

SIMULATION OF PROPAGATION OF SMALL FATIGUE CRACKS INTERACTING WITH GRAIN BOUNDARY

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ABSTRACT

The propagation behavior of microstructurally small fatigue cracks was numerically simulated on the basis of the plasticity-induced crack closure model. The effects of the frictional stress of dislocation motion and the strength of the grain boundary blocking on the crack closure behavior were analyzed. Then the simulation of the propagation of a crack nucleated in the weakest grain was conducted by assuming that the crack growth rate was controlled by the crack-tip opening displacement, $\Delta CTOD$. The grain size, the critical value of microscopic stress intensity factor at grain boundary and the frictional stress were given as random variables following two-parameter Weibull distributions. When the crack approached adjacent grains with higher frictional stresses, $\Delta CTOD$ decreased and the crack opening stress increased. The grain boundary and the higher frictional stress act as a resistance to crack propagation. When compared at the same stress intensity range, $\Delta CTOD$ increased with increasing stress ratio. The effect of the stress ratio on the fatigue limit was analyzed on the basis of the results of the simulation. The relation between the fatigue limit and the value of the applied mean stress is nearly identical to the modified Goodman relation.

KEYWORDS

Fatigue, Crack closure, Crack opening displacement, Fatigue limit, Mean stress, Simulation, Microstructure, Grain boundary

INTRODUCTION

The propagation behavior of long fatigue cracks is uniquely determined by the stress intensity range, ΔK . On the other hand, small fatigue cracks show anomalously high irregular propagation rates when compared with long cracks at the same ΔK [1-6]. This fast propagation rate has been ascribed to microstructural effects, premature crack closure and macroplasticity [3,5]. The irregular propagation behavior is caused by the microstructural inhomogeneity. In our previous study [7,8], a model for crack-tip slip band blocked by a grain boundary based on the continuously distributed dislocation theory was proposed. The statistical nature of propagation behavior of small cracks could be successfully derived by the Monte Carlo simulation. However, the effect of the crack closure was not taken into account.

In this study, the propagation behavior of microstructurally small fatigue cracks was numerically simulated. The effects of the frictional stress of the adjacent grain and the strength of the neighboring grain boundary on the crack closure behavior were analyzed. Then the simulation of the propagation of a crack nucleated in the weakest grain was conducted. The effects of the microstructural parameters and the value of the applied mean stress on the propagation behavior and on the fatigue limit were investigated.

CRACK CLOSURE ANALYSIS

Analytical Model

The interaction model between grain boundary and slip band ahead of a crack-tip is shown in Fig. 1. The isolated crack is located at the center of a grain and the slip band is assumed to spread in the same plane. In the figure, L_{q-1} is the distance between the $(q-1)$ th grain boundary and the center of the crack. σ_q^f is the frictional stress of the q th grain. When the slip band is blocked by the grain boundary, the stress field near the tip of the slip band has a singularity with the following intensity [7]:

$$K^m = \beta \sigma \sqrt{\pi c}$$
$$\beta = 1 - \frac{2\sigma_1^f}{\pi\sigma} \cos^{-1}\left(\frac{a}{c}\right) \quad (1)$$

The microscopic stress intensity factor, K^m , increases with crack length. When K^m becomes larger than the critical value of the strength of the grain boundary, K_c^m , the slip band propagates into the adjacent grain as shown in Fig. 1 (c). When the crack becomes large, the slip bands spread into several grains (Fig. 1 (d)). When the crack-tip and the tip of the slip band are in the j th and the q th grain, respectively, the plastic zone size, $\omega (= c-a)$, can be calculated by [7]

$$\frac{\pi}{2} \sigma_{\max} - \sigma_j^f \cos^{-1}\left(\frac{a}{c}\right) - \sum_{k=j+1}^q (\sigma_k^f - \sigma_{k-1}^f) \cos^{-1}\left(\frac{L_{k-1}}{c}\right) = 0 \quad (2)$$

where σ_{\max} is the maximum applied stress.

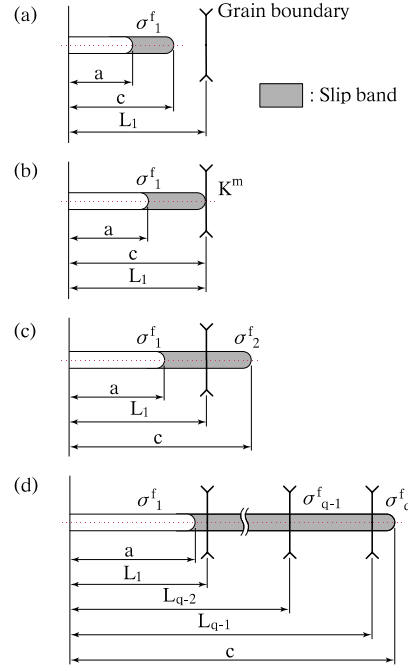


Figure 1. Slip band model.

