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SCALING OF QUASI-BRITTLE FRACTURE : MORPHOLOGY OF CRACK SURFACES AND ASYMPTOTIC ANALYSIS.

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ABSTRACT

Recently, the *anomalous* scaling properties, observed on crack surfaces of quasi-brittle materials, was proposed to be related to the experimental *R*-curve behavior and size effect on the critical energy release rates [1]. On this basis, an energy-based asymptotic analysis allows to extend the link between anomalous roughening of crack surfaces to the nominal strength of structures. The established relation represents a smooth transition from the case of no size effect, for small structure sizes, to a power law size effect which appears as a decrease of the linear elastic fracture mechanics theoretical one, in the case of large sizes. This predicted size effect is confirmed by fracture experiments on wood.

KEYWORDS

Scaling, Quasi-brittle materials, Roughening, Asymptotic analysis, Energy release, Size effect.

INTRODUCTION

Fracture of quasibrittle materials such as concrete, wood, tough ceramics or various rocks and composites, is characterized by the effect of structure size on its nominal strength [2]. In these materials, a large fracture process zone (FPZ) with microcracking damage develops inducing stress redistributions which greatly increases the effective fracture energy. This particular behavior, known as the resistance curve (*R*-curve), leads to the release of the stored energy by the stable macrocrack growth, before the maximum load is reached, and is at the origin of the size effect.

A recent study has shown that the *R*-curve behavior of quasi-brittle materials could be linked to the morphology of crack surfaces [1]. This link between macroscopic fracture properties and the fractal nature of cracks at the microscale is based on the analysis of the complete scaling behavior of the local fluctuations of crack surfaces obtained in two quasi-brittle materials : granite [3] and wood [4]. The scaling law (called *anomalous* scaling [5,6]) needed to describe accurately the crack developments in the perpendicular and parallel directions of crack propagation involves three scaling exponents, one is the universal local roughness exponent $\zeta_{loc} = 0.8 - 0.9$ (*i.e.* independent of the fracture mode and of the material [7]) and two other exponents which are material dependent, the global roughness exponent ζ and the dynamic exponent z. The measured values of the global roughness exponent are $\zeta = 1.2$ in granite [3] and respectively 1.35 and 1.60 in the case of two species of wood (pine and spruce [4]). Moreover, it has been shown in [3,4], that fracture surfaces starting from a straight notch, exhibit anomalous roughness development for crack length increments $\Delta_a \ll \Delta a_{sat}$. This is followed by a stationnary regime where the magnitude of the roughness saturates and appears to be dependent on the specimen size for crack length increments $\Delta_a \gg \Delta a_{sat}$. On this basis, the revisited Griffith criterion [1] which takes into account the energy required to create a rough surface at the microscale, leads to a *R*-curve behavior (1) linked to the anomalous roughness development of fracture surfaces. In (1), G_R is the resistance to crack growth and 2γ the specific surface energy (considered as an average value taking into account the heterogeneous character of the material). The terms A and B are prefactors and l_o is the lower cutoff of the fractal range of the fracture surface (*i.e.* the characteristic size of the smaller microstructural element relevant for the fracture process). Note that the term under square root is dimensionless.

$$G_R(\Delta a \ll \Delta a_{sat}) \simeq 2\gamma\sqrt{1+\left(\frac{AB^{\zeta-\zeta_{loc}}}{{l_o}^{1-\zeta_{loc}}}\right)^2 \Delta a^{2(\zeta-\zeta_{loc})/z}} \quad (1)$$

$$G_{RC}(\Delta a \gg \Delta a_{sat}) \simeq 2\gamma\sqrt{1+\left(\frac{A}{{l_o}^{1-\zeta_{loc}}}\right)^2 (C\,L)^{2(\zeta-\zeta_{loc})}} \quad (2)$$

