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STUDY ON CRACK-NUCLEATING DAMAGE OF HYDRAULIC CONCRETE

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ABSTRACT
Based on the concept of renormalization group, a methodology of crack-nucleating damage of hydraulic concrete is studied in this paper. Damage state depended on the jump of multi-scaling and two-leveling phase changes is defined by a fractal. The damaging models of considering the influences of aggregate size, aggregate interface, crack nucleation, fractal dimension, etc. on the damage are established by energy-equilibrium relationship under the state in the crack-nucleating damage.

1. INTRODUCTION

Concrete is a makeup material that is made of cement, sand, gravel, and water, etc. The interfaces between the different mediums create lots of inherently weak joints and/or micro- and macro-cracks, as well as voids, or say defects in generality. A concrete body contaminated by the defects informs a non-ordered system. The damage [1-3] gets up firstly at the weakly jointed places at which the crack nucleation is produced by a crack-nucleating mechanism [4], and the damage evolution starts and diffuses with growth of the micro-cracks after nucleating. Thus, the concrete is furthermore described as a man-made material whose physical structure is discontinuous, non-homogeneous, and irregular. The damage and the damaged diffusion in concrete may be categorized by a multi-scale-level mechanism, and the configuration of the damaged front is defined as a fractal. Following these definitions, the makeup of the concrete is shown in figures 1a-1c.

2. FRACTAL LENGTH OF DAMAGE-NUCLEATING CRACKS

At arbitrary damage-scaling level D_i, let the activated micro-cracks in concrete inform a set called as Group, $\Re_i$. To a defect as shown in figure 1d, its configuration is assumed to be a fractal round, whose circumference is featured with a fractal line [5].

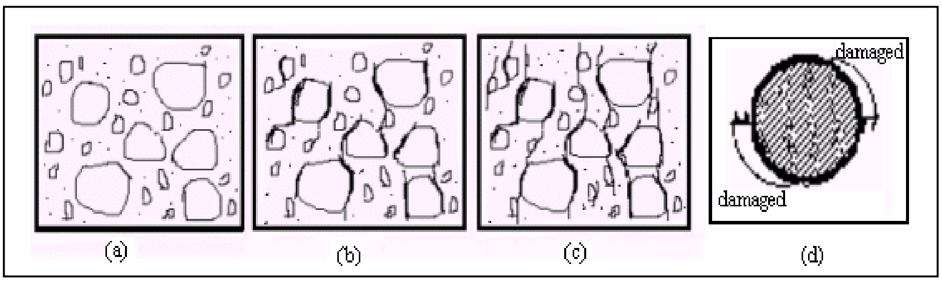


Figure 1 Concrete defects, nucleating and developing damage, and fractal round

The nucleating damage, going with nucleation of the micro-cracks, starts firstly at a place in the circumference of the fractal rounds when the strain or the stress exceeds the nucleating-damage threshold of the concrete. Let the fractal round be a damaged unit consisting of 4 quadrants, and a micro-crack appears stochastically at arbitrary quadrant. Obviously, if the micro-cracks are simultaneously produced at 4 quadrants, the damaged unit is in damaged state. Subsequently, if the micro-cracks are measured at the symmetrical quadrants, say quadrant 1 and 3 or quadrant 2 and 4, the damaged unit is also considered to be in damaged state. Thus, using the concept of the K. G. Wilson's renormalization group, a twice magnification transformation of renormalization group is created as:

$$p_i' = \Re_{i+2}(p_i) = p_i^4 + 2\times 2p_i^2\left(1-p_i^2\right) \tag{1}$$

in which p_i is priori crack-nucleating probability, p_i' is probability after twice renormalization magnification. A physical interpretation of this semi-group transformation is a twice-amplifying relationship of the measuring scale of the cracks. Therefore, the critical jump point of damage phase change can be obtained by the following formula:

$$p_c^4 + 2\times 2p_c^2\left(1-p_c^2\right) - p_c = 0 \tag{2}$$

Its closed solutions are $p_c = 0,\ \ 1,\ \ 0.618\ \ -1.618$, respectively. It is clearly that the solutions 0，1 represent the zero-damaged and certainly damaged states, respectively, whilst minus number should be dropped out because of no physic meaning（a probability does not appear in a minus number）. Thus, the probability of the damage phase change is $p_c = 0.618$, and the expectation value of nucleating damage at the defined damaged unit is stated as the following:

$$E_c = \left[4p_c^4 + 2\times 2p_c^2\left(1-p_c^2\right)\right]/\,p_c = 4p_c \tag{3}$$

The value, substituting the critical value $p_c = 0.618$, is $E_c = 2.472$. One can note that $E_c = 2.0$ can be obtained to the classical fracture mechanics.

