# ROUGHNESS DEVELOPMENT OF MORTAR FRACTURE SURFACES

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#### ABSTRACT

From mode I fracture tests on mortar, we describe the morphology of fracture surfaces using quantitative fractography. Results obtained are in good agreement with previous works on others quasibrittle materials: granite and wood. We have shown the anisotropy of the roughness of such surfaces. It means that fracture surfaces exhibit different behaviour along the crack propagation direction and along the transverse direction. Indeed, roughness of profiles parallel to the initial notch and perpendicular to the crack propagation direction exhibits self-affine behaviour. This self-affinity is characterized by a single exponent called local roughness exponent which seems to be universal, i.e., independent of the material and of the specimen shape. Along the crack propagation direction, the roughness evolves as a function of the distance from the initial notch and this development can be described by an "anomalous" scaling law. This law is characterized by two regimes of roughness growth: one affects length scales smaller than a correlation length, and the other affects larger length scales. These regimes can be described by use of two more exponents called global roughness exponent and dynamic exponent. Hence, the fractal dimension, obtained from the local roughness exponent, is insufficient to characterize the crack surface morphology as a whole.

#### **1 INTRODUCTION**

The fracture of quasibrittle materials such as mortars is characterized by the development of a microcracked fracture process zone ahead of the main crack. The damage development in the fracture process zone leads to particular fracture properties such as Rcurve behaviour and size effect. On the basis of the previous works of López [1] and Morel [2] on other quasibrittle materials, granite and wood, the morphology of fracture surfaces of a mortar was studied using a suitable tool: quantitative fractography. We show that the roughness of such surfaces is anisotropic, i.e. it must be described differently in the crack propagation direction and in the transverse direction.

### 2 EXPERIMENT

In this section, we describe the experimental setup used for fracture tests in mode I mortar specimens. We use four points bending notched beams. The length of the beams is 1400 mm; their height and thickness are 140 mm. The initial notch is performed with a steel sheet (thickness 0.4 mm) pulled out when the mortar is 24 hours old. The notch is 70 mm long. Topographies of the obtained fracture surfaces are recorded using an optical profilometer. Each map (more than  $10^6$  points) is built up with profiles parallel to the initial notch (*x* direction, see Fig. 1). The sampling step along profiles is 20 µm. The first profile is close to the straight notch (approximately zero roughness) and two successive profiles are separated by 50 µm along the direction of crack propagation (*y* direction).



Figure 1: Mortar beam. The outline and the initial notch have been underlined for clarity.



Figure 2: Topography of a mortar fracture surface. The side length of this map is 130 mm.

3 MORPHOLOGY OF THE FRACTURE PROFILES: SELF-AFFINE BEHAVIOUR Many works have been dedicated to the statistical characterization of the fracture surfaces morphology. These studies concern various loading conditions and materials such as metals (Bouchaud [3]; Dauskardt [4]; Imre [5]), ceramics (Mecholsky [6]; Måløy [7]), glass (Daguier [8]), various rocks (Schmittbuhl [9] and [10]), sea ice (Weiss [11]), or wood (Engøy [12]; Morel [2]). They have shown that fracture surfaces exhibit self-affine scaling properties in a large range of length scales (see Bouchaud [13] and Bouchaud [14] for reviews). If the roughness of a profile, at a distance *y* from the notch, is characterized by the root mean square of the height fluctuations  $\Delta h(l,y)$  estimated over a window of size *l* along the *x* axis then  $\Delta h(l,y) \sim l^{\zeta loc}$  (Fig. 3). The scaling exponent  $\zeta_{loc}$  is called the *local* roughness exponent. The correlation length  $\zeta$  corresponds to the size of the self-affine domain. At larger length scales, roughness saturates. Despite the various test conditions, the local roughness exponent appears always close to 0.8. It has been suggested that it could be a universal exponent, i.e., independent of the fracture mode and of the material (Bouchaud [3]).



Figure 3: Roughness  $\Delta h$  as a function of the scale *l* in a log-log plot.

The local roughness exponent  $\zeta_{loc}$  can be estimated using three independent statistical methods: The root mean square variable bandwidth method (Schmittbuhl [9]; Bouchaud [13]), which is described above, the power spectrum method (Schmittbuhl [9]) and the average wavelet coefficient method (Simonsen [15]). All these methods consistently give:  $\zeta_{loc}=0.78$ . This result is in good agreement with previously reported results (Bouchaud [13], [14]) and confirms the universality assumption.

