ICF100322OR

PLASTIC AND BUCKLING BEHAVIORS OF TUBULAR BEAMS UNDER COMBINED BENDING AND TORSION

G. H. Nie, R. Shi and R.J. Zhang

Key Laboratory of Solid Mechanics of MOE, Department of Engineering Mechanics,
Tongji University, Shanghai 200092, P.R. China

ABSTRACT

This paper presents theoretical, numerical and experimental analyses on plastic collapse and buckling behaviors of tubular cantilever beams under combined bending and torsion. A special set-up is designed and used to carry out experimental analysis for different ratios of bending and torsion (*M/T*=infinity, 2,4/3,1,0, respectively). All of tubular beam specimens are metal tubes of outer diameter 50mm and wall thickness 1.5mm. A concentrated force causes the combined load, and changing the torque arm thus induces various twisting moments. The two characteristic relations between the force and the transverse displacement at the free end (tip deflection), and between twisting moment and corresponding angle of twist have been measured. Local buckling occurring around the root of beam has been observed. On the other hand, based on the principle of complementary energy, a beam-model is proposed and applied to acquire the corresponding characteristic relations which are correlative to a load parameter (i.e. ratio of bending and torsion). Meanwhile, finite element method is used and a four-node shell-type element is chosen to predict buckling modes of the tubular beams. The computational results are compared with that of theoretical model and experimental data. Finally, some discussions on resulting errors are also given.

KEYWORDS

Tubular beams, bending and torsion, plasticity, buckling.

INTRODUCTION

Recently tubular beams or thin-walled circular tubes have been widely used in pipe networks, offshore and bridge structures. Compared with behaviors of solid cross-sectional beams, tubular beams show very different behaviors. For example, for tubular beams under bending [1,2], there will be cross-sectional flattening or ovalization, and local kinks occurring on the compression side. These phenomena can not be observed in the beams with solid cross-sections. For tubular beams under combined bending and twisting

[3,4], the elastic and plastic behaviors have been studied. It appears that there exists little work on plastic collapse and buckling behaviors of thin-walled circular tubes.

The present work presents plastic and buckling behaviors of tubular cantilever beams under combined bending and torsion. Experimental analysis for different ratios of bending and torsion has been performed. Observations on deformations and buckling are illustrated and experimental data are compared with the results by both finite element method and a simple beam model.

EXPERIMENTS

Material Properties

All tubular specimens are circular, metallic tubes of outer diameter 50mm and wall thickness 1.5mm. The stress-strain relation is obtained from tension test, as shown in Figure1.

A Designed Special Set-up

Figure 2 display a special set-up designed for combined bending and twisting. The length of tubular beam is 240mm, its one end is clamped in a supporting block mounted onto the base of the testing machine while the other end is connected to a rigid torque arm of length 360mm. A concentrated force causes the combined load, and changing the torque arm thus induces various twisting moments.

