ON THE SIZE SCALING OF CLEAVAGE TOUGHNESS IN THE TRANSITION: A SINGLE VARIABLE EXPERIMENT AND MODEL BASED ANALYSIS

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ABSTRACT

A systematic investigation of the effects of specimen size on cleavage fracture toughness of a typical pressure vessel steel in the transition is reported. Size dependence of toughness may arise from two basic mechanisms. The first is related to the total volume of material acted on by high stress fields near a blunting crack tip, which is a function of the crack front length (B) in small scale yielding (SSY). We call this the statistical stressed volume (SSV) effect. The second is due to SSY constraint loss (CL), which depends on the ligament length (b) as well as B. Until now, it has not been possible to quantify the individual and combined effects of SSV and CL (or B versus b) size scaling laws, or even to verify fully the existence of the underlying mechanisms. In order to develop a single variable database on size effects, a complete B-b matrix of fracture specimens, fabricated from a single plate of steel from the Shoreham pressure vessel, was tested at a common set of conditions. The B ranged from 8 to 254 mm and b from 3.2 to 25.4 mm. The database was analyzed using 3-dimensional finite element simulations of the crack tip fields, calibrated to the local fracture properties of the Shoreham steel. The finite element based analysis shows that both SSV, giving rise to a $B^{-1/4}$-type scaling, as well as CL, evaluated in terms of in-plane areas within critical stress contours, can sometimes play a significant role in the size scaling of transition toughness. The significance of these results to the recently approved Master Curve standard (ASTM E 1921-97) is discussed briefly.

KEY WORDS: Cleavage toughness, statistical and constraint loss effects, fracture models, Master Curve

INTRODUCTION

There has been significant progress in understanding the mechanisms and mechanics mediating the fracture toughness ($K_{IC}$) of steels in the cleavage transition regime that qualitatively rationalizes observations of both large scatter and explicit effects of specimen size and geometry. This understanding is based on: a) the concept of critically stressed regions containing a statistical distribution of cleavage trigger particles local to a blunting crack tip; and b) realistic finite element (FE) simulations of the corresponding stress fields, including 3-D computations, for conditions that deviate substantially from SSY [1,2,3]. Statistical considerations dictate that fracture toughness increases with decreasing volume of material under high stress, giving rise to an inverse power dependence on the crack front length, or thickness, B, in standard edge cracked specimens. We will refer to this as the statistically stressed volume (SSV) effect. Reductions in the crack tip stress amplitude associated with tri-axial constraint loss (CL) can also result in increasing toughness with decreasing specimen dimensions [3] outside the SSY regime. The challenge is to define the limits of SSY, which is complicated by the fact that both SSV and CL effects can appear to have quite similar dimensional scaling in some regimes [4].

Unfortunately, the body of data to quantify the individual and combined contributions of SSV and CL to size effects is rather sparse. A recent review found that the available fracture toughness database is generally inconclusive, since it is dominated by specimens with simultaneous variations in all dimensions, and most
often has ambiguous levels of constraint [5]. Further, the magnitude of CL depends on the detailed micromechanics of cleavage, hence, on the material and microstructure.

These fundamental issues have significant practical implications to the recent ASTM standard master curve (MC) method for measuring $K_{\text{jc}}$ in the transition (ASTM E 1921-97) [6]. The MC method uses an explicit SSV adjustment of $K_{\text{jc}}(B)$ to a common reference $B_r = 25.4$ mm to account for size effects as $K_{\text{jcr}} = K_{\text{min}} + [K_{\text{jc}}(B) - K_{\text{min}}][B/B_r]^{1/4}$, where $K_{\text{min}} = 20$ MPam is an assumed minimum toughness. A relatively favorable constraint limit is used to ‘censor’ data affected by CL, and a statistical procedure is used to include the censored data in determining a reference temperature ($T_o$) at $K_{\text{jc}}$ of 100 MPam. The maximum toughness limit for uncensored data is determined from a non-dimensional CL factor $M = E\sigma_{ys}/K^2$ at $M = 30$, where $\sigma_{ys}$ is the yield stress, $E$ is the elastic modulus and $b$ is the ligament length.

