MODELISATION OF DELAYED FAILURE OF WOOD BEAMS

M. Chaplain and G Valentin
Laboratoire de Rhéologie du Bois de Bordeaux, 33612 Cestas , France

ABSTRACT

The predictions of the delayed failure of wooden beams are an important problem in timber engineering especially for beams containing stress concentrations (as notches or tapered end-notches or openings). Several theories have been developed to provide duration of load (DOL) of timber structures. The damage theory is probably the most employed one to predict the time to failure of wood elements. However, the fracture mechanics approach developed to provide the crack growth can be performed to obtain the time to failure as well. The viscoelastic crack model (VCM) based on the crack growth in an orthotropic viscoelastic medium is studied in this paper. For finite dimension beams or beams containing singularities (notches, holes, knots...), a correct expression of the stress intensity factor can be introduced in VCM. This expression is depending on the shape of the beam and of the kind of loading and it is also depending on the elastic stiffness properties of the material. It can be obtained by finite element calculations. Simplified model, noted VCMS can be derived from VCM under the assumption that the stress intensity factor is only function of the crack length. Compared to experiments on notched Laminated Veneer Lumber (LVL) beams, VCM and VCMS give fair predictions of the time to failure. Some predictions are also given for beams containing a hole displaying a more complex behavior versus initial crack length. Simulations are also carried out to study the influence of the initial crack length on the delayed failure.

1 INTRODUCTION

The fracture mechanics approaches (Schapery [1], Nielsen [2], Valentin [3, 4]), developed to provide the crack propagation, can be performed to obtain the time to failure The presented model, noted VCM (Viscoelatic Crack Model) (Valentin [3]), is based on the Schapery theory. The crack grows in an orthotropic viscoelastic medium and a damage area is assumed to exist at the crack tip. This fracture mechanics model introduces a creep function (viscoelastic compliance). The establishment of this function is explained in this paper. It is limited to the growth of crack along the grain direction in opening mode (mode 1).The duration of load predicted by the model are compared to experimental results of bending tests on notched Laminated Veneer Lumber beams (LVL, KERTO S made by Finn Forest). Long term loading experiments were creep test (constant load) and step by step loading tests (Boyancé [5], Gustafsson [6]). Simulations are also carried out on beams with a hole for different initial crack length.

2 VISCOELASTIC CRACK MODEL (VCM)

Equations giving the crack velocity in a viscoelastic media were firstly developed by Schapery [1, 7] and its application to wood fracture were made in the LRBB. This theory is based on the Barenblatt's crack approach. In the neighborhood of the crack, the material is divided into two regions (Fig. 1): a process zone (length α) (1) which can be highly damaged and nonlinear viscoelastic and a region surrounding the process zone (2) where the material is considered as linear viscoelastic. The crack length used in the model includes the length of the process zone.

The distribution of the cohesive stress (σ_c) along the failure zone is not necessary uniform (no assumption on the behavior of the material). The requirement of the finite stress (Barenblatt hypothesis) at the crack tip yields to the equation (1) in opening mode.

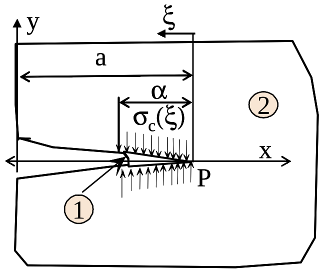

Figure 1 : Barrenblatt crack model

$$K_I = \sqrt{\frac{2}{\pi}} \cdot \int_0^\alpha \frac{\sigma_f(\xi)}{\sqrt{\xi}} d\xi = \sqrt{\frac{2\alpha}{\pi}} \cdot \sigma_m I_1 \text{ with } I_1 = \int_0^\alpha \left[\frac{\sigma_c(\xi)}{\sigma_m} \frac{1}{\sqrt{\alpha.\xi}} \right] d\xi \text{ and } \sigma_m = \underset{0<\xi<\alpha}{\mathrm{Max}}(\sigma_c(x)) \quad (1)$$

For orthotropic viscoelastic medium containing a crack along a material axis (longitudinal direction), a relation between stress intensity factor K_I and the fracture energy G_I has been established:

$$2G_I / K_I^2 = \kappa^v(t^v) \qquad \text{with} \qquad t^v = \frac{\lambda_n^{1/n}\alpha}{\dot{a}} \quad (2)$$

$\kappa^v(t)$ is a reduced creep compliance in mode I along the natural axis in TL configuration and t^v is a reduced time. $\dot{a}$ is the crack velocity.