Assuming the measuring scale δ is as much as n^{-1} times of the diameter of the fractal round studied, namely $d = n\delta$, the fractal dimension of the damage group, D_f, can be expressed as:

$$D_f = \frac{\ln E_c}{\ln n} = \frac{\ln E_c}{\ln \frac{d}{\delta}} \tag{4}$$

The relationship between the fractal dimension and the measuring scale is shown in figure 2.

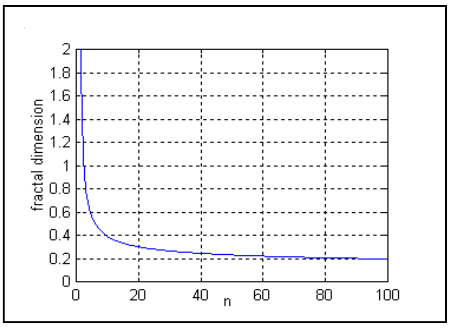


Figure 2 Fractal versus measuring scale

It follows the Mandelbort's definition of fractal line, herein the circumference, L_f, of the fractal round can be stated as:

$$L_f(\delta) = L_0(\delta)\delta^{1-D_f} \tag{5}$$

As an example, if the nucleating cracks appear at the quadrant 1 and 3 of the fractal round shown in figure 1d, the length, $L_c(\delta)$, of the micro-cracks after nucleating is as following:

$$L_c(\delta) = \frac{1}{2}L_0(\delta)\delta^{1-D_f} \tag{6}$$

If $L_0(\delta)$ is defined as the circumference length of the regulation round, namely $L_0(\delta) = \pi n\delta$, the expectation length of the micro crack at the damaged unit is written as:

$$L_c(\delta) = E_c \pi n \delta^{2-D_f} / 4 \tag{7}$$

Taking， one can get a formula that expresses the crack length in respective with the measuring scale and the fractal dimension, namely:

$$L_c(\delta) = 0.618\pi n\delta^{2-D_f} \tag{8a}$$

or,

$$L_c(\delta) = 0.618\pi n^{D_f-1} d^{2-D_f} \tag{8b}$$

3. DAMAGE-NUCLEATING MODELING

In case of the state of crack-nucleating damage, the equilibrium condition of the unit-thicken concrete body can be stated as:

$$\left(\frac{\delta_0}{d}\right)^2 \sigma_f + \sigma\left(C_3 + \frac{4}{\pi}\sqrt{\frac{\delta_0}{d}}(1-C_3)\right)^2 = \sqrt{\frac{4E\psi \cdot 1}{\pi(1-\nu^2)L_c}} \tag{9}$$

in which δ_0 is the thickness of cement-grouting film between the defect (fractal round) and concrete, σ_f failure strength of concrete, σ loading stress, C_3 material constant to be determined by test, ψ energy containing in unit volume of concrete, ν Poisson's ratio, and E initial elastic modulus. It can be seen that the right-hand side of the equation is defined by simulating Griffith's law. Introducing the foregoing definitions to the fractal round and the length of the micro-crack, equation (9) can be changed as:

$$\left(\frac{\delta_0}{d}\right)^2 \sigma_f + \sigma\left(C_3 + \frac{4}{\pi}\sqrt{\frac{\delta_0}{d}}(1-C_3)\right)^2 = \sqrt{\frac{6.472E\psi \cdot 1^{2-D_f}}{\pi^2 n(1-\nu^2)\delta^{2-D_f}}} \tag{10a}$$

or,

$$\left(\frac{\delta_0}{d}\right)^2 \sigma_f + \sigma\left(C_3 + \frac{4}{\pi}\sqrt{\frac{\delta_0}{d}}(1-C_3)\right)^2 = \sqrt{\frac{6.472E\psi.1^{2-D_f}}{\pi^2 n^{D_f-1}(1-\nu^2)d^{2-D_f}}} \tag{10b}$$