The overall objective of this work is to develop and analyze a single variable database on size effects on $K_{\text{jc}}$ in a typical pressure vessel steel. Specifically, we seek to assess, and physically model, the individual and combined contributions of SSV (B) and CL (b and B) mechanisms.

**EXPERIMENTAL: A SINGLE VARIABLE SIZE SCALING STUDY**

A full test specimen matrix included variations in B for a set of constant b, and variations in b for a set of constant B. The nominal values of B and b were 8, 16, 32, 64, 127 and 254 mm (B), and 25.4, 12.7, 6.3 and 3.2 mm (b), respectively. A section of ASTM A533 Grade B Class 1 plate from the decommissioned Shoreham vessel was machined into single edge notched three point bend specimens (SENB) in the L-S orientation with span (S) to width ratio S/W=4 and a/W 0.5. Individual specimen types were taken from randomized locations within the original steel plate. Pre-cracking procedures were developed to ensure straight crack fronts at a constant 1/4 plate thickness depth. Based on preliminary measurements of $T_o$, using 1T C(T) specimens, all quasi-static loading rate tests were carried out at -91°C. With minor or necessary exceptions, including atypical specimen geometry (e.g., large and small B/W ratios), the fracture tests followed the basic ASTM E 1921-97 procedures. For $b=25.4$ mm, $M$ is greater than 100 up to 175 MPam, reasonably assuring negligible to modest CL effects in all but the thinnest specimens. The large range of B from 8 to 254 mm produces a maximum variation of a factor of about 2 in the mean toughness for SSV scaling. The variations in b were used to explore a wide range of constraint conditions, down to M values of less than ten.

Eight tests were carried out at each matrix point. Other characterization studies include: a) fracture tests on the 1T C(T), pre-cracked Charpy (PCC) bend bars with a/W of about 0.1 and 0.5, and 1/2 and 1/3-sized PCC bend bars; b) tensile properties at a wide range of temperature; c) metallography, micro-hardness and quantitative fractography. Further details on the experimental procedures will be given in future publications.

**EXPERIMENTAL RESULTS AND DATABASE**

In all cases, fracture was by cleavage initiation with little or no stable crack growth. Figure 1a shows a composite plot of the unadjusted toughness data for various specimens ($K_{\text{sp}}$) plotted against log B (in mm). For clarity, only the maximum and minimum $K_{\text{sp}}$, connected by lines, along with the $K_{\text{sp}}$ average for a given B, are shown. The heavy dashed line is a least square fit to the average $K_{\text{jc}}$ data for the largest $B = 25.4$ mm specimens using the MC form as $K_{\text{min}} + [K_{\text{jc}} - K_{\text{min}}][B/B_r]^{1/4}$. In this case $p = 0.26 \pm 0.09$, in almost exact agreement with the nominal SSV prediction. While they are somewhat difficult to see in this very busy plot, three other trends can be observed: a) the minimum $K_{\text{sp}}$ is relatively independent of B and b; b) both the average and maximum $K_{\text{jc}}$ generally increase with decreasing B; and c) both the average and maximum $K_{\text{jc}}$ tend to be higher for smaller b. Figure 1b shows the SSV adjusted $K_{\text{ssv}}$ averaged for the six B plotted on an

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1. The $B^{1/4}$ size scaling fundamentally derives from the fact that for SSV, the in-plane area within a specified stress contour varies as the applied $K^4$. Thus, the corresponding volume within a specified stress contour scales with $BK^2$. Assuming a critically stressed volume cleavage criteria gives rise to the B$^{1/4}$-type scaling.
2. Other important specimen dimensions are the crack depth (a), width (W) and ligament length (b = W-a).
3. Due to problems in reliable pre-cracking, the specimens at the matrix point b = 3 mm, B = 64 mm were not tested.
4. Note, while we have characterized constraint in terms of the in-plane ligament dimension b, loss of out-of-plane (transverse) plane strain constraint also occurs in thin (small B) specimens.
5. The fractography-fracture reconstruction efforts include optical, scanning electron and confocal microscopy. These techniques are being combined to identify fracture initiation sites and process zone damage development sequences.
6. The maximum stable crack growth in the smallest specimens with the highest toughness was less than 5% of the ligament.
expanded scale against b. Clearly, significant CL effects may occur at nominal values of $M_{\text{nom}}$ (also shown, assuming $K_{Jc} = 100 \text{ MPam}$) greater than 30. However, given the large range of specimen geometry in this database, additional analysis of the SSV and CL effects requires application of more detailed size scaling models.

