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THE EFFECT OF CONSTRAINT ON THE FIELD OF INTERFACE CRACKS BETWEEN STRENGTH MISMATCHED MATERIALS IN ELASTIC-PERFECTLY PLASTIC PLANE STRAIN CONDITIONS

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ABSTRACT

The structure of the elastic-perfectly plastic plane strain fields for a crack located on the interface between elastically similar but strength mismatched elastic-perfectly plastic materials has been investigated. Attention is focussed on the role of the non-singular T stress on the structure of the near mode I asymptotic fields. Analytic solutions have been developed by expressing the fields as combinations of elastic and plastic sectors. In the plastic sectors, the stresses are analysed in terms of slip-line field theory while the stresses in the elastic sectors are determined from semi-infinite elastic wedge solutions. The analytic solutions have been verified numerically using plane strain modified boundary layer formulations.

KEYWORDS

Mismatch, T stress, elastic-perfectly plastic, slip-line fields

INTRODUCTION

Fracture mechanics originates from studies of cracks in isotropic homogeneous solids. However, in welded structures defects may occur on the interface between solids which are elastically similar but plastically dissimilar. Mismatch problems of this type have been investigated for strain hardening materials by Zhang and co-workers [1], and are widely discussed by Schwalbe and Koçak [2]. Insight into the structure of the fields can however be obtained by simplifying the plastic response to perfect-plasticity. In this context Sham, Li and Hancock [3] have developed analytic solutions for the contained yielding fields of an interface crack between strength mismatched solids for the complete range of normal versus shear tractions on the interface. In the limit, when the yield strengths are identical these solutions make contact with known homogenous elastic-plastic solutions. Notably in mode I the Prandtl field which is the non-hardening limit of the HRR fields is recovered. Similarly in mode II the Hutchinson [4] field is recovered. It is however known that for homogenous solids, the mode I fields are not unique, and depend on a non-singular terms in the Williams expansion [5], which Rice [6] has denoted T.

The possibility that plasticity might not always encompass the tip, allowing the possibility of elastic sectors, was originally discussed by Nemat-Nasser and Obata [7]. In this context Du and Hancock [8] demonstrated that the Prandtl (or HRR) field only occurs when the T stress is positive and plasticity encompasses the crack tip. If T is non-positive elastic sectors appear on the crack flanks and lead to a loss of constraint (mean stress) in the sectors ahead of the crack. In the strength mismatch problem Ganti and Parks [9] have shown that the near mode I fields are also affected by the T stress. The present work examines the role of T on the near mode I fields for strength mismatched solids. Analytic solutions are developed which are compared with numerical solutions from modified boundary layer formulations.

GEOMETRY AND MATERIAL RESPONSE

A crack is defined along the interface between two elastically similar but strength mismatched solids under plane strain conditions. Cylindrical axes (r,θ) are centred at the crack tip with the crack flanks located at $\theta = \pm \pi$. The material above the crack ($\pi \geq \theta \geq 0$) has a uniaxial yield stress σ_0, while the material below the crack, ($\pi \leq \theta \leq 0$) has a higher yield stress $M\sigma_0$ ($M \geq 1$). Both materials are elastic-perfectly plastic, obeying a Mises yield criterion and an associated flow rule. The solids are elastically identical with a shear modulus G and an elastic response close to incompressibility. Solutions are initially presented for the homogeneous problem, M = 1 and an example is given of the mismatch problem, M = 1.2. Fuller sets of solutions are presented by Lowe and Hancock [10]. For both the homogeneous and the mismatch problem the role of the non-singular T stress has been examined.

ANALYTIC METHOD

The analytic solutions derive from an analysis by Rice [11] which describe the asymptotic field in terms of elastic and plastic sectors. Rice [11] shows that the assumption that the crack tip stresses are finite allows the equilibrium equations and the plane strain yield criterion to be written in the form;

$$\frac{\partial \sigma_m}{\partial \theta} \cdot \frac{\partial \sigma_{r\theta}}{\partial \theta} = 0 \tag{1}$$