If the problem studied is focused on a big scale granule diameter of concrete aggregate, namely considering $d >> \delta_0$, the first term of the left hand side of equation (10a) or (10b) can be dropped out, and a crack-nucleating damage formula is obtained as:

$$\sigma_{dc} = \left(C_3 + \frac{4}{\pi}\sqrt{\frac{\delta_0}{d}}(1 - C_3) \right)^{-2} \sqrt{\frac{6.472 E\psi .1^{2-D_f}}{\pi^2 n^{D_f - 1} (1 - \nu^2) d^{2-D_f}}} \tag{11}$$

The energy stated in equation (11) can be divided into two parts. One is of a contribution of deteriorated damage, due to deterioration of the material characteristics by crack nucleation; other is of releasing energy due to formation of the micro-cracks at the damaged area. To the former, Helmholtz's free-energy expression is employed, namely:

$$\rho\psi_d = \frac{1}{2}(\lambda + 2\mu)\varepsilon_{ii}\varepsilon_{jj} - \mu(\varepsilon_{ii}\varepsilon_{jj} - \varepsilon_{ij}\varepsilon_{ji}) + C_1 D_{di}\varepsilon_{ij} D_{dj}\varepsilon_{kk} + C_2 D_{di}\varepsilon_{ij}\varepsilon_{jk} D_{dk} \tag{12}$$

in which ρ is mass intensity of material, λ, μ stand for Lame' constants, and C_1, C_2 represent material parameters to be determined by test and related to the damaged extent. For the sake of simplification, the tension stress state is only considered in this paper, and after a lengthily deducing, the deteriorated damage strain energy, ψ_d, can be got as:

$$\psi_d = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} \left[\frac{1}{2}\left(1 - \frac{2\nu^2}{1-\nu}\right)\varepsilon^2 + \frac{C_1 + C_2 - 2\nu C_1}{3(B_1 - B_3)}\varepsilon^3 - \frac{1-\nu}{8}\frac{C_1^2}{(B_1 - B_3)^2}\varepsilon^4 \right] \tag{13}$$

and the releasing energy due to cracking as:

$$\psi_c = \frac{\pi E(1-\nu)^2 L_c}{4(1+\nu)^2(1-2\nu)^2} \left[\left(1 - \frac{2\nu^2}{1-\nu}\right)\varepsilon + \frac{C_1 + C_2 - 2\nu C_1}{(B_1 - B_3)}\varepsilon^2 - \frac{1-\nu}{2}\frac{C_1^2}{(B_1 - B_3)^2}\varepsilon^3 \right]^2 \tag{14}$$

in which B_1, B_3 are material parameters. If the hyperbolic damage surface [6] is employed to express developing damage inside concrete body with loading, the following relationship is used, namely:

$$(B_1 - B_3) = \frac{\varepsilon}{D_d^2} \tag{15}$$

Thus,

$$\psi_d = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} \left[\frac{1}{2}\left(1 - \frac{2\nu^2}{1-\nu}\right) + \frac{(C_1 + C_2 - 2\nu C_1)}{3} D_d^2 - \frac{(1-\nu)}{8} C_1^2 D_d^4 \right]\varepsilon^2 \tag{16}$$

$$\psi_c = \frac{\pi^2 E(1-\nu)^2 n^{D_f - 1} d^{2-D_f}}{6.472(1+\nu)^2(1-2\nu)^2} \left[\left(1 - \frac{2\nu^2}{1-\nu}\right) + (C_1 + C_2 - 2\nu C_1) D_d^2 - \frac{1-\nu}{2} C_1^2 D_d^4 \right]^2 \varepsilon^2 \tag{17}$$

Substitution of equation (16) and/or (17) into equation (11) obtains the following damage constitute models describing concrete in tensile state.

Model I, unique consideration of damage-nucleating energy (by substituting equation (17) only)

$$\sigma_{dI} = \frac{\left[\left(1 - \frac{2\nu^2}{1-\nu}\right) + (C_1 + C_2 - 2\nu C_1) D_d^2 - \frac{1-\nu}{2} C_1^2 D_d^4 \right]}{(1+\nu)(1-2\nu)\left(C_3 + \frac{4}{\pi}\sqrt{\frac{\delta_0}{d}}(1 - C_3) \right)^2} \sqrt{\frac{1-\nu}{1+\nu}} E\varepsilon, \quad \varepsilon \ge \varepsilon^{th} \tag{18a}$$