Using a power law equation (3) for the creep compliance in equation (2), the following crack velocity is obtained (VCM):

$$\kappa^v(t) = A_o + A_2.t^n \quad (3)$$

$$\frac{da}{dt} = \frac{\pi}{2}\left[\frac{A_2\lambda_n}{2G_I\left(1-\frac{K_I^2}{K_{Ic}^2}\right)}\right]^{1/n_2} \frac{K_I^{2(1+1/n)}}{(\sigma_m I_1)^2} \quad (4)$$

with $K_{Ic} = \sqrt{2G_1 / A_o}$ is the critical stress intensity factor, λ_n is depending on n value (Schapery [1]): A_0 , A_2 and n are parameters of the of the reduced creep compliance $\kappa^v(t)$

Equation (4) shows clearly that the velocity becomes unbounded when K_I approaches K_{Ic} from below: so, failure is obtained when $K_I=K_{Ic}$. The equation (4) is numerically solved by a Runge-Kutta method. In this study, the product $\sigma_m I_1$ is assumed to be constant during the crack growth. Moreover, it was experimentally observed that G_1 can be supposed to be independent of the crack length or crack speed.

This general expression of the model can be simplified. Under the assumption that the length of the damage zone α is the same during the crack propagation or assuming that (σ_m/I) is constant, the following relation can be deduced using equations (1) and (2) :

$$\frac{da}{dt} = \dot{a} = A.K^N \tag{5}$$

This simplified relation, frequently used in other materials, has also been established in early experiments on slow crack growth on wood (Mindess [8]).
For a constant load, under the assumption that the stress intensity factor is equal to $K_I = \sigma(t)\sqrt{\pi a}$, the equation (6) can be integrated to obtain the time to failure "t_r" under the constant load σ. For a long term test and as N>2 (N≈20 for the tested wood), the following expression is obtained (VCMS):

$$\frac{t_f}{t_{ref}} = \left(\frac{\sigma}{\sigma_{ref}}\right)^{-N} = \left(\frac{SL}{SL_{ref}}\right)^{-N} \tag{6}$$

t_{ref} represents the time to failure obtained during a reference (long) creep test under σ_{ref}.
SL is the stress level, it is the ratio between the applied stress (σ) and the strength of a given beam (σ_s): SL=σ/σ_s and $SL_{ref}=\sigma_{ref}/\sigma_s$. SL is an adimensional term which takes into account the quality of the beam.

3 EXPRESSION OF THE STRESS INTENSITY FACTOR K_I

In mode I, the usual relationship between the stress intensity factor K_I, the applied stress σ and the crack length a in an isotropic media is the following:

$$K_I = \sigma\sqrt{\pi a} \tag{7}$$

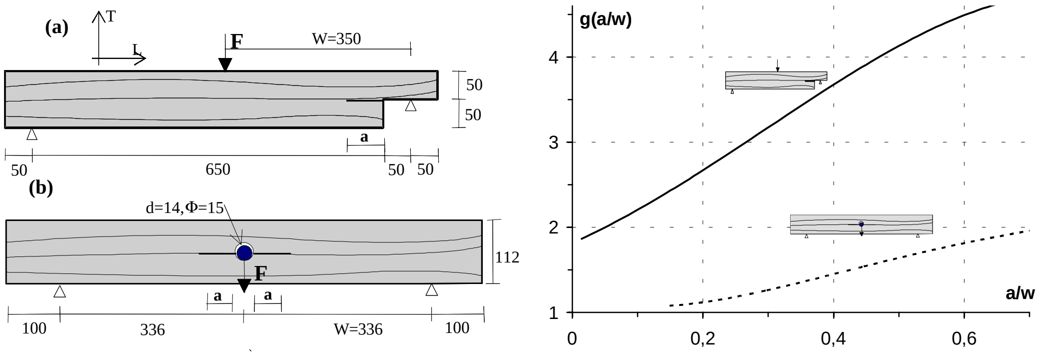

Figure 2 : Specimens geometry (thickness t =45 mm) and calibration function versus the relative crack length a/w

This expression is given for a finite crack (length 2a) in an infinite plate and is used in the fracture mechanics model developed by Nielsen [2] For finite dimension specimens, K_I is also depending on the specimen shape and on the elastic stiffness of the orthotropic material. The stress intensity factor is usually written as:

$$K_I = \frac{F}{B\sqrt{w}} g(a/w) \qquad \text{or} \qquad K_I = SL.\frac{F_s}{B\sqrt{w}} g(a/w) \tag{8}$$