On the other hand, on the zone where the roughness saturates (*i.e.* for crack length increments $\Delta_a \gg \Delta_{sat}$), the link [1] leads to a *critical* resistance to crack growth G_{RC} (2) where L is the characteristic size of the structure (homothetic structure) and C a proportionality coefficient. Note that in (1) and (2) the crack length increment Δa has to be understood as an elastically equivalent crack length increment in the sense of linear elastic fracture mechanics (LEFM).
The main consequence of the link between fracture mechanics and anomalous roughening of fracture surfaces [1] is the size effect on the critical resistance to crack growth (2) which is induced by the the dependence of the maximum magnitude of the roughness as a function of the structure size L. As shown in Fig. 1 for arbitrary values of the scaling exponents, the size effect on the critical resistance G_{RC} (2) exhibits two asymptotic behaviors. The transition happens for the the crossover length $L_C = 1/C\,({l_o}^{1-\zeta_{loc}}/A)^{1/(\zeta-\zeta_{loc})}$. For small structure sizes (*i.e.* $L \ll L_C$) where the roughness of fracture surfaces is weak, there is no size effect : $G_{RC} \simeq 2\gamma$, while for large structure sizes (*i.e.* $L \gg L_C$), which correspond to important fracture roughness, the critical resistance evolves as a power law : $G_{RC} \sim L^{\zeta-\zeta_{loc}}$.

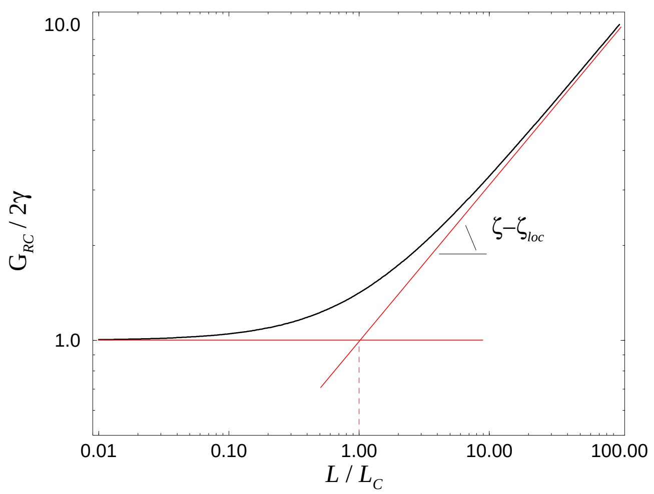

Figure 1: Size effect on the critical resistance to crack growth obtained for: $\zeta = 1.3$, $\zeta_{loc} = 0.8$, $A = 0.1$ and $l_o = 1$.

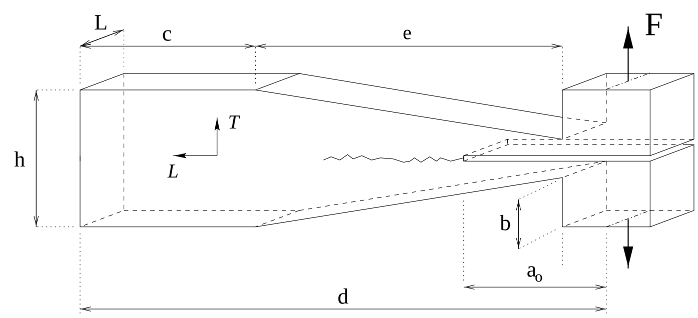

Figure 2: Geometrically similar TDCB fracture specimens of different sizes $L = 7.5, 11.3, 15.0, 22.5, 30.0$ and 60.0 mm.

The purpose of the present paper is to revisit the size effect relation on nominal strength of structures proposed by Bažant [8] by introducing the modifications induced by the link [1] between anomalous roughening of fracture surfaces and material fracture properties of quasi-brittle materials previously mentioned.