Model II, non-consideration of damage-nucleating energy (by substituting equation (16) only)

$$\sigma_{II} = \sqrt{\frac{6.472\left[\frac{1}{2}\left(1-\frac{2\nu^2}{1-\nu}\right)+\frac{\left(C_1+C_2-2\nu C_1\right)}{3}D_d^2-\frac{\left(1-\nu\right)}{8}C_1^2D_d^4\right]}{\pi^2 n^{D_f-1}\left(1+\nu\right)^2\left(1-2\nu\right)d^{2-D_f}\left(C_3+\frac{4}{\pi}\sqrt{\frac{\delta_0}{d}}\left(1-C_3\right)\right)^4}E\varepsilon}, \quad \varepsilon \geq \varepsilon^{th} \tag{18b}$$

Model III, integrated damage (by substituting equations (16) and (17))

$$\sigma_{dIII} = \sqrt{\sigma_{dI}^2 + \sigma_{dII}^2} \tag{18c}$$

4. DISCUSSIONS OF PARAMETERS C_1 AND C_2

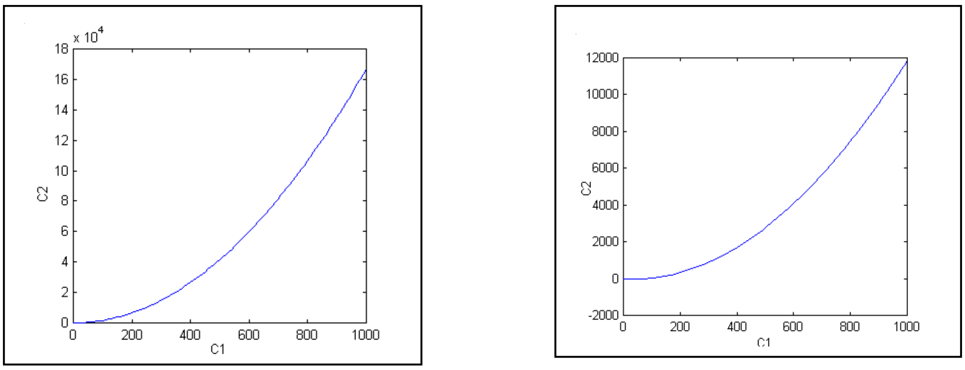
To illustrate the procedure to determine the material parameters of C_1 and C_2, 32 concrete specimens of 70.7mm×70.7mm×212.1mm, were done by a machine of MTS 810 TestStar, and 3-parameter Weibull distribution is used to describe the probabilistic characteristics of the strains. Based on the definition of the $P - D_d - \varepsilon$ curve of concrete [7], a regressive curve on $D_d - \varepsilon$, corresponding to the expectation probability $E_c = 50\%$ (closely to the foregoing theoretical value of 0.618), is obtained as:

$$\varepsilon = 803{D_d}^3 - 910{D_d}^2 + 501D_d + 73 \tag{19}$$

It is clearly when $D_d = 0$, the threshold strain of the damage is $\varepsilon^{th} = 73\mu\varepsilon$. In considering the case of $D_d = 0.2$, one can easily get the relevant value of the strain from equation (10) as $143.2\,\mu\varepsilon$. Thus, dynamic stress corresponding to this strain is approximately got as 2.7MPa by test constitutive relationship [8]. Taking $\delta_0 / d = 0.001$, $C_3 = 0.2$, $\nu = 0.16$, $E = 2.1\times10^4\,\text{MPa}$, as well as $d = 0.01$m, $n = 10$, and $D_f = 0.393$, the relationships between C_1 and C_2 are obtained, from equation (18a) and (18b), and shown in figures 3a and 3b, respectively, as:

$$C_1^2 - 4C_1 - 5.97C_2 + 133.4 = 0 \tag{20}$$

$$C_1^2 - 54.33C_1 - 79.60C_2 - 2802.98 = 0 \tag{21}$$



(a) (b)

Figure 3 C_1 versus C_2 in case of the above conditions

5. CONCLUSIONS

In consideration of the special characteristics of concrete makeup, a combination of the mechanisms of both damage and micro-crack nucleation is a way to study crack-nucleating damage of the hydraulic concrete. From the theoretical viewpoints, the crack-nucleating damage models established in this paper are suitable. It is surely that a result closer to the practice damage and fracture in engineering can be well predicted by the viewpoints subjected, if the further work is done, especially in experimental and comparing studies.

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