![Figure 1](image)

**FINITE ELEMENT MODELING OF TOUGHNESS AND SIZE SCALING**

Three-dimensional (3-D) finite element (FE) simulations of crack tip stress and strain fields were used to analyze the database described in the previous section, following the general approach of Nevalainen and Dodds [3]. This method assumes that cleavage occurs when a specified principle stress ($\sigma_I = \sigma^* \theta$) contour encompasses a critical volume ($V^*$) of material in front of the blunting crack tip\(^7\). Assuming B is fixed, the constraint adjustment is given by the ratio of the applied $K_{sp}$ for a finite specimen ($K_{sp}$) producing a particular stressed volume ($V_{sp}$), to the corresponding SSY $K_{ssy}$ that produces the same stressed volume (i.e., $V_{ssy} = V_{sp}$), or $K_{sp}/K_{ssy}$. However, we also wish to consider variations in B relative to a reference thickness, $B_r$. Thus, it is most convenient to evaluate $V_{sp}$ as $<A_{sp}>B$, where $<A_{sp}>$ is the stressed area averaged over the entire crack front. The corresponding reference $V_{ssy}$ is $A_{ssy}B_r$, where $A_{ssy}$ is the stressed area for plane strain conditions at a minimal level of global plastic deformation with no T-stress field. The measured toughness can be adjusted to SSY conditions at $B = B_r$, $K_{Jc}$, by dividing by the corresponding FE based $K_{sp}/K_{Jc}$ factor at $<A_{sp}>B = A_{ssy}B_r$. Following Nevalainen and Dodds, the SSY $A_{ssy}(K_{Jc}, \sigma/\sigma_{ys})$\(^8\) were computed based on an elastic boundary layer analysis using the general purpose FE code ABAQUS [7]. The mesh, consisting of 1200 two dimensional, eight-noded continuum plane strain elements, was very refined near the crack tip, and increased in size out to a large boundary radius. This ensured that the plastic zone was deeply embedded in an elastic zone bounded by imposed elastic displacements with a T-stress equal to 0.

A quarter-symmetry mesh composed of 2480 20-noded quadratic brick elements was used in the 3-D FE simulations for all the non self-similar specimen geometries. Wedge-shaped elements, with overlapping nodes at an initially sharp crack tip, were used to efficiently model blunting up to high levels of gross specimen plastic deformation, corresponding to a minimum M of less than 10. The $K_{sp}/K_{Jc}$ adjustment accounts for both SSV and CL mediated size-scaling effects\(^9\). In the SSY regime, $K_{sp}/K_{Jc} = [B/B_r]^{1/4}$, while the effects of constraint loss can be isolated by assuming $K_{sp}/K_{Jc} = [A_{ssy}(K_{Jc})/<A_{sp}(K_{Jc})>]^{1/4}$. For a given constitutive law, the CL adjustment depends only on $\sigma_I/\sigma_{ys} = \sigma^*/\sigma_{ys}$.

The $A_{ssy}$ can be represented in a compact non-dimensional form as $A_{0} = \log[A_{ssy}(\sigma_I/\sigma_{ys})\sigma_{ys}/K_J^{4/3}]$, and fitted to a polynomial $f(\sigma_I/\sigma_{ys})$ [3]. This relation can be used to evaluate $K_{Jc}$ at specified values of $\sigma_I = \sigma^*$, $A_{ssy} = A^*$ and $\sigma_{ys}$. Further, it has been shown that the predicted $K_{Jc}(T)$ curve shape for unirradiated steels

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\(^7\) Statistical variations in trigger particle microstructures result in a corresponding distribution of $\sigma^*-V^*$ leading to the inherent scatter of $K_{Jc}$ data in the transition.

\(^8\) The compact nomenclature in this paper uses $K_I$ rather than $J$, as $K_I = J\sqrt{E}$, where $E$ is the plane strain modulus.