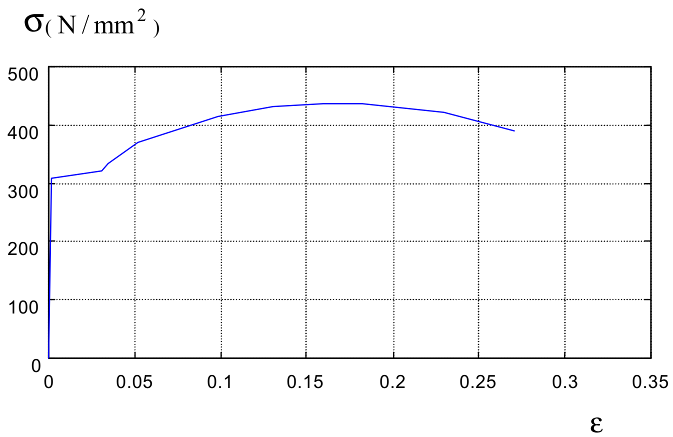

Figure 1: The stress-strain relation

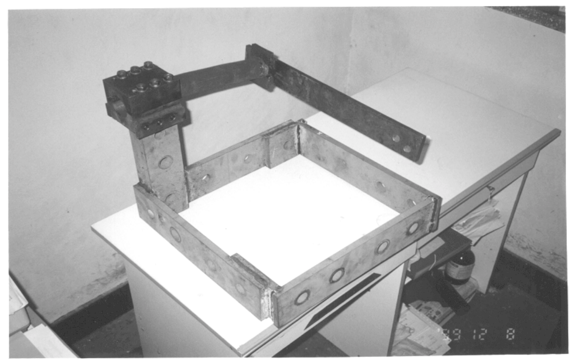

Figure 2: A designed set-up for experiment

Experimental Results

The curves for the concentrated force (P) and vertical displacement (δ), and twisting moment (T) and angle (φ) with the axis of the tube are illustrated in Figures 3 and 4 respectively. The values of half wave-lengths and angles of buckles are listed in Table 1. For two cases of pure bending or pure torsion, calculations of half wave-lengths of buckles are based on analyses by Yu *et al.* [1] and Yamaki [5] respectively. The computational results of angles of buckles are determined by using $\psi = \frac{1}{2}\arctan(T/M)$. It can be observed that a buckle is formed and developed in the region near the root when the value of force increases to a certain value. The cross-section of the tube will change eventually to be approximately oval. Figures 5 and 6 present deformations and buckling of tubular beams for various ratios of bending and torsion, i.e., $\infty, 2, 4/3, 1, 0$.

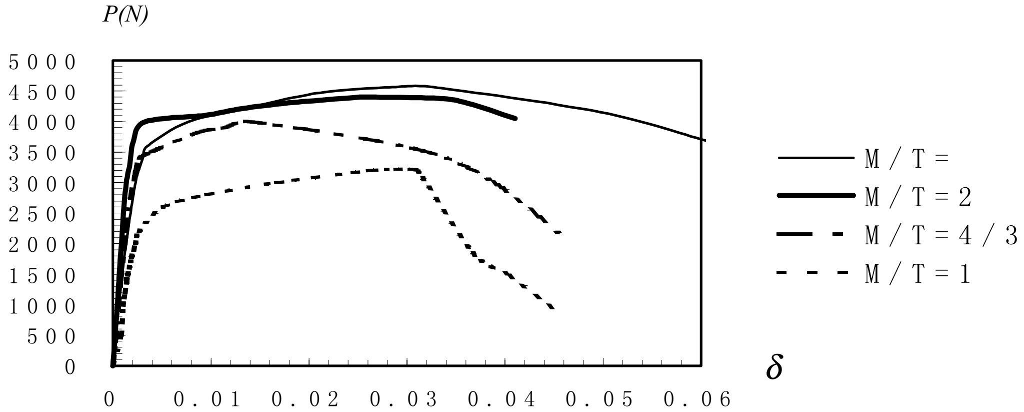

Figure 3: $P-\delta$ curves

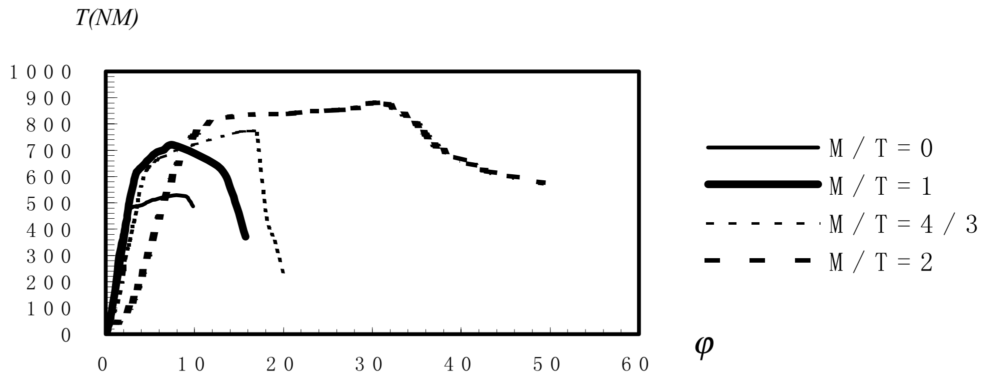

Figure 4: $T-\varphi$ **curves**

TABLE 1

HALF WAVE LENGTHS AND ANGLES OF BUCKLES

M/T		∞	2	4/3	1	0
λ (mm)	experiment	6.2	8	12	13	24
	theory	6				24
$\psi\,(^0)$	experiment	0	11-13	16-18	20-22	40-45
	theory	0	13.3	18.4	22.5	45