where F is the applied load, F_s is the strength of the specimen, SL is the stress level, B is the thickness and w is a characteristic length of the specimen. The calibration function g(a/w) depends on the type of loading too (traction, bending…). The function g has been computed by finite element method for notched beam and for beam with a hole (Fig. 2 a, b) using the average characteristic obtained on the LVL tested material. For both specimens, because of the orthotropy of wood, the mode of failure is not pure mode I but a mixture of mode I and II. However, in this study, we consider that the failure is only produced by the predominant mode which is mode I.
For the LVL material, the critical stress intensity factor K_{Ic} has been found equal to 0,66 MPa $\sqrt{m}$. As the strength of the elements is depending on the length of the initial crack length a_o, the stress level SL is also depending on a_o. Thus, the evolution of SL versus the critical crack propagation (Δa_c=ac-a_o), for different initial crack length a_o, is illustrated on figure 3. For a given SL, for notched beams, when a_o increases, the crack propagation decreases. For beams with a hole, when a_o increases Δa_c increases up to 100 mm, then decreases. For an infinite plate, when a_o increases, Δa_c always decreases and the crack length a_c increases.

4 MODELISATION

4.1 Expression of reduced viscoelastic compliance $\kappa^v(t)$

The reduced creep compliance κ^v is determinate using the visco elastic orthotropic visoelastic creep compliance moduli S_{ij} of the material. (here 1 is the longitudinal direction and 2 the tangential direction).
Using the quasi-elastic method, the expression of κ^v becomes (in TL configuration):

$$\kappa^v = 2.\left[\sqrt{\frac{S_{22}}{S_{11}}} + \frac{2.S_{12} + S_{66}}{2.S_{11}}\right]^{1/2} \left[\frac{S_{11}.S_{22}}{2}\right]^{1/2} \tag{9}$$

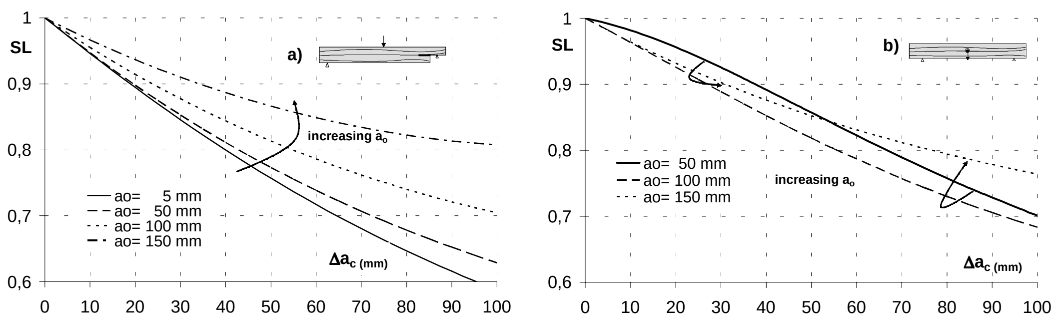

Figure 3 : SL versus the critical crack propagation Δac for several initial crack length a_o.
a) notched beam b) beam with a hole

The viscoelastic compliances used in this paper were obtained on LVL tensile specimens (Chaplain [9]). Short term test and long term tests (creep tests) have been carried out to determine elastic and vicoelastic properties of wood. The reduced compliance needed in the fracture mode I, is identified to a 3-parameters power law or by a 2-parameters power law (in the calculations, the expressions of the compliance S_{ij} are expressed by a 3-parameters or 2-parameters power law).

4.2 Notched beams

The figure 4 presents the experimental time to failure obtained under constant loading (circle points). These results are compared to the previsions of VCM and of the simplified model (eqn 6). At the beginning of the test, the initial crack length a_o was very small (around 5 mm). The model VCM is applied with a_o=5 mm. However, calculations have also been made for a_o=100 mm to show that the real initial crack length has only a small effect on the duration of load.

As illustrated on figure 4a, VCM model is in a good agreement with the experimental results for smaller stress level (<0,9). However, VCM model is still optimistic for the high stress levels: in fact when the failure happens in least than 1 day, the "initial" loading has an influence on the time to failure which is not taken into account in the modelisation. The model VCM is not very influenced by the initial crack length a_o, the strength F_s of the beams decreases when a_o increases: the times to failure of the beams are similar whatever is a_o, but the beams do not carried out the same load.

The simplified expression of VCM, VCMS (eqn 6), is fitted using the value 0,8 for the reference stress level. This simple model can not be used for high stress level (<0,9) but it gives quite fair results for lower Stress Level as presented on figure 4a.

4.3 Beams with a hole

For beams with a hole experiments are in progress. The calculated duration of load under constant load are presented on figure 4b for different values of the initial crack length a_o. When initial crack is smaller than 100 mm, the duration of load decreases when a_o increases and when a_o is upper 100mm, the initial crack and the time to failure evolve in the same way. The evolution of SL (fig 3b) explains this evolution.