\(^9\) Note that since the details of the FE computations and analysis procedure were somewhat different for the SSV boundary element and 3-D specimen cases, the $A_{ssy}/A_{sp}$ approaches unity at $M \approx 200$. Thus, no constraint correction is applied for specimens in which the measured $M \geq 200$. 

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with $T_o$ below 0°C is consistent with the MC shape assuming that $\sigma^*$ and $A^*$ are constant (independent of T) using a generic empirical $\sigma_y(T)$ relation [8]. Thus, the model can be used to fit SSY $K_{lc}(T)$ data to estimate the $\sigma^*\cdot A^*$. The results of fitting are shown in Figure 2a, where the filled circles are data from 1T C(T) specimens and the open diamonds are for PCC specimens SSV adjusted to $B_r = 25.4$ mm (a/W 0.5 in both cases). Note that this data has not been CL adjusted, which would generally be small. The solid line is the MC shape with $T_o = -91^\circ$C. The heavy dashed curve is the predicted $K_{lc}(T)$ curve for $A^* = 3.7 \times 10^{-8}$ m$^2$ and $\sigma^* = 1790$ MPa - $\sigma*/\sigma_y = 591$ MPa. 3. The light dashed lines show the results for fits with a $\sigma*/\sigma_y$ ratios of 2.75 and 3.25. An additional fitting criterion for $\sigma^*/A^*$ was that the predicted $K_{lc}(T)$ match the MC shape in the lower shelf-knee regime (in this case down to $-196^\circ$C). Thus, the nominal estimate of $\sigma*/\sigma_y = 3$ is certainly reasonable, but not completely unique. Figure 2b shows a 3-D plot of the effect of $B/W$ and M on CL in terms of the $K_{sp}/K_{ssy}$ for $\sigma*/\sigma_y = 3$. Additional details regarding the FE simulations and use of alternative statistical models, as well as the insight gained from the other characterization studies, will be presented in future publications.

ANALYSIS AND EVALUATION OF THE DATABASE

The SSV and CL adjusted $K_{Jcr}$ are shown in Figure 3a plotting the average $K_{Jcr}$ versus $B$ at the 4 ligament (b) lengths. In all cases, the toughness averaged over all $B$ at each b is slightly less than the preliminary estimate of 100 MPam. The overall average at all B and b is 91.1±4 (at 1 SD) MPam. The $K_{Jcr}$ average over $B$ for the three largest b are nearly identical at 93±0.2 MPam, while the corresponding average is only 85.4 MPam for the smallest b = 3.2 mm. The overall dependence of the average $K_{Jcr}$ on B at various b, as well as on b at various B, is weak and statistically insignificant.

Figure 3b plots the cumulative distribution of all 180 of the individual adjusted $K_{Jcr}$ data points. The distributions are nearly identical for the three largest b (48 data points each) with a standard deviation (SD) of 23.0±1 MPam. The corresponding distribution for the 40 data points at the smallest b = 3.2 mm is somewhat narrower and shifted to slightly lower $K_{Jcr}$. However, in both cases, the data are well represented by a normal cumulative distribution function (CDF), shown by the solid and dashed line for the appropriate average $K_{Jcr}$ and SD. Figure 3c plots a cumulative distribution of the averages at each of the 23 B-b combinations. The standard deviation of this adjusted data set is ±10 MPam. These data are also
reasonably consistent with a normal CDF, shown by the solid line. The largest deviations are for two of the small b = 3.2 mm specimens with somewhat atypical B/W ratios of about 2.5 and 5. A Monte Carlo simulation, assuming a population of $K_{Jcr}$ data with a SD of 23 MPam, gave a corresponding SD for the averages for the B-b combinations of 8.1 MPam, close to the observed value of 10. Figure 4 shows Weibull plots of all the $K_{sp}$, $K_{ssv}$ and $K_{Jcr}$, along with corresponding least square (dashed lines). The unadjusted $K_{sp}$ data are for different specimen sizes, hence, are not expected to follow a Weibull distribution. However, to the extent that the SSV and CL adjustments ‘work”, size-independent Weibull distributions, with a slope approaching $m = 4$, can be anticipated for $K_{ssv}$ and $K_{Jcr}$. Thus the low Weibull slope for $K_{ssv}$ of $m = 3.17$, and deviations at higher toughness, again suggest that the SSV adjustment alone is not sufficient to account for size effects in this dataset. However, the combination of a SSV and CL adjustment results in Weibull slope of $m = 3.85$ and smaller deviations at high $K_{Jcr}$. At intermediate $K_{Jcr}$, the slope is almost exactly equal to the theoretical value of 4.

