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PREDICTION OF PROBABILITY DISTRIBUTION OF NOTCH STRENGTH OF CERAMICS

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Abstract Based on the expression for notch strength of brittle material and the fundamental principle of probability theory, correlations between the statistical characteristic parameters of notch strength and those of flexural strength of ceramics are obtained and checked by test results. It's shown that for the normal distribution, the mean value and the standard deviation of the notch strength are, respectively, K_t times less than those of flexural strength, where K_t is the theoretical stress concentration factor. For the log-normal distribution, the mean value of logarithm of notch strength is equal to that of flexural strength minus the logarithm of K_t, and the standard deviation of the logarithm of notch strength keeps equal to that of flexural strength as K_t increases. For Weibull distribution, the product of characteristic value, i.e. the scale parameter of Weibull distribution of notch strength and K_t is equal to the scale parameter of Weibull distribution of flexural strength and the shape parameter of notch strength keeps constant as Kt increases. Based on the above results, procedures are developed to predict the probability distribution of notch strength of ceramics from that of the flexural strength. It is shown that the notch strength of ceramics follows the normal distribution, the log-normal distribution and Weibull distribution, and all of the above distributions can be predicted. The predicted results of three probability distributions of notch strength of ceramics mentioned above are in good agreement with test results. The possibility of predicting the probability distribution of notch strength of ceramics is of practical importance in the design and reliability assessment of ceramic structure elements.

Keywords: ceramics, notch strength, probability distribution, prediction

1. Introduction

Discontinuity of profile always exists in structural elements due to the need of joining and the structure design, and may be regarded as notch where the stress concentration occurs. So, it's necessary and important to investigate and predict the notch strength and its probability distribution in order to meet the requirement of the evaluation of strength and reliability assessment of ceramics elements[1-7].

Ref.[6] offered a method to predict the strength distribution of the notched specimen of ceramics from the crack length distribution, however, because the critical crack length and the effective surface in both notched and smooth (plane) specimens are difficult to be determined by experiment, the method is hard to be applied in engineering. In addition, the results obtained in ref.[6] are doubtful that the notch strength increases with the increasing of stress concentration factor of notch specimen of ceramics.

In the present study, different test results of notch strength of alumina ceramics given in ref.[7] are analyzed based on the formula for the notch strength of brittle materials given in ref.[8] and the general principle of probability theory. Furthermore, the correlations between the characteristic parameters of probability distribution of the notch strength and those of the flexural strength are obtained, which offers the possibility of predicting the probability distribution of notch strength of ceramics from test results of flexural strength of smooth specimens. So it is of practical importance in the design and the reliability assessment of ceramics element, without the need to determine the notch strength of specific stress concentration factor, which are expensive and time consuming.

2. Expression for notch strength of ceramics

Based on the assumptions that the crack initiation at notch tip may occur due to the fracture of a hypothetical material element at notch root and the fracture of notched element occurs right after crack initiation without any sub-critical crack propagation because of the low values of K_{IC} and the critical crack length of brittle materials, a formula for notch strength of brittle materials was developed as follows[8]:

$$\sigma_{bN}=\sigma_f/K_t \qquad (1)$$

where σ_f is the fracture strength of brittle material, K_t is the theoretical stress concentration factor, σ_{bN} is the fracture stress of notched element, i.e. notch strength.

Equ (1) has been checked by the test result of notch strength of brittle metal[8]. Wang, et al found that the test data of notch strength and fracture strength of ceramics under bending condition also followed the above-mentioned equation[3]. But under bending condition, σ_f in eqn.(1) should be replaced by the flexural strength of ceramics ,σ_{bb} ,as shown in eqn.(2) :

$$\sigma_{bN}=\sigma_{bb}/K_t \qquad (2)$$

eqn.(2) shows the quantitative relationship between the flexural strength and the bending notch strength of ceramics. So, the quantitative relationship between the probability distribution of notch strength and that of flexural strength of ceramics can be achieved by using eqn.(2).