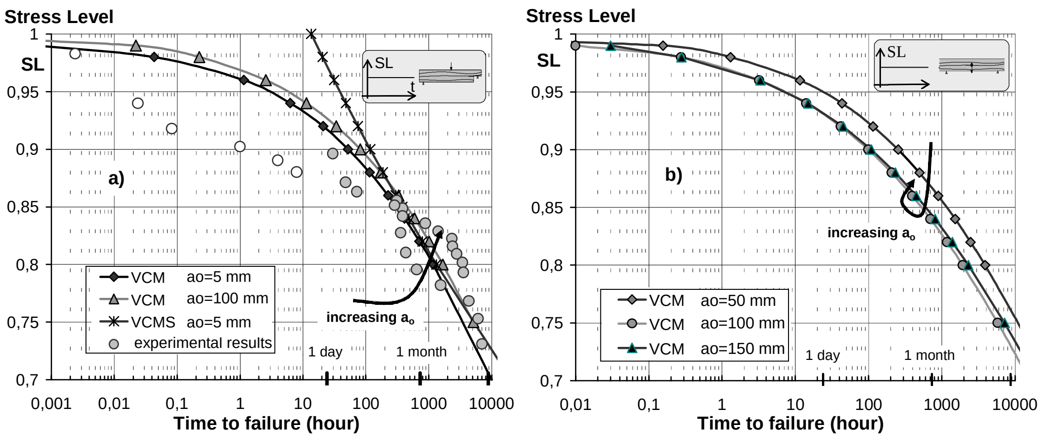

Fig. 4 : Comparison of models VMC prevision under creep tests
a) notched beam b) beam with a hole

We also observe that when a_o is greater than 100 mm, the initial crack length a_o does not have a lot of influence on the time to failure, but of course, a_o has a high influence on the strength of the beams. The influence of the shape of the beams on the duration of load is well observed too: for notched beams, the time to failure against SL, increases with a_o, it is the opposite for beams with a hole (when a_o is smaller then 100 m).

5 CONCLUSION

The presented model VCM takes into account the shape of the beams, the kind of loading and also the elastic stiffness by the way of a calibration function, for finite dimension beams or beams containing singularities (notches, holes...). The predictions of VCM are similar to the experimental results under creep tests. A simplified model (VCMS) can be derived from VCM. Fitted on experimental results, it is in fair agreement with the experimental results for low Stress Level (or long term test).

The influence of the initial length of the crack a_o on the strength or the delayed failure has been analyzed. Effectively, the strength decrease when a_o increases. The initial crack length also influences the duration of load : for notched beams a longer initial crack length leads to longer time to failure, for beams with a hole, when a is smaller than around 100 mm, the duration of load decreases when the initial crack length increases. This last result must be confirmed by experiments.

REFERENCES

[1] Schapery, R. A., A theory of crack initiation and growth in viscoelastic media: I. Theoritical development, II. Approximate methods of analysis , International Journal of Fracture, 11: 141-159, 369-388,1975.
[2] Nielsen, L. F., Wood as a cracked viscoelastic material, Part I: theory and applications. Proc. International Workshop on Duration of Load in Lumber and Wood Products, Richmond, Canda, 67-78,1985.
[3] Valentin, G., Chaplain, M. (2001): Effects of relative humidity on crack growth and fracture of Laminated Veneer Lumber. Proceeding of the first international Conference of the European Society for Wood Mechanics, Lausanne, Switzerland, EPFL, 243-253.
[4] Valentin, G., Bostrom, L., Gustafsson, P.J., Ranta-Maunus, A. and Gowda, S., Application of fracture mechanics to timber structures, RILEM state-of-the-art. Research notes 1262, VTT, 1991.
[5] Boyancé, P., Modélisation de la rupture différée d'un matériau orthotrope viscoélastique en environnement naturel. Application à un composite à base de bois : le LVL, Thèse de l'Université de Bordeaux I, n°2145, 1999.
[6] Gustafsson, P.J., Hoffmeyer, P. and Valentin, G., DOL behaviour of end-notched beams, Holz als Roh - und Werkstoff, 56(5): 307-317, 1998
[7] Brockway, G.S. and Schapery, R.A., Some viscoelastic crack growth relations for orthotropic and prestrained media. Engineering Fracture Mechanics, 10: 453-468, 1978.
[8] Mindess, S., Nadeau, J.S., Barret; J.D., Slow crack growth in douglas-fir, Wood Science (8)1,pp.389-396,1975
[9] Chaplain, M., Bouadjel, R. and Valentin, G., Cracking and rupture of notched LVL beams, Proceeding of the second international Conference of the European Society for Wood Mechanics, Stockholm, Sweden, pp 143-148, 2003.