Microstructural Parameters

Microstructural parameters used in this study are the grain size, d , the critical microscopic stress intensity factor, K^m_c , and the frictional stress, σ^f . Those parameters were given as random variables following two-parameter Weibull distributions. The random variables were generated by using their mean value (denoted by suffix μ) and a variance (denoted by suffix v). A fatigue crack is assumed to initiate from the weakest grain having the largest value of the following strength parameter Γ [9]:

$$\Gamma_k = d_k (\Delta\sigma - 2\sigma_k^f) \quad (3)$$

where $\Delta\sigma (= \sigma_{\max} - \sigma_{\min})$ is the applied stress range, d_k is the grain size. The mean frictional stress, σ_{μ}^f , and Young's modulus, E , are assumed to 400 MPa and 206 GPa, respectively.

Crack Propagation

In our previous study [7,8], we simulated the propagation behavior of microstructurally small fatigue cracks without taking into account of crack closure. The results correspond to the propagation of Stage I cracks. In this study, cracks are assumed to propagate in a Stage II manner under the influence of crack closure. Fatigue crack propagation behavior was simulated to evaluate the development of the plasticity-induced crack closure with crack growth by using an analytical closure model proposed by Newman [10]. The initial crack, a_i , and the plastic zone, ω_{\max} , at the maximum applied stress were divided into 20 and 40 elements, respectively. The amount of crack extension was prescribed to be $\Delta a = 0.002 \omega_{\max}$.

Budiansky et al. [11] analyzed the plasticity induced crack closure of semi-infinite cracks and showed that $\Delta CTOD$ was related to the effective stress intensity range, ΔK_{eff} , as

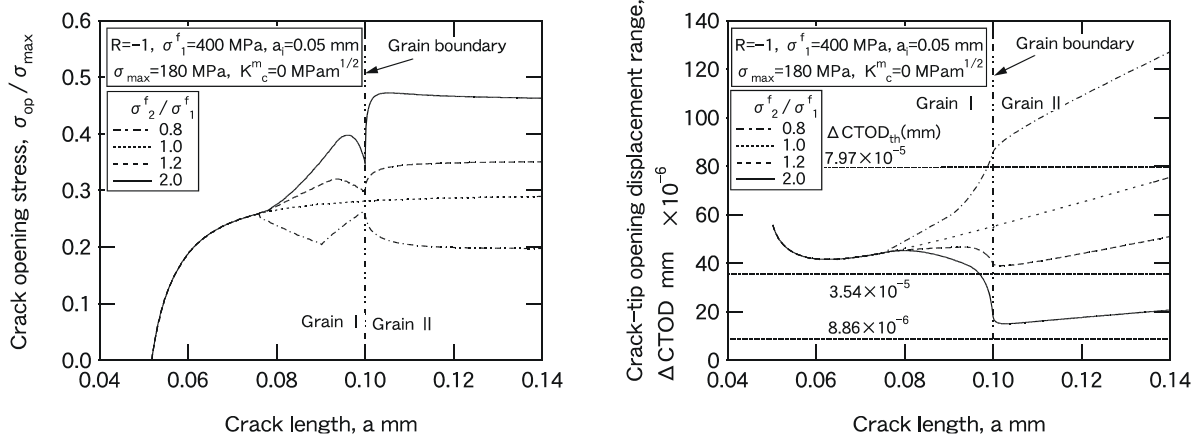
$$\frac{\Delta CTOD}{CTOD_{\max}} = 0.73 \cdot \left(\frac{\Delta K_{\text{eff}}}{K_{\max}} \right)^2 \quad (4)$$

The threshold value of $\Delta CTOD$ is evaluated by Eq. (4). By assuming the threshold value of ΔK_{effth} as 1, 2 and 3 MPam^{1/2}, the threshold value of $\Delta CTOD$ is calculated to 8.86×10^{-6} , 3.54×10^{-5} and 7.97×10^{-5} mm.

