RECENT DEVELOPMENTS IN THE WEIBULL STRESS MODEL FOR PREDICTION OF CLEAVAGE FRACTURE IN FERRITIC STEELS

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ABSTRACT

This paper reviews recent developments in the Weibull stress model for prediction of cleavage fracture in ferritic steels. The procedure to calibrate the Weibull stress parameters builds upon the toughness scaling model between two crack configurations having different constraint levels and eliminates the recently discovered non-uniqueness that arises in calibrations using only fracture toughness data obtained under small scale yielding (SSY) conditions. The introduction of a non-zero threshold value for Weibull stress in the expression for cumulative failure probability is consistent with the experimental observations that there exists a minimum toughness value for cleavage fracture in ferritic steels, and brings numerical predictions of the scatter in fracture toughness data into better agreement with experiments. The calibrated model predicts accurately the toughness distributions for a variety of crack configurations including surface crack specimens subject to different combinations of bending and tension.

Keywords: Cleavage fracture, Failure probability, Weibull stress, Calibration, Surface crack

1. INTRODUCTION—THE WEIBULL STRESS MODEL FOR CLEAVAGE FRACTURE

In the ductile-to-brittle transition (DBT) region of ferritic steels, transgranular cleavage initiated by slip-induced cracking of grain boundary carbides often triggers the brittle fracture event which results in catastrophic failure of structural components. Due to the highly localized character of the failure mechanism and microstructural inhomogeneity of the material, the cleavage fracture toughness often exhibits a large amount of scatter and a strong sensitivity to the local stress and deformation fields [1, 2]. This complicates greatly the interpretation of fracture toughness data to define meaningful values for application in fracture assessments of structural components, and has stimulated a rapidly increasing amount of research on micromechanical descriptions of the cleavage fracture process. The Weibull stress model originally proposed by the Beremin group [3] based on weakest link statistics provides a framework to quantify the relationship between macro and microscale driving forces for cleavage fracture. They introduced the scalar Weibull stress (σ_w) as a probabilistic fracture parameter, computed by integrating a weighted value of the maximum principal (tensile) stress over the process zone of cleavage fracture (i.e., the crack front plastic zone). The Beremin model adopts a two-parameter description for the cumulative failure probability

$$P_f(\sigma_w) = 1 - \exp\left[-\left(\frac{\sigma_w}{\sigma_u}\right)^m\right], \tag{1}$$

with

$$\sigma_w = \left[\frac{1}{V_0}\int_{\overline{V}} \sigma_1^m dV\right]^{1/m}. \tag{2}$$

Here parameters m and σ_u denote the Weibull modulus and the scale parameter of the Weibull distribution. Moreover, m defines the shape of the probability density function for microcrack size [3]. In Eq. (2), $\overline{V}$ represents the volume of the cleavage fracture process zone, V_0 defines a reference volume to normalize the integral units and σ_1 is the maximum principal stress acting on material points inside the fracture process zone.

The Weibull stress thus defines a local, crack front parameter to couple remote loading with a micromechanical model that incorporates the microcracks in a weakest link philosophy. Under increased remote loading described by J (or K_J), differences in evolution of the Weibull stress, $\sigma_w = \sigma_w(J)$, reflect the potential strong variations in crack-front fields due to the effects of constraint loss and volume sampling. The inherently 3D formulation for σ_w defined by (2) readily accommodates variations in J (or K_J) along the crack front in a weighted sense.

The Weibull stress concept enables construction of a toughness scaling model between crack configurations exhibiting different constraint levels. Based on equal probabilities of fracture, the scaling model requires the attainment of the same Weibull stress value to trigger cleavage in different specimens, even though the J values may differ widely [4, 5, 6]. The quantitative relationships enable the simple transfer (or scaling) of critical J values from one geometry and loading condition to another to accommodate constraint and volume sampling differences.

2. CALIBRATION OF THE WEIBULL STRESS MODEL

The applicability of the Weibull stress model to predict failure probability and/or to scale fracture toughness values between different crack configurations relies on the calibrated values of Weibull parameters, m and σ_u. Initial efforts to calibrate Weibull parameters used notched (round) bars tested at lower-shelf temperatures (e.g., [3]) and assumed identical values apply for defect assessments in the DBT region. Anticipating that m and σ_u may very well depend on temperature, stress-strain gradient and plastic strain levels present in cracked components operating in the DBT region, many researchers (e.g., [7]) proposed to calibrate m and σ_u using fracture toughness data measured at a temperature comparable to the application.

