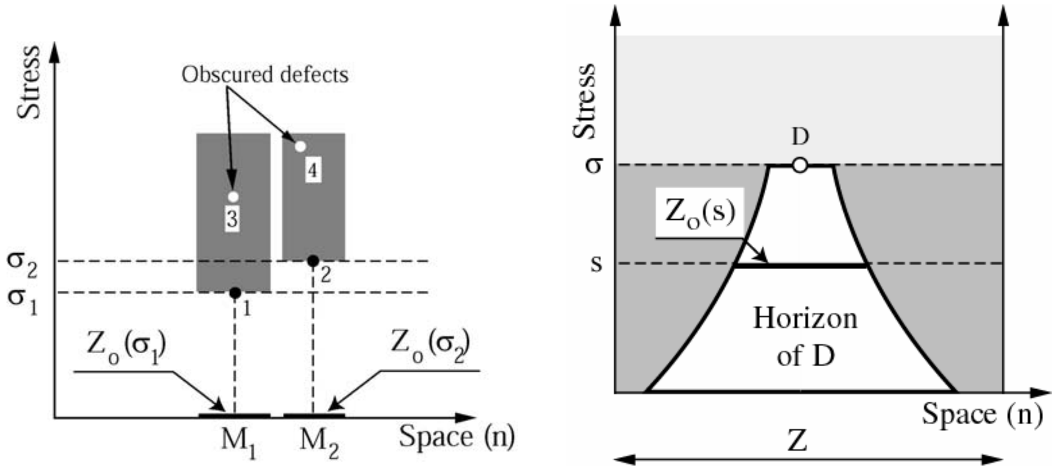
$$\lambda_t(\sigma) = \lambda_0 \left(\frac{\langle \sigma \rangle}{\sigma_0} \right)^m \tag{4}$$

where m and σ_0^m / λ_0 are material-dependent parameters, and $\langle \bullet \rangle$ the Macauley brackets. By making the weakest link assumption, the failure probability P_F is the probability of finding at least one critical crack in a uniformly loaded domain Ω

$$P_F = P(N_s \geq 1, \Omega) = 1 - P(N_s = 0, \Omega) = 1 - \exp\left[-Z\lambda_0 \left(\frac{\langle \sigma_1 \rangle}{\sigma_0} \right)^m \right] \tag{5}$$

Equation (5) corresponds to a two-parameter Weibull model in which m is the Weibull modulus and σ_0^m / λ_0 the scale parameter. In this setting, the material microstructure is modeled by the Weibull parameters. Equation (5) gives the scatter of the fatigue limits for a uniformly loaded volume.

In the following, the weakest link hypothesis is no longer made even though the *same* microstructure description is used (i.e., a Poisson point process is considered). Of all initial defects, potential initiation sites, only a fraction can create mesocracks. When the local stress intensity factor for a defect exceeds the threshold stress intensity factor (e.g., the propagation criterion is satisfied), a mesocrack is initiated in mode I. The presence of a crack modifies the local stress field in its vicinity. Two different zones can be distinguished; namely an amplification zone around the crack tip and a relaxation zone around the crack mouth. In a crack network, the interaction of these zones affected by the stress relaxation and other defects must be modeled. In the present section, all cracks of the network initiate during the first cycle of loading. The density of mesocracks is independent of the number of cycles but the growth of these initiated cracks is a function of the number of cycles via a propagation law and will affect the density of propagating mesocracks. The crack initiation condition can be represented on a space-stress level diagram (Fig. 2a). The spatial position of the microcracks is represented as a simple abscissa (instead of a three-, two- or one-dimensional representation) of an x-y graph where the y-axis represents the stress to initiation of a given site.



-a- -b-

Figure 2: -a- Depiction of two mesocrack initiations and obscuration for two microcracks.
-b- Horizon for a given location D.

A first initiation occurs at M_1 for a stress level equal to σ_1. The initiated mesocrack creates a stress relaxation zone or an "obscured zone" $Z_o(\sigma_1)$. For a higher initiation stress level σ_2, the second mesocrack will be initiated at M_2, which is far from the obscuration zone of mesocrack M_1. The second initiated mesocrack creates its own obscured zone $Z_o(\sigma_2)$. The third and fourth sites do not create mesocracks because they are obscured by the first and second mesocracks.

