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THE ANTI – PLANE SHEAR FIELD FOR CRACK IN INFINITE SLAB OF A NONLINEAR DAMAGE MATERIAL

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ABSTRUCT

In this paper, the constitutive equation of nonlinear damage material is given .The basis equation for founding solution of mode III crack is established, using transformation of coordinates .The analytical and numerical results are obtained .The shape and the scale of damage zone where the material completely fails are determined.

KEY WORDS

Nonlinear damage material, solution of near tip crack for Mode III crack

INTRODUCTION

The field of stress and strain near tip-crack is an important research problem of fracture mechanics because it controls the crack growth. There are many kinds of structures of crack –tip field [1-3] , that depend upon the material nature and the loading condition .All of the existing solutions to the crack tip fields are based on the singular analysis, i.e. either stress or strain possesses singularity at the crack tip. But the real behavior of materials does not permit the singular, at finite stress and strain the material always fails. Therefore, the singular solution is not valid when the crack tip is really approached. In order to reveal the true feature of the crack tip fields, we must consider the real character of materials. When strain reaches certain critical value, the strength of material will completely vanished, so that the concept of damage mechanics mast is introduced [4].

There are continuous models of internal damage parameters, which can be incorporated into crack analysis. For instance, Bui and Ehrlachar[5] Proposed a simple model to analysis the dynamic steady state propagation of a damage zone in elastic and plastic solids and got exact solutions for the small scale damage model in elastic material and for the strip problem, in mode III loading. There was not singularity in the solution of stresses and strains. With Krajcinovic's[6] assume, Popelar and Hoagland[7] discussed distribution of damage field of mode III crack ,where the relation of damage variable and strain is linearity.

This paper is concerned with an infinite slab containing a semi – infinite crack, which is subjected to the anti – plane shear K_{III} field at infinity. First, the constitutive equation of nonlinear damage material is given, which is that $t = G(1 - D^n)g$, where t is the effective stress, n is the softening parameter, damage factor D, depends on the effective strain, $D = (Gg / k)$, where G is the shear module, k is the damage module and the g is effective strain.. When $n = 1$, the material is linear damage material[6]. Secondly, the basis equation is given for founding solution of mode III crack, using transformation of coordinates. Finally, the analytical and numerical results are obtained .The shape of damage zone; stress distribution and the scale of damage zone are discussed. When $n = 1$, the result is same as that given by C.H.Popela[5]

THE BASIS EQUATION

The constitutive equation

The evolvement equation given by Krajcinovic [6] is

$$s = E(1 - D)e$$

$$D = EeH(e)/K \qquad 1$$

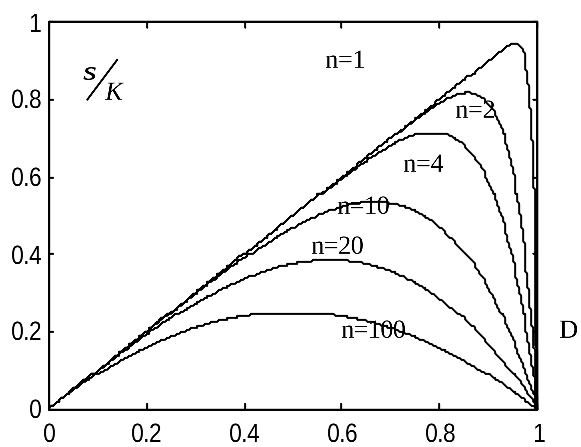
Where E is Young's modulus $H(e)$ is Heaviside's function. This constitutive equation describes damage process of concrete, rock and brittle materials. We assume that the constitutive equation of materials in the uniaxial tension case is

$$s_e = E(1 - D^*)e_e$$

$$D^* = \left(\tfrac{E}{K} e_e\right)^n = D^n \qquad (2)$$

where s_e is effective stress e_e is effective strain in the uniaxial tension case, Eq. 2 is