ANOMALOUS ROUGHENING AND SIZE EFFECT ON NOMINAL STRENGTH

In the frame work of reference [8], the size effect (for two-dimensional problem) in geometrically similar structures can be described by using the nominal stress :

$$\sigma_N = \frac{P}{d\,L} \quad (3)$$

where P is the external load applied on the structure (considered to be a load independent of displacement), L is the characteristic size of the structure and d is the ligament length (Fig. 2). When $P = P_u$, *i.e.* the ultimate load, σ_N is called the nominal strength of the structure.

In the case of a quasi-brittle material, the nominal strength σ_N depends on the crack length a corresponding to the ultimate load P_u such as : $a = a_o + \Delta a_{sat}$ where a_o is the length of the initial crack and Δa_{sat} id defined as the critical effective length of the fracture process zone (under P_u). In order to obtain dimensionless energy release expression, we introduce the relative crack length : $\alpha = \alpha_o + \theta$, where $\alpha_o = a_o/d$ and $\theta = \Delta a_{sat}/d$ are dimensionless variables. The main problem to establish any size effect relation on nominal strength is to determine the effective length of the process zone Δa_{sat}. Indeed, the dependence between Δa_{sat} (or in other terms the crack length increment below which the *R*-curve (2) applies) and the specimen size does not appear clearly in the roughness analysis [3,4]. A possible way to resolve this problem consists in considering that the material failure of a quasi-brittle material is not only characterized by the specific surface energy 2γ [1] per unit surface of the actual crack (Eq. 1 and 2), but also by a critical damage energy release rate G_d per unit volume of *damaged* material (*i.e.*, per unit volume of FPZ). Thus, one can assume that failure at maximum load is obtained for the energy balance :

$$G_d\, V_{FPZ} = \; 2\gamma\, A_r(\Delta a_{sat}) \tag{4}$$

where $A_r(\Delta a_{sat})$ is the real crack surface corresponding to the effective crack length increment Δa_{sat}. The volume of the fracture process zone can be estimated as $V_{FPZ} = L\, \Delta {a_{sat}}^2/n$ where $L\Delta a_{sat}$ corresponds to the projected crack surface and $\Delta a_{sat}\;/n$ is the *height* of the FPZ (n is a constant, *i.e.* independent of the size L).

On the other hand, at the maximum load, the crack corresponding to the crack length increment Δa_{sat} propagates if its energy release rate G (obtained at constant load P or σ_N) becomes equal to the critical resistance to crack growth G_{RC} defined in (2) :

$$G = \frac{1}{L}\left[\frac{\partial W^*}{\partial a}\right]_{\sigma_N} = \; G_{RC} \tag{5}$$

where the complementary energy W^* characterizes the energy stored in the structure. This energy can be expressed as a function of the relative crack length α : $W^* = {\sigma_N}^2 L\, d^{\,2} f(\alpha)/E$ where f is a dimensionless function characterizing the geometry of the structure. Thus, knowing the relative crack length for which the maximum load is reached, the nominal strength of the structure can be written as :

$$\sigma_N = \sqrt{\frac{E\, G_{RC}}{d\, g(\alpha)}} \tag{6}$$

in which $g(\alpha) = \partial f(\alpha)/\partial \alpha$ corresponds to the dimensionless energy release rate function.

On the other hand, in structures said to be of positive geometry (*i.e.* $\partial g(\alpha)/\partial\alpha > 0$) which is the restricted case of this study, the crack length increment Δa_{sat} at maximum load represents the limit of stability of the crack growth if the structure is under load control rather than displacement control.