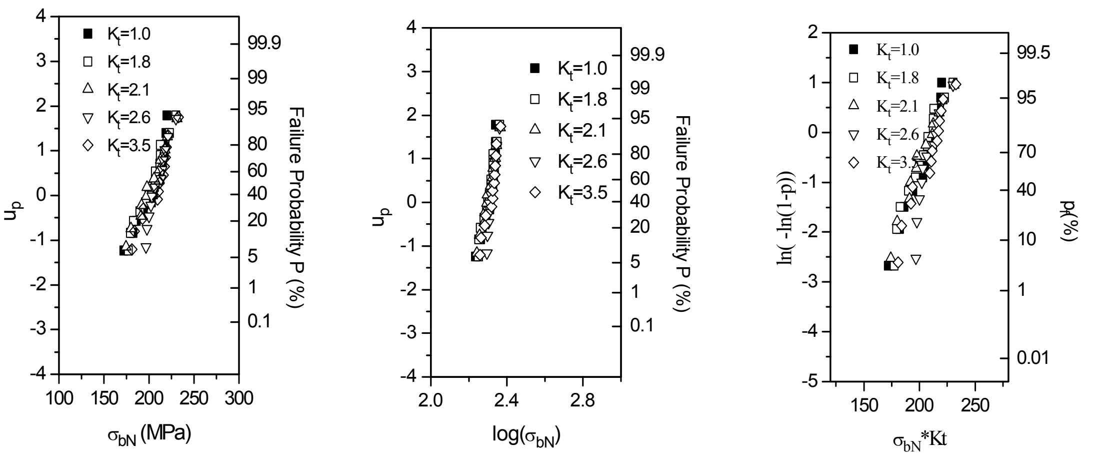
3. The probability distribution of flexural strength and notch strength of brittle ceramics

Eqn.(2) can be re-written as followings:

$$\sigma_{bN}*K_t=\sigma_{bb} \qquad (3)$$

It can be seen that $\sigma_{bN}*K_t$ follows the same probability distribution as that of σ_{bb}.

Generally, it was thought that the fracture strength of brittle ceramics follows Weibull distribution[6,7,9]. In ref.[3] , it is found that the flexural strength of ceramics also follows the normal distribution on the basis of statistical analysis of the test results of strength of alumina ceramics when the number of sample is small. Actually, re-analysis on the test data contain 127 samples[9] shows that the test results of the flexural strength of ceramics can simultaneously follow the normal distribution, the log-normal distribution and Weibull distribution. So, $\sigma_{bN}*K_t$ also follows the normal distribution , the log-normal distribution and Weibull distribution respectively.



(a) normal distribution (b)log-normal distribution (c) Weibull distribution

Fig.1 The probability distribution of unified notch strength and flexural strength of alumina ceramics

In ref.[7], the test results of the fracture strength of smooth specimen and notch strength of notched specimen of alumina ceramics under bending loading condition are given. After unifying by using the notch strength to times the stress concentration factor of the corresponding notch specimen, $\sigma_{bN}*K_t$ and σ_{bb} are simulataneously plotted in the same probability paper as shown in Fig.1. As may be seen from Fig.1, the unified notch strength and flexural strength of alumina ceramics almost form a straight line in the same probability paper, therefore, under the condition of normal distribution, log-normal distribution and Weibull distribution respectively, $\sigma_{bN}*K_t$ and σ_{bb} follows the same probability distribution as shown by eqn.(3). The above result is different from the results obtained in ref.[6] where the notch strength increases with the increasing of stress concentration factor of notch specimen of ceramics, so the unified notch strength.formed different lines from that of the flexural strength.

4. Prediction of probability distribution of notch strength from that of flexural strength

Since the dimension, direction and distribution of the defect in brittle ceramics are random, σ_{bb} and σ_{bN} in eqn.(3) are random variables. From ref.[10], it can be seen that the variations in the notch depth and notch root radius have little effect on the value of K_t. Thus, K_t can be taken as a constant .As a result, σ_{bN} can be regarded as a linear function of the random variable σ_{bb}, denoted as $\sigma_{bN}=\varphi(\sigma_{bb})$. The probability distribution density function of flexural strength such as $f(\sigma_{bb})$ can be relatively easy determined by the test result of smooth specimen. Then, on basis of eqn.(3) and the feature for distribution density function of linear function of random variable[11], the distribution density function of notch strength σ_{bN} can be determined as follows:

$$\phi(\sigma_{bN}) = |g(\sigma_{bN})| \bullet f(g(\sigma_{bN})) \quad (4)$$

where $g(\sigma_{bN})$ is the invert function of $\varphi(\sigma_{bb})$. As a result, the corresponding characteristic parameters of probability distribution of notch strength can be obtained.

