ON THE ESSENTIAL WORK OF FRACTURE: APPLICATION TO CHARACTERISE THIN POLYMER FILMS FRACTURE

L. Cousin-Cornet\textsuperscript{1}, M. Naït-Abdelaziz\textsuperscript{2}, C. Cazeneuve\textsuperscript{3}, G. Mesmacque\textsuperscript{2}

\textsuperscript{1} U.C.L, Unité P.C.I.M, 1348 Louvain-la-Neuve, Belgium
\textsuperscript{2} Laboratoire de Mécanique de Lille, CITÉ Scientifique, 59650 Villeneuve d'Ascq, France
\textsuperscript{3} Centre Charles Zviak, Département de Physique Appliquée, Laboratoire Matériaux et Surface, 92 580 Clichy, France

ABSTRACT

In this work, the Essential Work of Fracture (EWF) approach was used to characterise fracture of thin thermoplastic polymer films. The domain of validity in which the specific total work of fracture values, $w_f$, can be chosen to determine the specific essential work of fracture value, $w_e$, was first investigated. Then, the essential work of fracture is determined. Our results show that it is possible to assess the terms $w_e$ and $\beta w_{pl}$ via two distinct intrinsic equations. These relations have been established using a different way from the one used by Cotterell and Reddel.

KEYWORDS

Essential Work of Fracture, Polymer, Thin films, Stress state transition, Intrinsic relations.

INTRODUCTION

The essential work of fracture theory was first developed by Broberg [1] to characterise fracture of elastic-plastic materials. Considering a cracked thin sheet submitted to uniaxial loading, he proposed to divide the plastic zone at the crack tip in two distinct regions (Figure 1(a)) by means:

\begin{itemize}
  \item An inner region (end region) where the fracture process takes place (essentially by necking) and also called the fracture process zone (FPZ),
  \item An outer region where the material is fully plastified.
\end{itemize}

Following this idea, several authors [2,3,4] have therefore suggested to use such an approach to characterise thin polymer films fracture on DENT specimens (double edge notch in tension).
The total work to fracture $W_f$ (i.e., the total area under the load-displacement curve of a cracked specimen) is written as:

$$W_f = W_e + W_{pl}$$  \hspace{1cm} (1)

The essential work, $W_e$, is proportional to the ligament area $B.L.$, while $W_{pl}$ is function of the outer plastic zone volume. Thus, dividing eqn.1. by the ligament area leads to the following relationship:

$$w_f = w_e + \beta w_{pl} L$$  \hspace{1cm} (2)

which predicts a linear relation between $w_f$ and $L$. The linear fit is restricted to ligament lengths verifying some conditions we will further discuss and the specific essential work value, $w_e$, is obtained for $L = 0$ (Figure 1(b)).

**EXPERIMENTAL INVESTIGATION**

The material is a plasticised silicon acrylic belonging to the thermoplastic family. The fracture tests were performed on a universal tensile test device under a cross-head speed of 50 mm/mn at room temperature and 55% relative humidity. Note that a whole study of the mechanical behaviour of this material had shown that its properties are very sensitive to these parameters, revealing an elastic-viscoplastic behaviour [6]. The experiments are therefore carried out in an air-controlled room.

SENT specimens of about 0.4 mm thickness have been used with three different widths (18 mm, 24 mm, 30 mm), the length being constant and equal to 100 mm. The specimens were notched using a razor blade and several normalised crack length $a/w$ have been tested ($0.3 < a/w < 0.9$). The fracture experiments were filmed allowing crack initiation time to be accurately located. Figure 2 shows as an example a set of load-displacement records obtained in such conditions for this material, dots locating crack initiation.

**Figure 1:** (a) Schematic representation of crack tip separation in two distinct zones; (b) Linear relationship between the total specific work of fracture and the ligament.

**Figure 2:** Load-displacement curves ($w = 18$ mm)
RESULTS AND DISCUSSION

Conditions on ligament length

According to the literature [7,8], ligament lengths which can be used to determine \( w_e \) must satisfy the following conditions:

\[
3 - 5.B < L < \min (2 \, r_p, \, w/3) \quad (3)
\]

The lower bound is to ensure pure plane stress state. Moreover, \( L \) must be smaller than the minimum of \( 2 \, r_p \), because the ligament must be entirely yielded before the crack initiation occurs, and \( w/3 \), to prevent that results are not influenced by edge effects.