The space-stress level diagram is composed of the union of the obscured zones where no crack can be initiated and their complementary zones where any active site can initiate a mesocrack. Because different obscured zones may overlap (i.e., a flaw can be obscured by one or more other mesocracks), it is preferable to define the conditions of non-obscuration for a given defect by examining the *inverse* problem. It consists in considering the past history of a defect that would nucleate a mesocrack at a stress σ. The defect will create a mesocrack if no mesocrack exists in its *horizon*. For a given defect D, its horizon is defined as a space-stress zone in which a mesocrack will always shield D (Fig. 2b). Outside the horizon, a mesocrack will never obscure D. The initial distribution of defects can be separated into two parts and the average density of initiated mesocracks λ_m reads

$$\lambda_m(\sigma) = \lambda_t(\sigma) - \lambda_{obs}(\sigma) \tag{6}$$

where λ_{obs} denotes the density of obscured defects. The product $\lambda_m Z$ is the average number of initiated mesocracks in a zone of measure Z for a principal stress less than or equal to σ. Two conditions must be satisfied to initiate new mesocracks. A site must exist in the studied zone and it must not be obscured by any other existing mesocrack. The increment of initiated mesocrack density is related to that of λ_t by

$$\frac{d\lambda_m}{d\sigma}(\sigma) = \frac{d\lambda_t}{d\sigma}(\sigma)\left[1 - P_{obs}(\sigma)\right] \text{ with } \lambda_m(0) = 0 \tag{7}$$

where $1 - P_{obs}$ denotes the probability that no cracks exist in the horizon (i.e., non-obscuration probability). The variable $1 - P_{obs}$ can be split into an infinity of local events defined by the probability of finding at stress level s a new mesocrack during a stress increment ds in a zone $Z_{obs}(s)$. This probability increment is written by considering the intensity increment $d\lambda_t / ds$. These *independent events* can be used to provide an expression for P_{obs}

$$P_{obs}(\sigma) = 1 - \exp\left[-\int_0^\sigma \frac{d\lambda_t}{ds}(s) Z_{obs}(s) ds\right] \tag{8}$$

At the beginning of loading, there are not many interactions between the mesocracks and the new active site so that $\lambda_m(\sigma) \approx \lambda_t(\sigma)$ and as more and more mesocracks initiate, $\lambda_m(\sigma) \ll \lambda_t(\sigma)$. It is expected that the crack density saturates when $\sigma \to +\infty$ even though the total flaw density may approach infinity. The obscuration zone is the key parameter to model crack networks. This zone is proportional to the crack size *a*. As a first estimation, the obscured zone can be written as

$$Z_{obs}(s) = S\left[ka(s)\right]^n \tag{9}$$

where the constant k is independent of the Weibull modulus m [3], S a shape factor and a is proportional to s^{-2} [Eqn. (1)]. When $n = 1$, $S = 2$ and the value of k is equal to 1.334 , when $n = 2$, $S = \pi$ and the value of k is equal to 1.428 and when $n = 3$, $S = 4\pi/3$. In a linear elastic analysis, the shape of the obscured zone does not change during one loading cycle. In the framework of the present hypothesis (i.e., instantaneous nucleation), the crack network formation is independent of the number of cycles. As the applied number of cycles increases, the initiated mesocracks grow. The propagation of cracks is described by the change of the horizon

$$Z_{obs}(s,N) = S[ka(s,N)]^n \tag{10}$$

where N denotes the number of cycles. The propagation of mesocracks and their interaction can be represented in the same manner as their formation in a space-stress level diagram. Each mesocrack continues to propagate if it does not belong to any obscuration zone. The obscuration probability changes not only with the stress level s but also with the number of cycles N

$$P_{obs}(\sigma,N) = 1-\exp\left[-\int_0^{\sigma}\frac{d\lambda_t}{ds}(s)Z_{obs}(s,N)ds\right] \tag{11}$$

The density of mesocracks follows a modified differential equation

$$\frac{d\lambda_m}{d\sigma}(\sigma,N) = \frac{d\lambda_t}{d\sigma}(\sigma)\left[1-P_{obs}(\sigma,N)\right] \tag{12}$$

By using the crack growth law of mesocracks, the set of equations (10), (11) and (12) shows that the obscuration probability increases when number of cycles increases. Consequently, the density of active cracks decreases. This density corresponds to the predominant cracks in the network that shield more and more mesocracks.