When σ_{bb} follows the normal distribution, i.e. $\sigma_{bb} \sim N(\mu, \sigma^2)$,and its distribution density function is:

$$f(\sigma_{bb}) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(\sigma_{bb}-\mu)^2}{2\sigma^2}\right) \quad (5)$$

where μ and σ are, respectively, the mean value and standard deviation of the population of variable σ_{bb} . Then the distribution density function of σ_{bN} can be obtained according to eqn.(4):

$$\phi(\sigma_{bN}) = \frac{1}{\sqrt{2\pi}\left(\sigma/K_t\right)} \exp\left(-\frac{\left(\sigma_{bN}-\mu/K_t\right)^2}{2\left(\sigma/K_t\right)^2}\right) \quad (5)$$

Comparison of eqn.(5) with eqn.(6) tells that $\sigma_{bN} \sim N\left(\mu/K_t, \sigma^2/K_t^2\right)$ and the mean value and standard deviation of the population of variable, σ_{bN} , are, respectively, μ/K_t and σ/K_t , which are respectively, equal to $1/K_t$ times of those of σ_{bb} .

From eqn.(3) and the definition of the mean value and the standard deviation of the sample[21], the following equations should be satisfied:

$$\overline{\sigma}_{bN} = \overline{\sigma}_{bb}/K_t \quad (7)$$

$$s_{bN} = s_{bb}/K_t \quad (8)$$

where $\overline{\sigma}_{bN}$ and s_{bN} are, respectively, the mean value and the standard deviation of the sample of notch strength, $\overline{\sigma}_{bb}$ and s_{bb} are those for flexural strength, respectively. Since σ_{bb} and σ_{bN} both follow the normal distribution, and $\overline{\sigma}_{bb}$ (or $\overline{\sigma}_{bN}$) and s_{bb} (or s_{bN}) can be taken as the estimated values for μ (or μ/K_t) and σ (or σ/K_t) respectively. Thus, eqn.(7) and eqn.(8) are the expressions for characteristic parameters of probability distribution of notch strength of ceramics, which are both expressed by the corresponding characteristic parameters of flexural strength of smooth specimen divided by K_t .

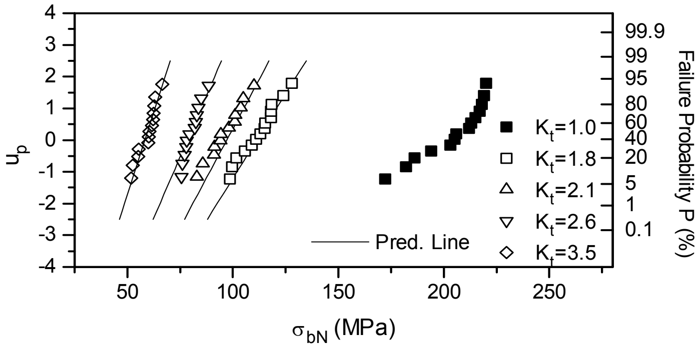
After logarithmic transformation, eqn.(3) becomes:

$$\log\sigma_{bN} = \log\sigma_{bb} - \log K_t \quad (9)$$

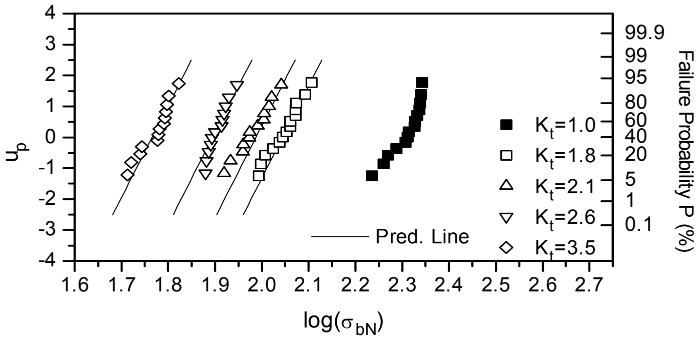
It indicates that $log\,\sigma_{bN}$ is still a linear function of the random variable $log\,\sigma_{bb}$. Therefore, σ_{bN}, the notch strength of ceramics should follow the log-normal distribution when σ_{bN} , the flexural strength of ceramics follows the log-normal distribution. Thus, it can be deduced from the similar analysis given in the above section that the characteristic parameters of log-normal distribution, such as the mean value and the standard deviation, of two random variables, i.e. σ_{bb} and σ_{bN} , should be correlated as follows:

$$\overline{\log\sigma_{bN}} = \overline{\log\sigma_{bb}} - \log K_t \tag{10}$$

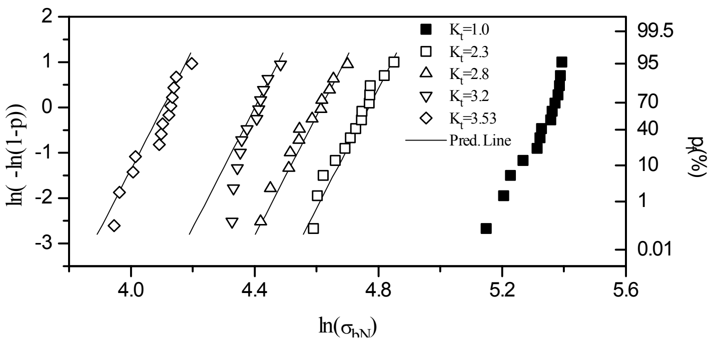
$$S_{log\,\sigma_{bN}} = S_{log\,\sigma_{bb}} \tag{11}$$



(a) normal distribution



(b)log-normal distribution



(c) Weibull distribution

Figure 2 The predicted lines of normal distribution, log-normal distribution, Weibull distribution and test results[7] of notch strength of alumina ceramics at K_t=1.8, 2.1, 2.6, and 3.5

When σ_{bb} follows the 2-parameter Weibull distribution , its density function can be expressed as:

$$f(x) = \frac{m}{\sigma_0}\left(\frac{x}{\sigma_0}\right)^{m-1} \exp\left[-\left(\frac{x}{\sigma_0}\right)^m\right] \tag{12}$$

where σ_0 is the scale parameter, i.e. the characteristic value of flexural strength when failure probability is 63%, and m is the Weibull modulus. According to eqn.(4), the density function of σ_{bN} is:

$$\varphi(\sigma_{bN}) = \frac{m}{\sigma_0/K_t}\left(\frac{\sigma_{bN}}{\sigma_0/K_t}\right)^{m-1} \exp\left[-\left(\frac{\sigma_{bN}}{\sigma_0/K_t}\right)^m\right] \tag{13}$$

Comparison of eqn.(21) with eqn.(22) suggests that σ_{bN} follows Weibull distribution equally. Let σ_{0N} and m_N be the characteristic value and Weibull modulus of notch strength respectively, then we have:

$$\sigma_{0N} = \sigma_0 / K_t \tag{14}$$

$$m_N = m \tag{15}$$

Based on the test results of the fracture strength of smooth specimen given in ref.[7] and its probablity distribution shown in Fig.1, the characteristical parameters of the flexural strength for normal distribution, log-normal distribution and Weibull distribution can be obtained respectively. Therefore the corresponding characteristical parameters of normal distribution, log-normal distribution and Weibull distribution of notch strength of notched specimen of alumina ceramics under bending loading condition can be determined according to the eqn.(7)-(8), eqn.(10)-(11) and eqn.(14)-(15) respectively, thereby, the probability distribution of notch strength of alumina ceramics can be predicted. Fig.2 show the test results of notch strength of alumina ceramics and the predicted probability distributions. As may be seen that the predicted probability distributions are in good agreement with the test results of notch strength of alumina ceramics. Therefore, the structure design and reliability assessment of notched elements made of ceramics can be simplified by using the conventional tests of fracture strength of smooth specimens instead of the costly and time consuming test of notched specimens of ceramics.

5. Conclusions

(1) The flexural strength of smooth specimens is K_t times of notch strength of ceramics, where K_t is the stress concentration factor of the notch specimen.

(2) Both notch strength and flexural strength under bending condition can simultaneously follow the normal distribution, the log-normal distribution and Weibull distribution.

(3) The characteristic parameters of the probability distribution function of notch strength can be determined from that of flexural strength and the stress concentration factor Kt.

(4) The normal distribution, the log-normal distribution and Weibull distribution of notch strength of ceramics can be predicted from that of flexural strength and the predicted results are in good agreement with test results.

(5) The possibility of predicting the probability distribution of notch strength of ceramics from test results of flexural strength of smooth specimens is of practical importance in the design and the reliability assessment of ceramics element, without the need to determine the notch strength of specific stress concentration factor, which are expensive and time consuming.

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