A BEAM MODEL FOR ANALYSIS OF PLASTIC BEHAVIOR OF TUBULAR BEAMS

A commonly used plane hypothesis is applied for the tubular beam. According to stress-strain relations for pure tension or pure shear of the used specimen, the expression for complementary energy per unit volume of the tube can be obtained from the following formula

$$U^* = \int_0^{\sigma} \varepsilon \, d\sigma + \int_0^{\tau} 2\gamma \, d\tau \tag{1}$$

Figure 5: Front view for ratios $M/T=\infty,2,4/3,1,0$ (from left to right)

Figure 6: Side view for ratios $M/T=\infty,2,4/3,1,0$ (from left to right)

Applying the principle of complementary energy, the corresponding characteristic relations, $P-\delta$ and $T-\varphi$, can be derived from the following two equations respectively

$$\delta(z)=\frac{\partial U^*}{\partial P} \tag{2}$$

$$\varphi=\frac{\partial U^*}{\partial T} \tag{3}$$

which z is the distance indicating positions of the cross-section. The expressions for the relations are omitted due to limitation of space.

NUMERICAL RESULTS AND COMPARISONS

In this paper finite element method is used and a four-node shell-type element is chosen to predict buckling modes of the tubular beams. Buckling modes for the roots of tubes for different ratios are shown in Figure 7.

Numerical results for $P-\delta$ and $T-\varphi$ relations are compared with that of theoretical model and experiment in Figures 8 and 9 respectively. A good agreement can be observed, especially between numerical results and experimental data.

ANALYSIS AND DISCUSSION

The behaviors of tubular beams under combined bending and torsion are complex. It is very important to develop a simple and valid theoretical model to characterize the plastic and buckling of tubes. Actually, the cross-section of the tube will not keep a plane during deformation. So, use of a beam model based on plane hypothesis is not exact for analysis of plastic behaviors of such structures. A further consideration on local buckling and flattening should be made toward an improvement of beam model.

ACKNOWLEDGEMENTS

The work is partially supported first by the Key Speciality of the Shanghai Education Commission and later by the National Natural Science Foundation through grant number 10072042, Foundation for Science and Technology and for University Key Teacher through grant numbers 99164 and GG-130-10247-1197 respectively by the Ministry of Education of China.