## 4 ROUGHNESS DEVELOPMENT: ANOMALOUS SCALING

Several recent studies have focused on the roughness development of fracture surfaces in quasibrittle materials (López [1]; Morel [2]; Morel [16]). They have shown that roughness evolves as a function of the distance y from the initial notch and that this development can be described by an "anomalous" scaling law (Schroeder [17]; Das Sarma [18]; López [19]; López [20]). This law introduces two more exponents: the global roughness exponent  $\zeta$  which controls the roughness growth at large length scales and the dynamic

exponent z which controls the increase of the self-affine correlation length as a function of the distance y to the initial notch.

$$\Delta h(l, y) = A \begin{cases} l^{\mathcal{G}oc} \boldsymbol{\xi}(y)^{\boldsymbol{\zeta} - \boldsymbol{\mathcal{G}oc}} & \text{if } l \ll \boldsymbol{\xi}(y) = \boldsymbol{B} \cdot y^{1/z} \\ \boldsymbol{\xi}(y)^{\boldsymbol{\zeta}} & \text{if } l \gg \boldsymbol{\xi}(y) \end{cases}$$
(1)

Anomalous scaling was so called as opposed to the most common observed Family-Vicsek scaling (Family [21]), which can be viewed as a particular case for which the global and the local roughness exponents have the same value:  $\zeta = \zeta_{loc}$ . When the self-affine correlation length  $\zeta$  reaches a value close to the profiles length (i.e. the thickness of the specimen), the roughness stops evolving. This saturation occurs as soon as the profiles are located at a distance  $y > y_{sat}$  from the initial notch. From that distance on, the correlation length  $\zeta(y) = \zeta(y_{sat}) = \zeta_{max}$  stays constant, and  $\Delta h$  only depends on *l*:

$$\Delta h(l < \boldsymbol{\xi}(\boldsymbol{y}), \boldsymbol{y} > \boldsymbol{y}_{sat}) \sim l^{\zeta_{loc}}$$
<sup>(2)</sup>

# 5 GLOBAL ROUGHNESS EXPONENT AND DYNAMIC EXPONENT

In figure 4,  $\Delta h(l, y)$  is plotted as a function of the distance y to the initial notch for various length scales *l*. The upper line corresponds to a length scale *l* which is larger than  $\zeta(y)$  for all values of y. According to eqn (1), the slope of that line in a log-log plot is expected to be equal to  $\zeta/z$ . The lower line corresponds to a length scale *l* smaller than  $\zeta(y)$  for all values of y. Each point of this line is in the self-affine domain and the anomalous scaling predicts the slope to be  $(\zeta-\zeta_{loc})/z$ . As  $\zeta_{loc}=0.78$ , the two slopes,  $\zeta/z=0.33$  and  $(\zeta-\zeta_{loc})/z=0.12$ , allow to compute  $\zeta=1.20$  and z=3.6. Table 1 summarizes the values of  $\zeta$  and z found for granite (López [1]) and several wood species (Morel [2]). The error bars are estimated to be 0.10 for the global roughness exponent and 0.20 for the dynamic one.

$$\Delta h(l, y < y_{sat}) \sim \begin{cases} y^{\zeta/z} & \text{if } l > \xi(y) \\ y^{(\zeta - \zeta_{loc})/z} & \text{if } l < \xi(y) \end{cases}$$
(3)

Table 1: Values for the global roughness exponent  $\zeta$  and the dynamic exponent z.

	Maritime pine	Norway spruce	Granite	Mortar
ζ	1.35	1.60	1.20	1.20
Z	2.3	3.0	1.2	3.6

Within the framework proposed by López [20], another estimate of the roughness exponents can be obtained from the determination of the scaling function g(u) defined as:  $g(l/y^{1/z})=\Delta h(l,y)/l^{\zeta}$ . According to the anomalous scaling law, g(u) is expected to scale as:

$$g(u) \sim \begin{cases} u^{-(\zeta - \mathcal{G}oc)} & \text{if } u \ll 1 \\ u^{-\zeta} & \text{if } u \gg 1 \end{cases}$$

$$\tag{4}$$



Figure 4: Representation of the roughness as a function of the distance y to the notch.



Figure 5: Data collapse with  $\zeta = 1.20$  and z = 3.6.

For all the profiles ranging between  $y_{min}$  and  $y_{max}$ , the experimental values of g(u) are plotted using values previously estimated for the roughness and dynamic exponents ( $\zeta$ =1.20 and z=3.6). The good collapse of data proves that these values are reasonable (Fig. 5). Moreover as  $\zeta$  is not equal to  $\zeta_{loc}$ , roughness grows at small length scales so fracture surfaces of mortar clearly exhibit anomalous scaling.

# 6 CONCLUSION

This work confirms that fracture surfaces morphology of quasibrittle materials is anisotropic. Along the direction parallel to the initial notch, the roughness of fracture surfaces exhibits a self-affine behaviour which seems to be universal, whereas the roughness growth along the crack propagation direction is material-dependent. The results reported here on mortar show that the roughness development follows the anomalous scaling law, i.e., that the extension of the scaling domain increases as a function of the distance to the initial notch both at small length scales and at larger ones. Hence, the fractal dimension, obtained from the local roughness exponent  $\zeta_{loc}$ , is insufficient to characterize the crack surface morphology as a whole.

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