This allows two possible forms for the plastic sectors. The first possibility is the mean stress, σ_m, is independent of angle and the Cartesian stresses are constant, defining a constant stress region characterised by straight slip lines. The second possibility is that the shear stress, $\sigma_{r\theta}$ is constant which gives rise to centred fans. When the yield criterion is not satisfied, an elastic sector occurs and the stresses can be derived from the semi-infinite wedge solutions of Timoshenko and Goodier [12]. The sectors are assembled in a manner consistent with continuity of tractions $\sigma_{\theta\theta}$ and $\sigma_{r\theta}$. Although continuity of stresses is not enforced, it may be shown that the sector boundaries within the same material require continuity of all stresses, while discontinuities in σ_{rr} and σ_{zz} across the interface are permitted. Mismatch problems are inherently mixed mode and although the remote elastic loading comprise pure mode I field the local field is mixed mode. To date no analytic ways of connecting the mixity of the remote and local fields have been proposed. However, this is straight forward in numerical solutions, and so the analytic solutions for the low levels of mismatched problems originate from defined ratios of shear to normal traction on the interface and the mean stress on the interface . In the homogeneous fields it is straightforward to determine values of T associated with different levels of mean stress ahead of the crack, but fully analytic ways of making this connection have not yet been developed. Thus the homogeneous solutions start from defined levels of the mean stress or constraint ahead of the crack.

NUMERICAL METHOD

A finite element method has been used to validate the analytical solutions by modelling the crack tip region as a modified boundary layer formulation, (Rice and Tracey [13]). The mesh consisted of 24 concentric rings of second order elements focussed on the crack tip. The tip was defined as the focus of a ring of collapsed elements with coincident but independent nodes. On the remote boundary of the model, Cartesian displacements, u_x, u_y, corresponding to the first two terms of the Williams expression [5] were applied.

$$u_x = \frac{K}{2G}\left(\frac{r}{2\pi}\right)^{\frac{1}{2}} \cos\left(\frac{\theta}{2}\right)\left[\eta - 1 + \sin^2\left(\frac{\theta}{2}\right)\right] + \frac{1+\eta}{8G} rT\cos\theta \qquad (2a)$$

$$u_y = \frac{K}{2G}\left(\frac{r}{2\pi}\right)^{\frac{1}{2}} \sin\left(\frac{\theta}{2}\right)\left[\eta - 1 + 2\cos^2\left(\frac{\theta}{2}\right)\right] + \frac{\eta-3}{8G} rT\sin\theta \qquad (2b)$$

Here $\eta = 3 - 4\nu$, G is the shear modulus and ν is Poisson's ratio. K is a loading parameter established by the far field conditions while three representative values of T have been chosen as 0, $+0.4\sigma_0$ and $-0,4\sigma_0$, where σ_0 is the uniaxial yield stress. All calculations were performed with a non-hardening elastic/perfectly plastic incompressible response at constant values of T. Stresses were extrapolated to the crack tip along radial lines at an angular frequency of 7.5 degrees around the tip. In order to permit stress discontinuities in the radial stress at the interface the extrapolation to the interface nodes was made from element sets completely contained within either the harder or softer materials, allowing the possibility of stress discontinuities at the interface.

RESULTS

The solutions to the homogeneous mode I problem are given in Figure 1 for three different levels of T stress. When the T stress is positive the fully constrained Prandtl field is recovered. This is the non-hardening limit of the HRR fields and is a fully constrained field characterised solely by J. However, when the T stress is non-positive, a family of unconstrained fields arise, as discussed by Du and Hancock [8] and Li and Hancock [14]. Crack tip constraint is lost ahead of the crack and an elastic wedge appears on the crack flanks. These unconstrained fields require a two parameter characterisation using either T, (Betegón and Hancock [15]) or Q, (O'Dowd and Shih, [16]) to characterise the level of constraint. The numerical and analytic solutions to the homogeneous mode I problem can be seen to agree very closely. Although only mode I solutions are presented here, similar solutions to the mixed mode problem have been given by Li and Hancock [14].

Lowe and Hancock [10] have developed a full range of analytic solutions to elastic-perfectly plastic mismatch problems with remote mode I loading. The problems considered span a wide range of mismatch, but the examples presented in the present work are restricted to relatively low levels of mismatch. Interest is focussed on a mismatch, M=1.2, shown in Figure 2. The fields can be seen to be simple variants of the homogeneous fields given in Figure 1. Plasticity surrounds the crack tip when the T stress is positive, and elastic wedges appear on the crack flanks when T is non-positive. The angular span of the wedges increases as T becomes more negative, as in the homogeneous field. The non-singular T stress is shown to have a constraint effect on the structure of the fields, in agreement with Ganti and Parks [9]. Although the remote loading is pure mode I, the interface exhibits both normal tractions and shear stresses. Equilibrium requires that the hoop and shear stresses are continuous across the interface however there is an allowable jump in the radial stress and a necessary discontinuity in the slip-lines across the interface.