$$s/K = D(1 - D^n) \qquad 3$$



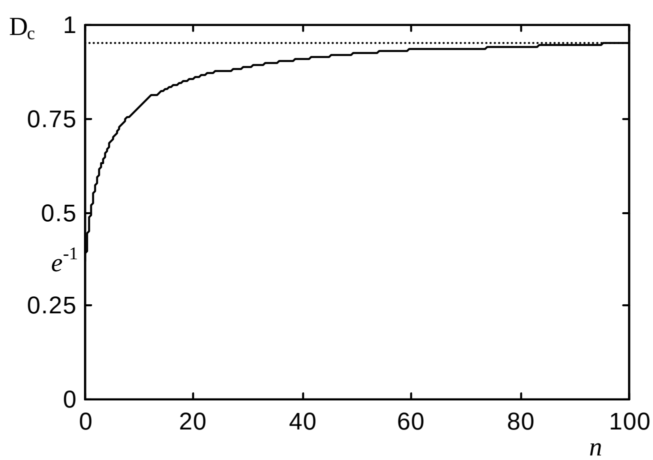


Figure 2: the relation of n and the damage critical

Figure 1 Curve of $s/K \sim D$

Curve of $s/K \sim D$ (Eq. (3)) is given by Fig.1.When stress reaches extremism the material is in Unstability State, soft and fails .The damage critical value satisfies that $\frac{ds}{dD} = 0$, then

$$D_c = \left(\frac{1}{n+1}\right)^{\frac{1}{n}} \qquad 4$$

As n increases, D_c increases the relation of n and the damage critical value is given by Fig.2

The basis equation of problem of Mole III crack

In anti-plane problem, stresses, $t_{xz} = t_x, t_{yz} = t_y$ satisfy the equilibrium equation:

$$\frac{\partial t_x}{\partial x} + \frac{\partial t_y}{\partial y} = 0 \qquad 5$$

The strain and displacement, $W = W(x, y)$, are given by

$$g_{xz} = g_x = \frac{\partial W}{\partial x}$$

$$g_{yz} = g_y = \frac{\partial W}{\partial y} \qquad 6$$

The compatibility equation is

$$\frac{\partial g_x}{\partial y} = \frac{\partial g_y}{\partial x} \qquad 7$$

The constitutive equation:

$$t_x = G\left(1 - D^n\right)g_x$$

$$t_y = G\left(1 - D^n\right)g_y \qquad 8$$

Where g is effective shear strain, $g = \left(g_x^2 + g_y^2\right)^{\frac{1}{2}}$ and

$$t = G\left(1 - D^n\right)g \qquad (9)$$

Where $D = \left(\frac{G}{K}g\right)$

The boundary conditions are

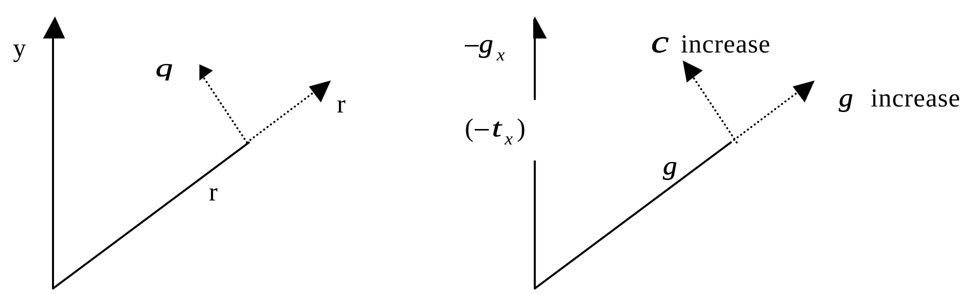
$$t_q = 0 \quad on \quad q = p$$

$$W = 0 \quad on \quad q = 0$$

$$t_y - i\,t_x \rightarrow \frac{K_{III}}{\left[2p(x - iy)\right]^{1/2}} \quad when \quad |x + iy| \rightarrow \infty \qquad (10)$$

THE BASIS SOLUTION OF MODE III CRACK

Using transformation of co-ordinates (Fig.3), we can take variable (g_x, g_y) instead of (x, y) [8].



The equilibrium equation 5 becomes

$$\frac{\partial x}{\partial \boldsymbol{t}_x}+\frac{\partial y}{\partial \boldsymbol{t}_y}=0 \qquad 11$$

The compatibility equation 7 is

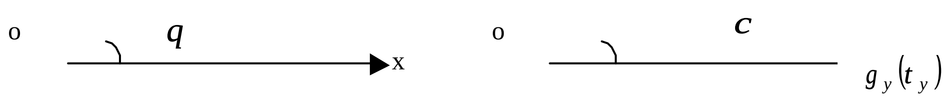


Fig.4 Physics and strain plane

$$\frac{\partial x}{\partial g_y}=\frac{\partial y}{\partial g_x} \qquad 12$$

Introducing the strain function $y(g_x, g_y)$

$$x=\frac{\partial \boldsymbol{y}}{\partial g_x} \qquad y=\frac{\partial \boldsymbol{y}}{\partial g_y} \qquad 13$$