Previous studies have shown that fracture toughness values in the DBT region follow a Weibull distribution, $P_f(J_c) = 1 - \exp[-(J_c/\beta)^\alpha]$, where β defines the toughness value at 63.2% failure probability and α quantifies the scatter. Reliable estimates of the Weibull slope α require a large number of measured toughness values (J_c) in the data set while as few as 6-10 J_c-values suffice to establish the characteristic toughness value β with high confidence levels [8]. The conventional calibration method employs an iterative procedure to determine m and σ_u such that the micromechanics model (1) predicts the measured toughness distribution. Because the calibrated values of m and σ_u depend on the measured toughness distribution (i.e., both α and β), the experimental data set must contain a large number of J_c-values. However, most experimental data sets do not satisfy this requirement. Experimental sets of J_c-values often have limited number of specimens (6-10) and therefore, large uncertainties must be expected in m and σ_u determined in this manner. Moreover, experimental programs usually employ deep-notch SE(B) specimens or C(T) specimens which fail under small scale yielding (SSY) conditions. Both theoretical studies and experimental results suggest that $\alpha = 2$ for SSY. Gao *et al.* [5] have shown that the conventional calibration method leads to non-unique values of m and σ_u under SSY conditions, i.e., many (m, σ_u) pairs can be found such that the microscopic model (1) predicts the same failure probability as the macroscopic model.

Recognizing these problems of the conventional calibration method, we propose a new approach to calibrate m and σ_u [5]. This approach requires testing of two sets of specimens giving rise to different constraint levels at fracture (e.g., SE(B) specimens with different a/W ratios). By using the toughness scaling model based on σ_w, the calibration process seeks the m-value which corrects the two sets of fracture toughness data to have the same statistical properties under SSY conditions, i.e., the two constraint-corrected SSY toughness distributions have the same β-value. A maximum likelihood estimate of β for the constraint-corrected SSY toughness distribution uses the theoretical value for α ($=2$), and 6-10 J_c-values in each set are sufficient to obtain β with high confidence. Once m is determined, σ_u is just the computed Weibull stress value at $J=\beta$ in the SSY configuration with the specified reference thickness.

In contrast to the conventional calibration method which attempts to find the values for m and σ_u by curve fitting the predicted P_f *vs.* J distribution to the experimental data, the new procedure adopts a fracture mechanics basis rather than a purely numerical fitting process. This approach has been successfully applied to calibrate m and σ_u for several ferritic steels [5, 9, 10].

3. THE THRESHOLD σ_w FOR CLEAVAGE FRACTURE

The Weibull stress model defined by Eqs (1-2) represents a pure weakest link description of the fracture event. This two-parameter model describes the unconditional cleavage probability that assumes no microcracks arrest (macroscopic cleavage fracture occurs once the critical microcrack experiences propagation). However, the unconditional probability has significant shortcomings to predict cleavage fracture [5, 11]. First, it implies that a very small K_I (stress intensity factor due to applied load) leads to a finite failure probability, which is not true in reality. Cracks cannot propagate in polycrystalline metals unless sufficient energy exists to break bonds, to drive the crack across grain boundaries and to perform plastic work. Consequently, there must exist a minimum toughness value ($K_{\min}$) below which cracks arrest. $K_{\min}$ has an experimentally estimated value of 20 $\text{MPa}\sqrt{\text{m}}$ for common ferritic steels under SSY conditions, independent of the crack front length. The value of $K_{\min}=20$ $\text{MPa}\sqrt{\text{m}}$ has been adopted by ASTM E-1921 [8]. Second, the unconditional probability often over-estimates the measured scatter of fracture toughness (see Anderson *et al.* [11] and Gao *et al.* [5] for examples).

Some researchers (e.g., Bakker and Koers [12], Xia and Shih [13], and others) introduce a threshold stress (σ_{th}) into computation of the Weibull stress to reflect the observed macroscopic threshold toughness. One such proposal for the integrand to compute the Weibull stress has the form $(\sigma_1-\sigma_{th})^m$. But rational calibration procedures for σ_{th} remain an open issue. Moreover, introduction of σ_{th} into the Weibull stress expression does not imply the existence of $K_{\min}>0$. A finite value of σ_w (and thus a finite value of failure probability) exists at a very small K-value even though $\sigma_{th}>0$ is introduced in the Weibull stress formulation.