As with the homogeneous problem, crack tip constraint is lost when the non-singular T stress is zero or negative. The extent of constraint loss is similar to that observed in the homogeneous problems (Zhang et al, [1]. Constraint loss is also accompanied by the appearance of elastic wedges on the crack flanks, as in the homogeneous problem. Sham, Li and Hancock [3], developed a set of fields which agreed closely with analytic solutions for almost the complete range of remote mixed mode loading. However discrepancies between the analytic solutions and the accompanying numerical solutions were noted for near mode I problems. Sham et al [3] attributed this to constraint effects associated with T. The present work confirms this conclusion and shows that the Sham, Li and Hancock solutions make full contact with the T positive fields in the near mode I region. As T becomes negative the angular span of the wedge increases. The T stress also has a weak but noticeable effect on the ratio of shear to normal tractions directly ahead of the crack. At high levels of mismatch the fields change from those shown in Figure 2. Elastic sectors appear even in the T positive loading cases, however the dominant feature that the Sham, Li and Hancock fields make contact with the T positive fields is retained while non-positive T stresses lead to both a loss of constraint and a change in the structure of the fields, as discussed in detail by Lowe and Hancock [10].

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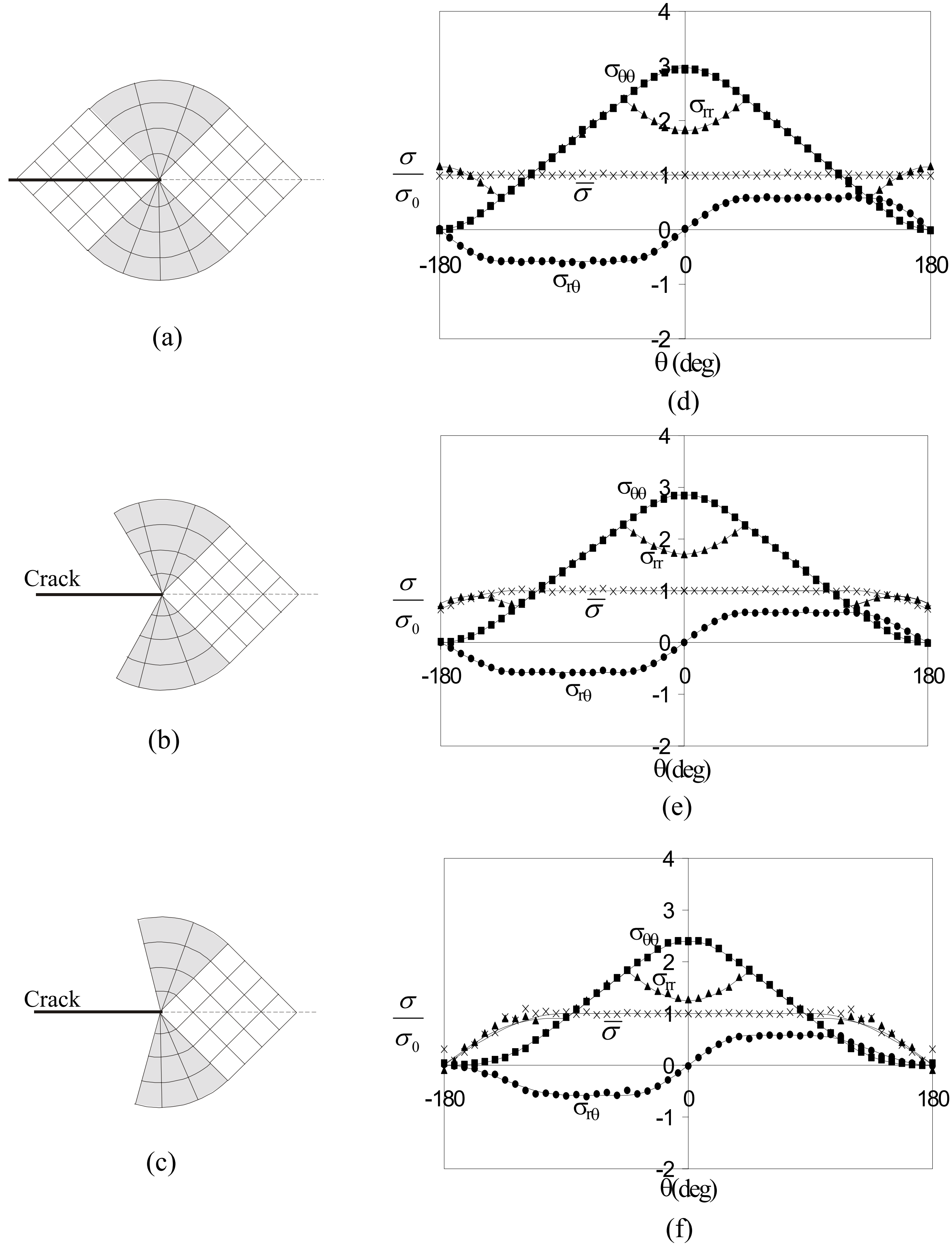

Figure 1 Slip-line fields for M=1 for (a) Positive T stress, (b) Zero T stress and (c) Negative T stress and asymptotic stress fields for M=1 for (a) Positive T stress, (b) Zero T stress and (c) Negative T stress. The solid lines represent the results from the analytic solutions, the symbols the results from the computational analysis.