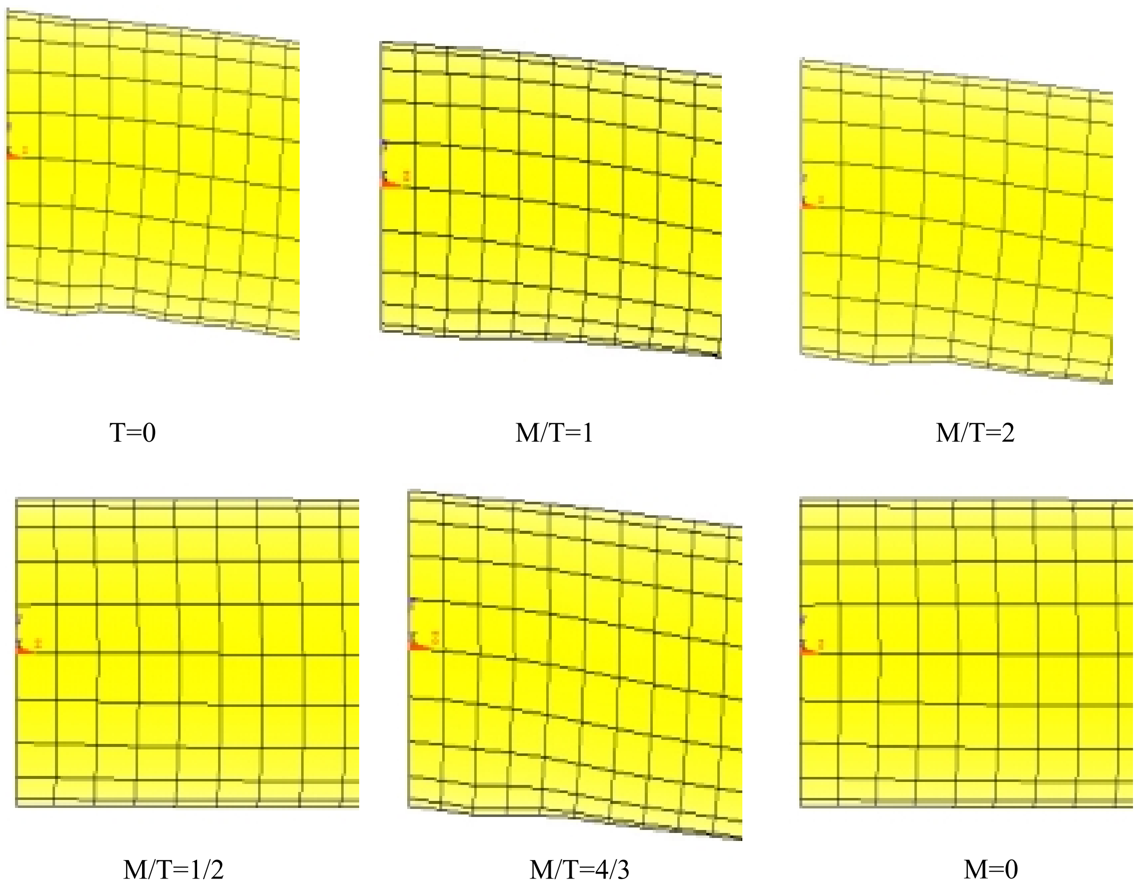

Figure 7: Buckling mode for the roots of tubular beams

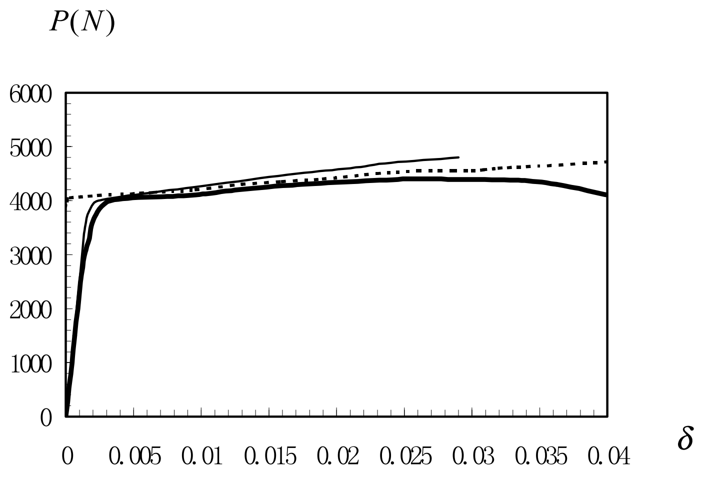

Figure 8: $P-\delta$ curves for M/T=2
(Bold solid line: experiment, thin solid line: FEM, broken line: theory)

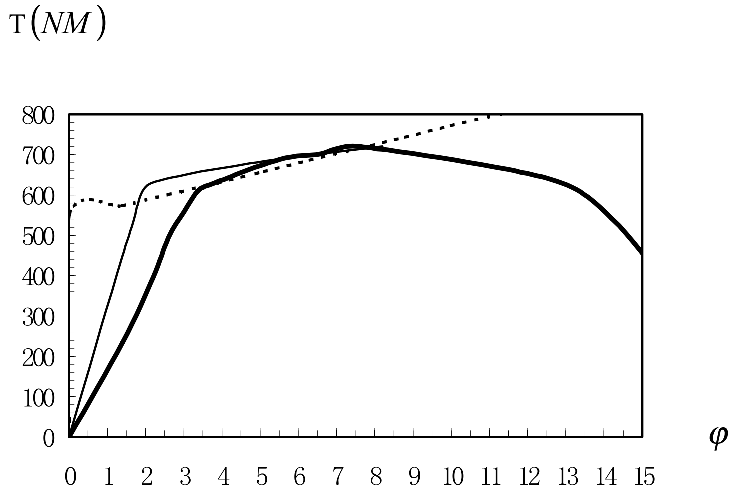

Figure 9: $T-\varphi$ curves for M/T=4/3

(Bold solid line: experiment, thin solid line: FEM, broken line: theory)

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