To introduce an explicit threshold toughness into the Weibull stress model, we propose a modified form for Eq. (1) given by

$$P_f(\sigma_w) = 1 - \exp\left[-\left(\frac{\sigma_w - \sigma_{w-\min}}{\sigma_u - \sigma_{w-\min}}\right)^m\right], \tag{3}$$

where $\sigma_{w-\min}$ represents the minimum σ_w-value at which macroscopic cleavage fracture becomes possible. Consistent with the definition of $K_{\min}$, we define $\sigma_{w-\min}$ as the value of σ_w calculated at $K=K_{\min}$ in the (plane strain) SSY model, where the SSY model has a thickness equal to the configuration of interest for which (3) is applied. Therefore, calibration of $\sigma_{w-\min}$ is straightforward and does not require any additional experimental data. According to this three-parameter Weibull stress model (3), the toughness scaling model between specimens having different geometries and loading conditions should be constructed at identical $\bar{\sigma}_w$-values, where $\bar{\sigma}_w=\sigma_w-\sigma_{w-\min}$. Gao *et al.* [5] and Gao and Dodds [6] provide detailed discussions about the three-parameter Weibull stress model and the toughness scaling method based on Weibull stress with $\sigma_{w-\min}>0$.

4. PREDICTION OF CLEAVAGE FRACTURE IN A PRESSURE VESSEL STEEL

This section describes an application of these recent developments in modeling cleavage fracture to predict the behavior for various crack configurations of an A515-70 pressure vessel steel, including surface crack specimens loaded by different combinations of tension and bending. Joyce and Link [14] and Tregoning (see Gao *et al.* [9]) recently performed extensive fracture tests on this material in the DBT region. The material has a Young's modulus of 200 GPa, Poisson's ratio of 0.3 and yield stress of 280 MPa at -7^oC and 300 MPa at -28^oC. Twelve plane-sided 1T C(T) specimens ($a/W = 0.6$) were tested at -28^oC and twelve plane-sided 1T SE(B) specimens ($a/W = 0.2$, $B \times 2B$ cross-section) were tested at -7^oC. In addition, seven bolt-loaded and seven pin-loaded surface crack specimens were tested at -7^oC. The pin-loaded specimen experiences a higher bending moment whereas the bolt-loaded specimen experiences predominantly tensile loading. All specimens failed by cleavage without prior macroscopic ductile tearing.

Fracture toughness data for the deep-notch C(T) specimens and the shallow-notch SE(B) specimens are used to calibrate the Weibull stress parameters. Because the C(T) specimens and the SE(B) specimens have different test temperatures, toughness values for the C(T) specimens are needed at -7^oC. Here, we employ the "master curve" approach of ASTM E-1921 [8] to adjust the C(T) toughness values for the temperature change. The "master curve" for ferritic steels makes possible the prediction of median fracture toughness (for 1T thickness) at any temperature in the transition region, provided the reference temperature (T_0) for the material has been determined from SSY fracture toughness data at a single temperature. The calibration for m is as follows: 1) Assume an m-value and compute the σ_w *vs.* K_J history for C(T) and SSY (plane strain) configurations respectively using the material flow properties at -28^oC. Scale the measured toughness values for C(T) specimens to the SSY configuration. Determine T_0 using the constraint corrected toughness values and estimate K_0 (K_J at 63.2% failure probability) at -7^oC (denote as K_0^A) according to ASTM E-1921; 2) Compute the σ_w *vs.* K_J history for SE(B) and SSY (plane strain) configurations respectively at -7^oC. Scale the measured toughness values for SE(B) specimens to the SSY configuration. Estimate K_0 for the constraint corrected toughness distribution and denote it as K_0^B; 3) Define an error function as $R(m) = (K_0^B - K_0^A)/K_0^A$. If $R(m) \neq 0$, repeat the above steps for additional m-values. The calibrated Weibull modulus, $m = 11.2$, makes $R(m) = 0$ for the A515-70 steel tested at the current conditions. After m is calibrated, $\sigma_{w-\min}$ and σ_u can be easily determined. At -28^oC, the values of $\sigma_{w-\min}$ and σ_u corresponding to the thickness of the C(T) specimen are 790 MPa and 1378 MPa. At -7^oC, the values of $\sigma_{w-\min}$ and σ_u corresponding to the thickness of the SE(B) specimen are 741 MPa and 1435 MPa.