Large-size asymptotic expansion of the size effect

As previously mentioned, the crack length increment Δa_{sat} corresponding to the maximum load P_u can be determined by Eq.(4). Moreover, for large structure sizes (*i.e.* $L \gg L_C$), the corresponding real crack surface A_r can be easily estimated from [1] as : $A_r \simeq \beta\, L\, \Delta a_{sat}\; (L/L_C)^{\zeta-\zeta_{loc}}$ where β is a constant (function of the scaling exponents). Substituting A_r in (4) yields the expression of the crack length increment :

$$\Delta a_{sat} = c^* \beta \left(\frac{L}{L_C}\right)^{\zeta-\zeta_{loc}} \tag{7}$$

where $c^* = n2\gamma/G_d$ can be consider as a material length. Thus, for large structure sizes, the relative crack length of the FPZ (*i.e.* $\theta = \Delta a_{sat}/d$) is expected to evolve as a power law $\theta \sim L^{\zeta-\zeta_{loc}-1}$ and $lim\;\theta = 0$ for $L \rightarrow +\infty$. In other terms, in large structures, the fracture process zone is expected to lie within only an infinitesimal volume fraction of the body and so $lim\,\alpha = \alpha_o$ for $L \rightarrow +\infty$. Note that this result is in agreement with the assumption made in [8]. Hence, $g(\alpha)$ being a smooth function, we may expand it into Taylor series around $\alpha = \alpha_o$ and Eq.(6) thus yields :

$$\sigma_N \;=\; \sqrt{\frac{E\, G_{RC}}{d}}\left[\, g(\alpha_o) + g_1(\alpha_o)\,\theta + g_2(\alpha_o)\,\frac{\theta^2}{2!} + g_3(\alpha_o)\,\frac{\theta^3}{3!} + ...\,\right]^{-1/2} \tag{8}$$

$$= \sigma_M \sqrt{\frac{\left(1 + (\frac{L}{L_C})^{2(\zeta-\zeta_{loc})}\right)^{1/2}}{\frac{L}{L_1} + (\frac{L}{L_C})^{\zeta-\zeta_{loc}} + b_2\frac{L_1}{L}(\frac{L}{L_C})^{2(\zeta-\zeta_{loc})} + b_3(\frac{L_1}{L})^2(\frac{L}{L_C})^{3(\zeta-\zeta_{loc})} + \ldots}} \tag{9}$$

where $g_1(\alpha_o) = \partial g(\alpha_o)/\partial\alpha$, $g_2(\alpha_o) = \partial^2 g(\alpha_o)/\partial\alpha^2$, ..., and $b_2 = g(\alpha_o)g_2(\alpha_o)/(2g_1(\alpha_o)^2)$, $b_3 = g(\alpha_o)^2 g_3(\alpha_o)/(6g_1(\alpha_o)^3)$, ..., and,

$$\sigma_M = \sqrt{\frac{2\gamma E}{mg(\alpha_o)L_1}} \tag{10}$$

$$L_1 = c^* \frac{\beta}{m} \frac{g_1(\alpha_o)}{g(\alpha_o)} \tag{11}$$

are all constants. The terms m in (10) is a proportionality coefficient between the ligament length d and the characteristic size L of the specimens ($d = mL$). Expression (9) provides a large-size asymptotic series expansion of the size effect but is expected to diverge for structure sizes $L \to 0$ as shown in Fig. 3.

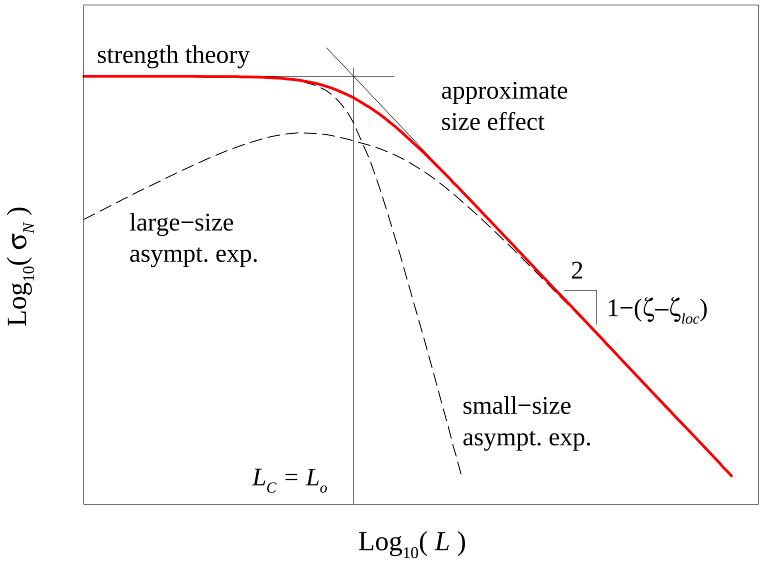

Figure 3: Approximate size effect on the nominal strength (solid curve) and asymptotic series expansions (dashed curves).