RESULTS AND DISCUSSION

Effect of Friction Stress

Fatigue crack propagation behavior was simulated to investigate the effect of the frictional stress of an adjacent grain on crack closure. Both the length of an initial Stage I crack and the distance between the crack-tip and the grain boundary were assumed to be 50 μm . Figure 2(a) shows the change of $\sigma_{\text{op}}/\sigma_{\text{max}}$ with crack length. The maximum applied stress is 180 MPa. The frictional stress of the first grain, σ_1^f , is 400 MPa, and $\sigma_2^f/\sigma_1^f=0.8, 1.0, 1.2$ and 2.0 in the second grain. There is no barrier at the grain boundary ($K_c^m=0$ MPam^{1/2}). The value of $\sigma_{\text{op}}/\sigma_{\text{max}}$ for $\sigma_2^f/\sigma_1^f>1.0$ becomes larger than that for $\sigma_2^f/\sigma_1^f=1.0$. When the extension of the plastic zone is constrained by the adjacent grain with a higher frictional stress, the crack opening stress increases with crack length. Namely, the effective component of the stress range decreases as a consequence of the constraint. The crack opening stress takes a local maximal value in the first grain before the crack tip reaches the grain boundary. When the crack propagates into the adjacent grain, the crack opening stress increases sharply, and takes the maximum value in the second grain. On the other hand, for the case of $\sigma_2^f/\sigma_1^f<1.0$, the behavior is in contrast with the result obtained for $\sigma_2^f/\sigma_1^f>1.0$. The change of $\Delta CTOD$ with crack growth is shown in Fig. 2(b). $\Delta CTOD$ first decreases due to the development of the residual stretch, then increases until the tip of the plastic zone reaches the grain boundary. For the case of $\sigma_2^f/\sigma_1^f>1.0$, $\Delta CTOD$ begins to decrease, as the plastic zone crosses the grain boundary. When the driving force of crack propagation is given by $\Delta CTOD$, the higher frictional stress of the neighboring grain act as a resistance of crack propagation. If the threshold value of $\Delta CTOD$ is assumed to be 3.54×10^{-5} mm, the crack becomes non-propagating at the crack length of 0.097 mm for the case of $\sigma_2^f/\sigma_1^f=2.0$. It is interesting to note that $\Delta CTOD$ takes a minimum value at the crack length of 0.103 mm in the second grain.



(a). Change of σ_{op} with crack length.

(b) Change of $\Delta CTOD$ with crack length.

Figure 2. Effect of frictional stress of adjacent grain.

Statistical Analysis of Crack Propagation

The same simulation was conducted to investigate the effects of the grain-boundary blocking and the applied mean stress on the threshold condition of small cracks. The frictional stress and the grain size were given

as random variables. The initial crack length is $a_i=5 \mu\text{m}$. The mean frictional stress and grain size are 400 MPa and 50 μm , respectively. The variances of those values are $(\sigma^f/\sigma_{\mu}^f)_v=0.2$ and $d_v=50 (\mu\text{m})^2$. The mean value and the variance of the critical value of the microscopic stress intensity factor are $(K_c^m/\sigma_{\mu}^f (\pi d_{\mu})^{1/2})_{\mu}=0.4$ and $(K_c^m/\sigma_{\mu}^f (\pi d_{\mu})^{1/2})_v=0.04$, respectively. The relation between $\Delta CTOD$ and ΔK is shown in Fig. 3(a). The applied stress is $\sigma_a/\sigma_{\mu}^f=0.5$. In the figure, the broken line indicates the relation obtained for the constant mean frictional stress and $K_c^m=0$, and has a slope of two in the log-log diagram. The results obtained from 30 kinds of random number sequence are plotted. The scatter diminishes as the crack propagates and converges to the broken line. This corresponds to the experimental results of propagation behavior of small fatigue cracks [6]. When the threshold value of ΔK_{effth} is assumed to be 1 MPa $\text{m}^{1/2}$, the threshold value of $\Delta CTOD$ becomes 8.86×10^{-6} mm. In this case, 24 cracks out of thirty are arrested at the first grain boundary. The abscissa was subdivided into 20, and the arithmetical averages of $\Delta CTOD$ and ΔK were calculated within each subdivision. Figure 3(b) shows the relation between the arithmetical average of $\Delta CTOD$ and ΔK . In the figure, the results obtained for the stress ratio of -1.5 and -0.5 were also plotted. The dotted lines in the figure are the results obtained for $\sigma_k^f=\sigma_{\mu}^f$ and $K_c^m=0$ under each stress ratio. When compared at the same ΔK , $\Delta CTOD$ increases with increasing stress ratio. Two and twenty eight cracks out of thirty are arrested under the stress ratio of -0.5 and -1.5, respectively. The number of arrested cracks decreases with stress ratio.