3. CONTINUOUS INITIATION

The microstructure is modeled in terms of sites where microcracks can be created. The sites are approximated by points of *density* λ_t. A Poisson point process of intensity λ_t is considered again. A power law function is assumed and leads to a Poisson-Weibull model [4-6]

$$\lambda_t(\Delta\sigma) = \lambda_0\left(\frac{\Delta\sigma}{\sigma_0}\right)^m \tag{13}$$

where $\Delta\sigma$ the stress amplitude. Among all these microcracks, there is only a fraction for which the mesoscopic initiation condition is satisfied. Let λ_{tI} denote the corresponding density that depends upon the stress amplitude $\Delta\sigma$ and the number of cycles N. For instance, a threshold $\Delta\sigma_u(N)$ accounting for *continuous* mesoscopic initiation can be considered

$$\lambda_{It}(\Delta\sigma,N) = \lambda_t\left(\langle\Delta\sigma-\Delta\sigma_u(N)\rangle\right) \tag{14}$$

Equation (14) shows that the initiation process needs a minimum number of cycles N_{min} (i.e., such that $\Delta\sigma - \Delta\sigma_u(N_{min}) = 0$) to create the first mesocrack. In Eqn. (14), the principal variable is N and $\Delta\sigma$ appears as a (constant) parameter. An obscuration domain of measure Z_{obs} around cracks has to be defined again. Following the results of Section 2, the obscuration domain of measure Z_{obs} is assumed to be proportional to the size a of propagating mesocracks

$$Z_{obs}(\Delta\sigma,N-N_I) = S[ka(\Delta\sigma,N-N_I)]^n \tag{15}$$

where N_I the number of cycles to mesoscopic initiation. It can be noted that the initial mesocrack size $a(\Delta\sigma,0)$ is different from zero and depends upon microstructural parameters [2]. By using this set of hypotheses, microcracks do not obscure each other (i.e., it is an intragranular process) and mesocracks obscure microcracks, thereby partly inhibiting mesocrack initiation. The increment of density of mesocracks, λ_m, can be related to that of λ_{tI} by

$$\frac{d\lambda_m}{dN}(\Delta\sigma, N) = \frac{d\lambda_{tI}}{dN}(\Delta\sigma, N) \times \left[1 - P_{obs}(\Delta\sigma, N)\right] \tag{16}$$

with $\lambda_m(\Delta\sigma, 0) = 0$ and P_{obs} the probability of obscuration [7]

$$P_{obs} = 1 - \exp\left[- \int_{N_{\min}}^{N} Z_0(\Delta\sigma, N - N_{\min}) \frac{d\lambda_{tI}}{dN_I}(\Delta\sigma, N_I) dN_I \right] \tag{17}$$

It can be noted that Eqn. (17) accounts for overlappings of obscuration zones. Furthermore, in the context of mathematical morphology, the above-described approach corresponds to a Boolean islands model [8,9]. Equations (14) to (17) show that by increasing the number of cycles, the probability of obscuration increases. At the same time, the number of initiated mesocracks increases too.

4. SUMMARY

A probabilistic model based upon a random distribution of initiation sites is proposed to describe different aspects of formation and propagation of crack networks in fatigue. The total potential initiation site distribution is modeled by a Poisson point process and the interaction between mesocracks is accounted for by considering obscuration zones. Closed-form solutions are derived to model the crack density as the function of stress amplitude and number of cycles. Instantaneous and continuous initiation was considered and leads to similar results in terms of obscuration probability and change of the mesocrack density.

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