Figure 1 compares the predicted failure probabilities for the C(T) and SE(B) specimens using the calibrated Weibull stress model (3) with the median rank probabilities for the measured J_c values. The dashed lines indicate the 90% confidence limits for the estimates of the experimental rank probabilities. To compute these confidence limits, we assume that the (continuous) P_f values from Eq. (3) provide the expected median rank probabilities for an experimental data set containing the number of measured J_c-values. The calibrated Weibull stress model predicts accurately the shape of the toughness distribution and captures the strong constraint effect on fracture toughness.

Finally, we apply the calibrated three-parameter Weibull stress model to predict the cumulative failure probability for cleavage fracture of the tested surface crack specimens. Because the crack front length of the surface crack specimen equals to $1.67 \times$ the crack front length of the SE(B) specimen, the values of $\sigma_{w-\min}$ and σ_u for surface crack specimens are slightly different from those for SE(B) specimens. Here, $\sigma_{w-\min} = 776$ MPa and $\sigma_u = 1470$ MPa. The model predictions capture the measured toughness distributions for both bolt-loaded and pin-loaded specimens, see Fig. 2, where the J-values for plotting are computed at the center-plane (the deepest point on the crack front). The pin-loaded specimen has a greater bending load and thus exhibits a higher failure probability at the same J-level

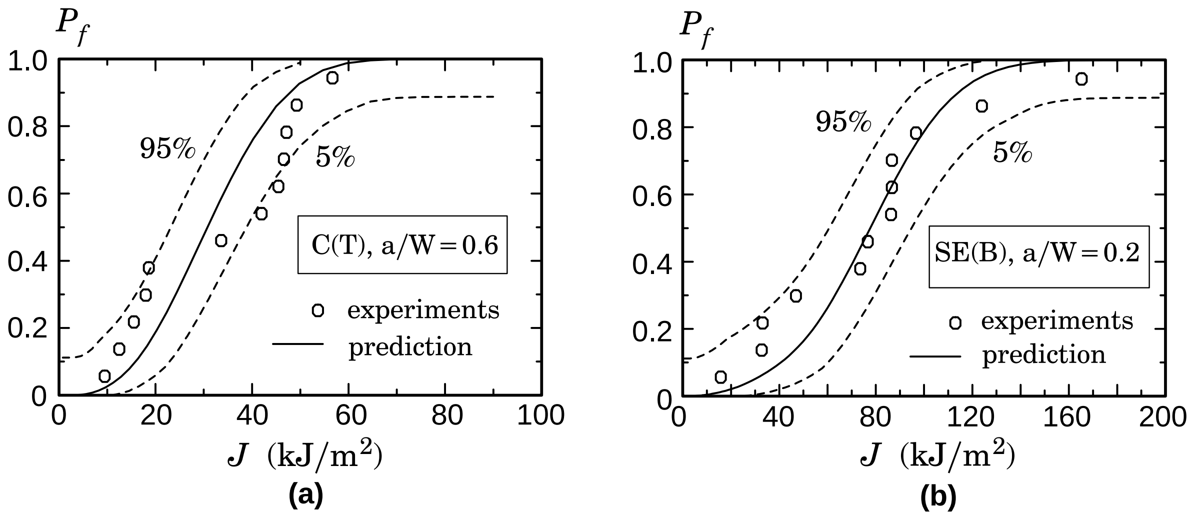

Fig. 1. Comparison of predicted cleavage probabilities (solid lines) with rank probabilities for measured J_c-values (symbols). The dashed lines represent the 90% confidence limits for the median rank probabilities. (a) deep-notch C(T) specimens; (b) shallow-notch SE(B) specimens.