Figure 4 shows the Haigh diagram which is the relation between the stress amplitude and the mean stress at the fatigue limit. The fatigue limits are calculated as the fracture probability of 50%. The threshold value are given as $\Delta CTOD_{\text{th}}=8.86 \times 10^{-6}$, 3.54×10^{-5} and 7.97×10^{-5} mm. The fatigue limit decreases linearly with the mean stress irrespective of the threshold value. The dot-dash line is the modified Goodman relation obtained by assuming $\sigma_B/\sigma_{\mu}^f=1.15$ [12]. The slope is close to the simulated results.

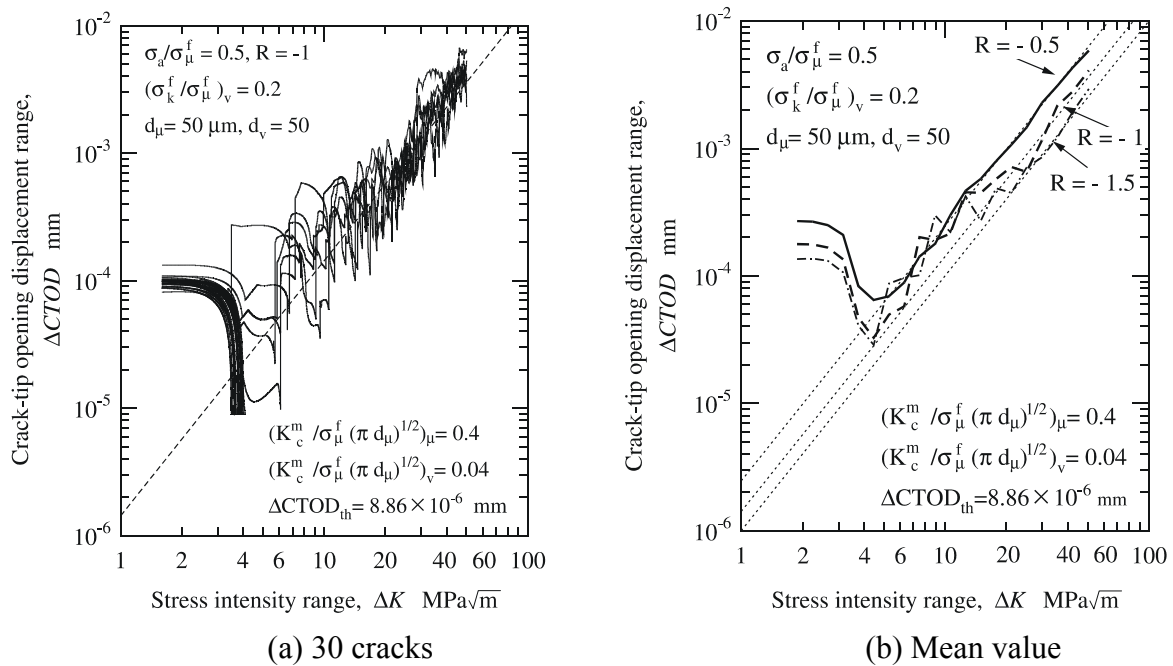


Figure 3. Relation between $\Delta CTOD$ and ΔK .

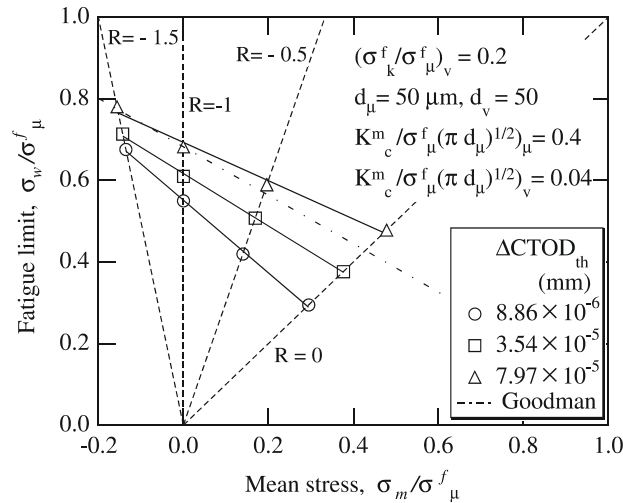


Figure 4. Haigh diagram.

CONCLUSION

The propagation behavior of microstructurally small fatigue cracks was numerically simulated on the basis of the plasticity-induced crack closure model.

- (1) When the crack approached grains with higher frictional stresses, $\Delta CTOD$ decreased and the crack opening stress increased. The grain boundary blocking and higher frictional stress act as a resistance to crack propagation.
- (2) The scatters of $\Delta CTOD$ diminished as the crack length becomes longer. When compared at the same stress intensity range, $\Delta CTOD$ increased with stress ratio.
- (3) The relation between the fatigue limit and the applied mean stress is nearly identical to the modified Goodman relation.

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