Moreover, in (9) the terms of non zero powers in denominator vanish for $L \to \infty$ and so, the large-size first-order asymptotic approximation can be obtained by truncating the series after the linear term :

$$\sigma_N = \sigma_M \sqrt{\frac{\left(1 + (\frac{L}{L_C})^{2(\zeta-\zeta_{loc})}\right)^{1/2}}{\frac{L}{L_1} + (\frac{L}{L_C})^{\zeta-\zeta_{loc}}}} \tag{12}$$

Thus, the main consequence of the anomalous roughening for the size effect in the case of large sizes is that the nominal strength is expected to decrease as $\sigma_N \sim L^{-1/2+(\zeta-\zeta_{loc})/2}$ (as shown in Fig. 3) instead of the theoretical result of LEFM $\sigma_N \sim L^{-1/2}$ [8]. This difference originates in the fact that, in the case of an anomalous roughening, the critical resistance to crack growth G_{RC} (2) is expected to evolve as a power law $G_{RC} \sim L^{\zeta-\zeta_{loc}}$ for large structure sizes (as shown in Fig. 1) while in the theoretical case of LEFM, the critical resistance G_{RC} is assumed constant. Nevertheless, in the case where there is no anomalous roughening, *i.e.* $\zeta = \zeta_{loc}$ [1], the theoretical size effect of LEFM is recovered.

Small-size asymptotic expansion of the size effect

In Bažant's theory [8], no size effect is expected for small structure sizes ($L \to 0$); this is the domain of the strength theory. A possible justification is that, in small structures, the fracture process zone occupies the entire volume of the structure and hence, there is no stress concentration and so no distinct fracture at maximum load.

Such an argument can be also obtained by substituting the real crack surface A_r created in small structures into the the fracture criterion (4). Indeed, in small structure sizes (*i.e.* $L \ll L_C$), the roughness being negligible, actual crack surfaces are not so different from the projected one : $A_r \simeq L\Delta a_{sat}$. Hence, from (4), the crack length increment Δa_{sat} (or in other terms the effective fracture process zone size) tends to the material length : $\Delta a_{sat} = n2\gamma/G_d = c^*$ for $L \ll L_C$. Thus, when the material length $c^* = d - a_o$ (Fig. 2), the fracture process zone occupies the entire ligament of the structure.
On the basis of Bažant's theory [8] and in order to obtain a small-size asymptotic expansion of the size effect, let us now introduce a new variable and a new function :

$$\eta = \frac{1}{\theta} = \frac{d}{\Delta a_{sat}}\,, \qquad \psi(\alpha_o, \eta) = \frac{g(\alpha_o + \theta)}{\theta} = \eta\, g(\alpha_o + 1/\eta) \tag{13}$$

The function $\psi(\alpha_o, \eta)$ corresponds to the dimensionless energy release rate function of the inverse relative size of the fracture process zone. Substituting (13) into (6) and expanding $\psi(\alpha_o, \eta)$ into Taylor series about the point $(\alpha_o, 0)$ since $lim\ \eta = 0$ when d or $L \to 0$, leads to the nominal strength :

$$\sigma_N = \sqrt{\frac{E\, G_{RC}}{c^*}} \left[\psi(\alpha_o, 0) + \psi_1(\alpha_o, 0)\,\eta + \psi_2(\alpha_o, 0)\,\frac{\eta^2}{2!} + \psi_3(\alpha_o, 0)\,\frac{\eta^3}{3!} + ... \right]^{-1/2} \tag{14}$$