compared to the bolt-loaded specimen. Fig. 2 shows two curves for the predicted failure probabilities of the bolt-loaded specimens. In Eq. (2), the principal stress (σ_1) value appearing in the Weibull stress integral can be assigned the current value at the loading level (J) or the maximum value experienced by the material point during the loading history. Of the four crack configurations examined in this work, the choice of σ_1 definition makes a difference only for the bolt-loaded surface crack specimen as shown. Consequently, the calibrated values of m, σ_u and $\sigma_{w-\min}$ do not depend on the choice of σ_1 definition. Constraint loss in the bolt-loaded configuration leads to a slight decrease in near-front stresses under large scale yielding, and thus use of the maximum σ_1 values raises the failure probability. Stresses have smaller values under large scale yielding but the process zone volume for cleavage continues to grow with crack front blunting which leads to monotonically increasing failure probabilities. The prediction that includes the history effect provides a slightly better agreement with the experimental data for this very low constraint configuration.

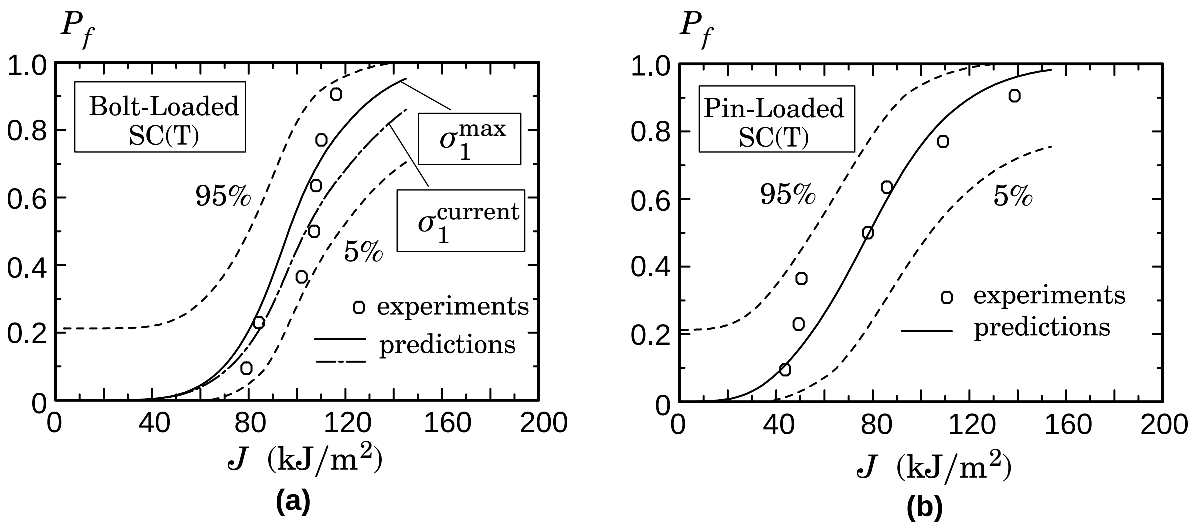

Fig. 2. Comparison of predicted toughness distributions (solid lines) for both bolt-loaded and pin-loaded SC(T) specimens with experimental data. The symbols represent rank probabilities for the measured J_c-values and the dashed lines represent the 90% confidence limits for the rank probabilities. (a) bolt-loaded specimens; (b) pin-loaded specimens.

5. CONCLUDING REMARKS

This work applies recent developments in the Weibull stress model to predict cleavage fracture in an A515-70 pressure vessel steel. The procedure to calibrate the Weibull stress parameters builds upon the toughness scaling model between two crack configurations having different constraint levels and exhibits very strong sensitivity to m. It eliminates the recently discovered non-uniqueness that arises in calibrations which use only deep-notch SE(B) or C(T) data. The calibrated Weibull modulus for the A515-70 steel at -7^oC is $m = 11.2$. The introduction of a non-zero threshold value for Weibull stress ($\sigma_{w-\min}$) in the expression for cumulative failure probability reflects an approximate treatment of the conditional probability of propagation in the DBT region and is consistent with the experimental observations that there exists a minimum toughness value for cleavage fracture in ferritic steels. It brings numerical predictions of the scatter in fracture toughness data into better agreement with experiments. Calibration of the threshold Weibull stress makes use of the generally accepted, minimum toughness value for ferritic steels and requires no additional experimental effort. The calibrated three parameter Weibull stress model accurately predicts the toughness distributions for all specimen configurations and captures the strong constraint effect on cleavage fracture due to differences in crack geometry and loading mode (bending *vs*. tension).

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