Figure 4 Weibull plots of the $K_{sp}$ (far right), $K_{ssv}$ (middle) and $K_{Jcr}$ (far left) data (offset for clarity).

These results lead to two major conclusions. First, the Shoreham plate is relatively homogeneous at the sampling location with a mean reference toughness of about 92±8 MPam at –91°C. Second, the combination of the CL and SSV adjustments produce a self-consistent $K_{Jcr}$ data set. Thus, together with this database, the FE model-based adjustments provide a basis to distinguish the separate and combined effects of SSV and CL.

Figure 5a shows a decomposition of the adjustments for the various b and B. The unadjusted $K_{sp}$ data point, furthest to the left, is connected to the adjoining $K_{ssv}$ data point to its immediate right. Note, this accepts a primary B$^{-1/4}$ scaling adjustment, which reduces $K_{ssv}$ relative to $K_{sp}$ for B < 25.4 mm and has the opposite effect for B > 25.4 mm. The next point to the right reflects the additional CL adjustment that is needed, in some cases, to produce a fully size-independent to $K_{Jcr}$ data set, based on the FE $\sigma^* - A^*$ local fracture model. The solid and dashed lines are the average adjusted $K_{Jcr}$ and the ±10 MPam standard deviation respectively. As expected, the SSV adjustment increases with those B that deviate more from the reference $B_r = 25.4$ mm, and CL adjustments are most significant at small b and B.

Figure 5 a) The $K_{sp}$, $K_{ssv}$, $K_{Jcr}$ adjustment map, with increasing B in going from left to right; b) the CL adjustment vs. M for various B/W.

Figure 5b plots the CL adjustments ( = $K_{ssv}$–$K_{Jcr}$) versus M on a log scale (for clarity). The adjustments for the cases with atypically large and small B/W (< 0.5 and > 5) suffer somewhat (high B/W) to significantly (low B/W) larger CL at a given M. The mechanics leading to higher CL with very large B/W
will be discussed in future publications. The data points with B/W ratios from about 0.63 to 2.5, more typical of specimen configurations, are similar. These results suggest that CL begins at M = 100 and becomes significant (CL adjustments ≥ 10 MPam) for M from about 50 to 80. Note, a similar analysis using a σ*/σys = 3.25, rather than the nominal value of 3, suggests that CL begins at M slightly less than 100, and becomes significant for an M from about 30 to 50. Further, the M marking significant CL may be lower for compact tension specimens, with a smaller T-stress compared to bend bars.

The individual data sets can also be used to derive To, which average -84±9.3°C, and are also reasonably represented by a normal distribution. The highest To values are again for the b=3.2 specimen group with B/W = 2.5 and 5. Additional results on the effect of specimen size on the evaluation of To will be reported in the future.

SUMMARY AND FUTURE WORK

The results of this study present a remarkably clear picture of size scaling effects on KJc in the cleavage transition. They demonstrate that SSV and CL effects are both significant in some cases. Modeling the combined effects of SSV and CL, using FE simulations of crack tip stress fields, provides a basis for adjusting KJc data to a common reference size and SSY condition. The database and analysis supports the MC method approach to adjusting toughness data to account for the effect of B. However, the results also indicate that the use of M = 30 may not always fully account for CL.

The results and conclusions reported here should be considered preliminary. As well as additional modeling and statistical evaluations, information and physical insight emerging from both the broader characterization and toughness testing efforts will be incorporated in future data analysis. Further, additional testing may be warranted to fill in some gaps and resolve some uncertainties, such as the apparently low values of KJcr for two B at the smallest b.

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10 Because of length limits in this paper, we have not discussed a number of important issues. These issues include interactions between SSV and CL effects and the trends observed in the lower tail of the KJcr distribution in relation to the proposal of a threshold toughness of 20 MPam. These and other topics will be covered in future publications.