$$= \sigma_{M'} \sqrt{\frac{\left(1 + (\frac{L}{L_C})^{2(\zeta - \zeta_{loc})}\right)^{1/2}}{1 + \frac{L}{L_2} + c_2 (\frac{L}{L_2})^2 + c_3 (\frac{L}{L_2})^3 + ...}} \tag{15}$$

where $\psi_1(\alpha_o, 0) = \partial\psi(\alpha_o, 0)/\partial\eta$, $\psi_2(\alpha_o, 0) = \partial^2\psi(\alpha_o, 0)/\partial\eta^2$, ..., and $c_2 = \psi_2(\alpha_o, 0)\psi(\alpha_o, 0)^2/(2\psi_1(\alpha_o, 0)^2)$, $c_3 = \psi_3(\alpha_o, 0)\psi(\alpha_o, 0)^3/(6\psi_1(\alpha_o, 0)^3)$, ..., and,

$$\sigma_{M'} = \sqrt{\frac{2\gamma E}{\psi(\alpha_o, 0) c^*}}\,, \qquad L_2 = c^*\, \frac{\psi(\alpha_o, 0)}{m\, \psi_1(\alpha_o, 0)} \tag{16}$$

are all constants. The small-size asymptotic series expansion of the size effect on the nominal strength (15) is plotted in Fig. 3. As shown in Fig. 3, the nominal strength tends to a constant when $L \to 0$ (*i.e.* $\sigma_{M'}$ as expected in the case of a strength theory) but diverges from the large-size asymptotic behavior of the size effect (9) for $L \to \infty$ (which coincides to the straight line of slope $-1/2 + (\zeta - \zeta_{loc})/2$).

Approximate size effect
Now, the main problem consists in interpolating between the large-size (9) and the small size (15) asymptotic series expansion in order to obtain an approximate size effect valid everywhere. In our case the intermediate behavior seems very difficult to determine because the the first two terms at the numerators of the two asymptotic expansions are different. Nevertheless, it is interesting to observe in Fig. 3 that a satisfactory approximate size effect can be obtained by truncating the small-size asymptotic series expansion after the linear term :

$$\sigma_N = \sigma_{max} \sqrt{\frac{\left(1 + (\frac{L}{L_C})^{2(\zeta - \zeta_{loc})}\right)^{1/2}}{1 + \frac{L}{L_o}}} \tag{17}$$

where the contants σ_{max} and L_o will be discussed in the following. Indeed, this size effect curve is seen to represent the transition between a horizontal asymptote, characterizing the strength theory for which there is no size effect, and a decreasing asymptote, corresponding to a power law of exponent $-1/2 + (\zeta - \zeta_{loc})/2$ characterizing the decrease of the LEFM size effect caused by the growth of the critical energy release rate for large structure sizes. A possible justification is that, in the large-size first order expansion (12), the second term in numerator has no influence on the large size asymptotic behavior of the size effect but induces a divergence at small sizes.
However, one limitation of the approximate size effect (17) is that the values of σ_M and $\sigma_{M'}$ (characterized by σ_{max}), and the values of L_1 and L_2 (characterized by L_o) would be surely different if these values were to be determined from large size data or from small size data. The approximate size effect (17) only gives the shape

of the size effect relation on the nominal strength but does not allow for a determination of the parameter values σ_{max} and L_o. Only the crossover length L_C and the scaling exponents ζ_{loc} and ζ are univoquely determined from the roughness analysis.