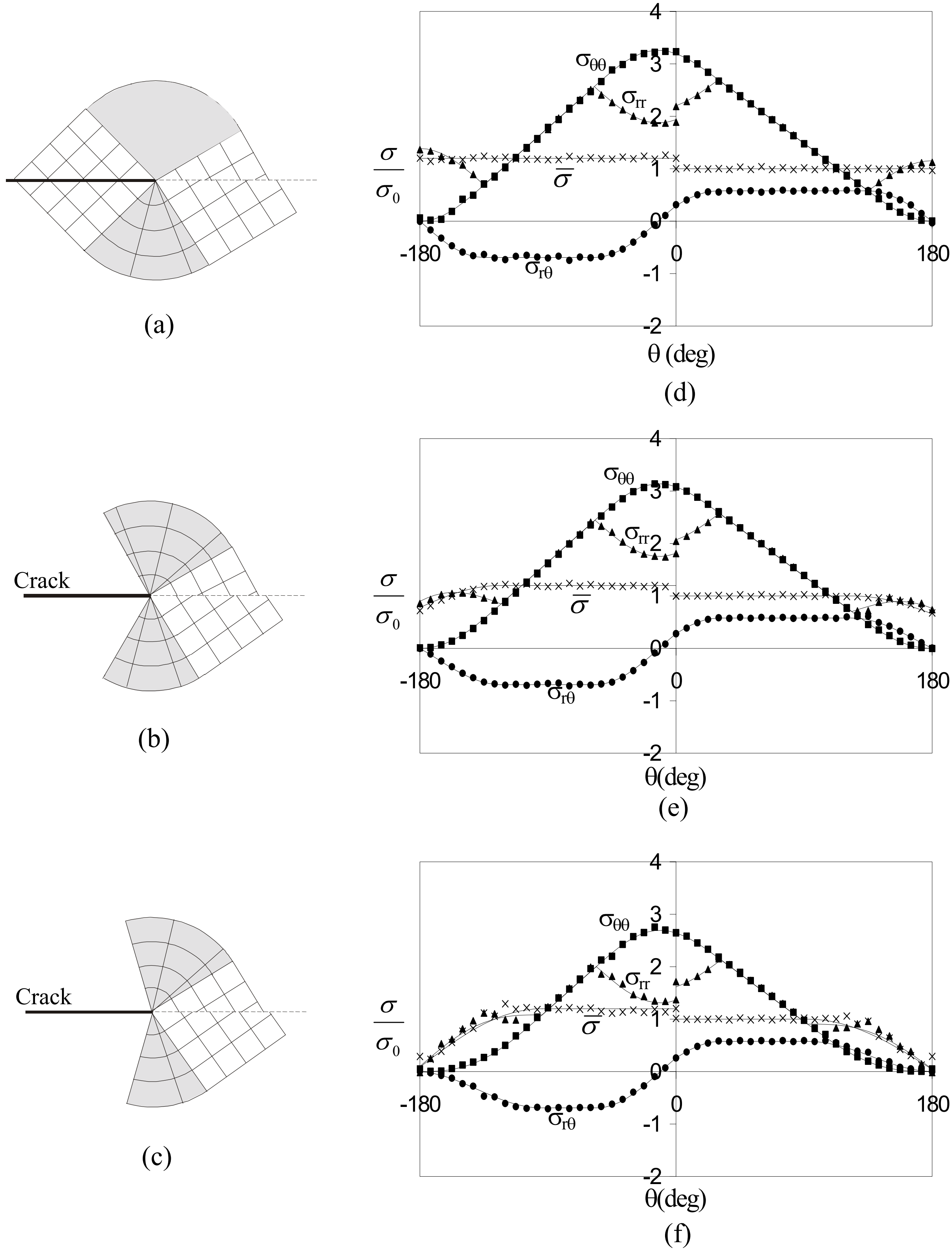

Figure 2 Slip-line fields for M=1.2 for (a) Positive T stress, (b) Zero T stress and (c) Negative T stress and asymptotic stress fields for M=1.2 for (a) Positive T stress, (b) Zero T stress and (c) Negative T stress. The solid lines represent the results from the analytic solutions, the symbols the results from the computational analysis.