Then Eq. 12 is satisfies. Taking note of transformation of co-ordinates, we have express of x and y

$$x=-\sin \boldsymbol{c}\frac{\partial \boldsymbol{y}}{\partial g}-\frac{\cos x}{g}\frac{\partial \boldsymbol{y}}{\partial x}$$

$$y=\cos x\frac{\partial \boldsymbol{y}}{\partial g}-\frac{\sin x}{g}\frac{\partial \boldsymbol{y}}{\partial x} \qquad 14$$

In the problem of mode III crack, $\boldsymbol{y}(g, x)$ satisfies

$$\frac{1-D^n}{1-(n+1)D^n}\frac{\partial^2 \boldsymbol{y}}{\partial D^2}+\frac{1}{D}\frac{\partial \boldsymbol{y}}{\partial D}+\frac{1}{D^2}\frac{\partial^2 \boldsymbol{y}}{\partial \boldsymbol{c}^2}=0 \qquad 15$$

When $D<D_c$ $\frac{d\boldsymbol{t}}{dg}>0$ Eq. 15 is elliptic and a well-behaved small-scale yielding solution is attainable.

Assume that

$$\boldsymbol{y} = f(D)\sin \boldsymbol{c} \qquad 16$$

Where $f(D)=D\boldsymbol{j}(D)$ and $\boldsymbol{j}(D)$ satisfies

$$\frac{d\boldsymbol{j}}{dD}=\frac{A}{D^3(1-D^n)} \qquad 17$$

A is unknown constant. We can obtain solution of Eq. 17 .

When $n=1$, we have

$$j(D)=-\frac{1}{2D^2}-\frac{1}{D}+\ln\left(\frac{D}{1-D}\right)+C \qquad 18$$

When $n=2$, we have

$$j(D)=-\frac{1}{2D^2}+\ln D-\frac{1}{2}\ln\left(1-D^2\right)+C \qquad 19$$

When $n \neq 1,2$ we have $nm-3 \neq -1$ i.e. when $n \neq 1,2$ there are not terms of D^{-1}, we have

$$\frac{1}{D^3\left(1-D^n\right)}=D^{-3}+D^{n-3}+D^{2n-3}\cdots\cdots+D^{nm-3}+\cdots\cdots \qquad 0<D<1$$

$$j=C-\frac{1}{2D^2}+\frac{D^{n-2}}{n-2}+\frac{D^{2n-2}}{2n-2}+\cdots\cdots+\frac{D^{Mn-2}}{Mn-2}+\cdots\cdots \qquad 20$$

Where C is unknown constant, and series in Eq. (20) is convergence, when $D \in (0,1)$.

DISCUSSION

The shape of damage zone

With Eq. 14 and 20 we have

$$x = X(D)+R(D)\cos 2c \qquad (21)$$
$$y = R(D)\sin 2c$$

Where
$$X(D) = -\frac{G}{2K}(f^{'}+f/D)$$
$$R(D) = \frac{G}{2K}(f^{'}-f/D)$$

The damage zone, defined by (21), is a set of circles for $0<D<D_c$ with their centers on x>0 ,y=0. There are two unknown constants , A and C, in solution ,which are determined by boundary condition and the damage zone where the material completely fails, respectively. By Eq. 10 , we have

$$A=\frac{\mathrm{K}_{\mathrm{III}}^{2}}{2p\ KG} \qquad 22$$

When $D=D_c$ by $X(D_c)=R(D_c)$ C satisfies

$$\frac{df}{dD}=0 \qquad 23$$

When n=1 C = - 4 ,this is same as result in [7]

Fields of stresses and strains

With Eq. (21 and expression of y we obtain

$$\begin{aligned} &\sin(2c - q) = b \sin q \\ &r = R(D)[b^2 + 2b\sin(2c) + 1] \end{aligned} \qquad 24$$

Where $b = X(D)/R(D)$

$$\begin{aligned} &g_r = g\sin(q - c) \\ &g_q = g\cos(q - c) \end{aligned} \qquad 25$$

For certain geometry point, (r, q) by Eq. 24 , (c, D) are determined and the field of strains is given.

As n increases $R(D_c)$ decreases. When $n \to \infty$

$$R(D_c) = \frac{\mathrm{K}_{\mathrm{III}}^{\;2}}{2pK^2} \qquad 26$$

If we take that $K = t_s$, the yield strength when $n \to \infty$ the material is elastic then the radius of damage zone is the same as of the plastic radius in the elastic perfectly plastic material[3].

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