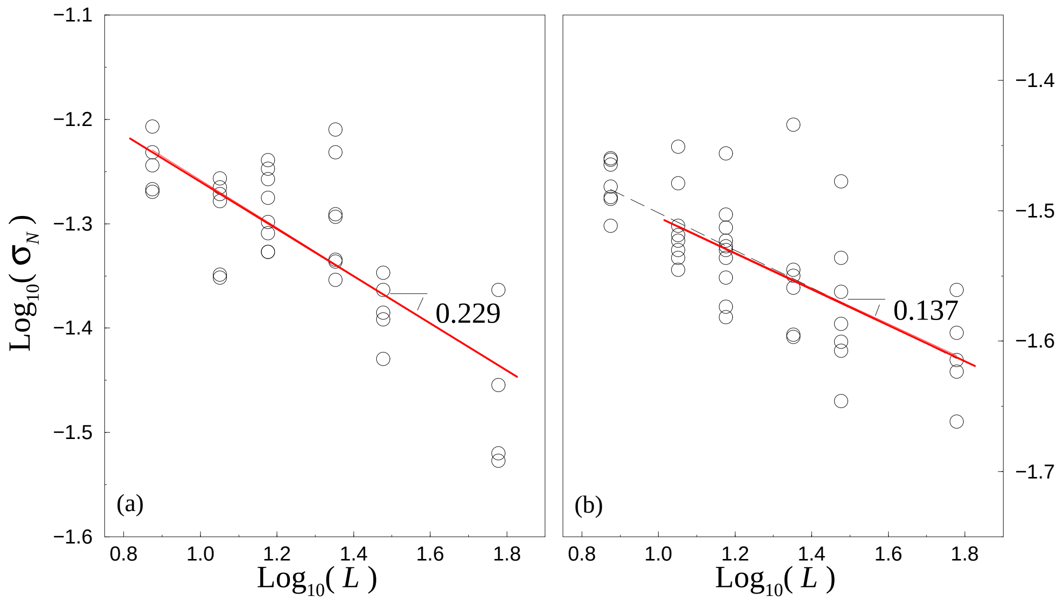

Figure 4: Size effect on nominal strength respectively for pine (a) and spruce (b). The expected slope from the roughness analysis [1] are $-\frac{1}{2}+\frac{\zeta-\zeta_{loc}}{2}=-0.265\pm0.085$ for pine (a) and -0.135 ± 0.085 for spruce (b).

On the other hand, it seems difficult to compare the crossover lengths L_1 and L_2, or in other terms L_o in Eq.17, to the crossover length L_C by means of their analytical expressions : the first one being deduced from a mechanical approach and the second one from a roughness analysis. However, from a physical point of view, both have the same meaning. For small structure sizes , *i.e.* $L \ll L_C$ or L_o, the energy release from the structure is negligible, while for large sizes, *i.e.* $L \gg L_C$ or L_o, the energy release is dominant. Hence, it seems reasonnable to assume that both crossover lengths are of the same order of magnitude (it is the case in Fig. 3 where it is assumed that $L_o = L_C$).

In Fig. 4, the nominal strength σ_N obtained from geometrically similar mode I fracture TDCB specimens is plotted versus the characteristic size L. With the assumption that $L_o = L_C$, it has been shown in [1] that the crossover length L_C was smaller than the sizes L of tested specimens. Hence, the size effect on nominal strengths is expected to evolve (see Eq. 17), as a power law $\sigma_N \sim L^{-1/2+(\zeta-\zeta_{loc})/2}$. The latter power law behavior is in good agreement with the observed experimental size effect in wood as shown in Fig. 4.

In conclusion, a modification of the Bažant's size effect law [8] has been proposed on the basis of the size effect on the energy release observed in the case of an anomalous roughening of fracture surfaces. It has been found that the approximate size effect relation (17) is slightly different from the *classical* size effect law [8] for large structure sizes, and predicts an asymptotic behavior $\sigma_N \sim L^{-1/2+(\zeta-\zeta_{loc})/2}$ instead of the size effect of LEFM, $\sigma_N \sim L^{-1/2}$. However, if the roughening of fracture surfaces is only slightly anomalous (*i.e.* $\zeta \to \zeta_{loc}$ [1]), the theoretical size effect of LEFM is recovered, which corresponds to the fracture behavior of a purely elastic brittle material.

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