Considering a rigid perfectly plastic material, Hill [9] showed that, for a double edge cracked sheet, a linear relationship exists between the reached maximum load, \( P_{\text{max}} \), and \( L \):

\[
P_{\text{max}} = p \cdot \sigma_y \cdot L \cdot B \quad (4)
\]

The author noted the existence of necking since the corresponding net-stress, \( \sigma_n \), is:

\[
\sigma_n = \frac{P_{\text{max}}}{(B \cdot L)} = p \cdot \sigma_y = \frac{2}{\sqrt{3}} \cdot \sigma_y
\]

(5)

So, if \( \sigma_n > p \cdot \sigma_y \), the stress state is triaxial and, if \( \sigma_n = p \cdot \sigma_y \), the pure plane stress state exists. The plastic-constraint factor \( p \) has then been found equal to \( 2/\sqrt{3} \) for DENT specimens. This result can be found using the limit load analysis assuming, as a first approximation, that the limit load is equal to the maximum recorded load, \( P_{\text{max}} \). Using this kind of analysis and the previous approximation in the case of SENT specimens, \( p \) is found equal to 1.

The evolution of the net-stress deduced from our experimental results as a function of the ligament length is shown in Figure 3. Two distinct zones are clearly pointed out: the first one where \( \sigma_n \) is decreasing with respect to ligament length while it remains approximately constant in the second part. The ligament length corresponding to the stress state transition is here about 4 mm which is higher than the recommended value expressed by eqn.3. Such a difference has been already noted [10], suggesting that this lower bound depends on the material nature.

![Figure 3: Net-stress evolution versus the ligament length](image)

The yield stress value of this material was measured under the same conditions of temperature and strain rate. \( \sigma_y \) was found equal to about 3 MPa allowing the plastic-constraint factor estimation when using eqn.4. The \( p \) value we have calculated is 1.18 which is higher than 1. Nevertheless, it must be noted that the theoretical value is issued from the limit load analysis which assumes a rigid perfectly plastic material. Since our material reveals a strain-hardening behaviour, even if the ligament is entirely yielded when the crack
initiation occurs, the stress is increasing yet because of strain-hardening. This may explain that the constraint factor is overestimated and therefore suggest that the limit load is lower than $P_{\text{max}}$. Nevertheless, provided that a sufficient number of fracture tests on specimens containing varying crack lengths are available, the methodology to determine the lower bound by analysing the evolution of the net-stress in the ligament seems an interesting approach since it only requires to get the yield stress of the studied material.

**Specific essential work of fracture determination**

The total specific work, $w_t$, is plotted versus the ligament length in Figure 4 for the three specimens widths we have tested.

![Figure 4: Specific total work of fracture evolution versus the ligament length](image)

A linear evolution of $w_t$ is highlighted when only retaining ligament lengths higher than 4 mm, i.e the above-mentioned lower bound. The linear fit leads to the essential specific work of fracture value which is here equal to $17.37 \pm 3.16$ kJ/m$^2$. Note that the upper limit has not been taken into account in this estimation since experimental values of $w_t$ do not clearly reveal any transition when increasing the ligament length. Elsewhere, neither $w_e$ nor the slope of the linear fit are significantly dependent on the specimen width [6].

**Analytical expressions of $w_e$ and $\beta w_{pl}$**

For a given specimen geometry the total work to fracture, $W_f$, can be written as follows:

$$W_f(L) = \gamma(L) \cdot P_{\text{max}}(L) \cdot u_f(L)$$  \hspace{0.5cm} (6)

Our experimental data are in a good agreement with eqn.6. The shape factor is found constant, equal to 0.78 [6].

Differentiation of eqn.6 leads to:

$$dW_f = \gamma \left[ \frac{\partial P_{\text{max}}(L)}{\partial L} u_f \ dL + P_{\text{max}} \frac{\partial u_f(L)}{\partial L} \ dL \right]$$  \hspace{0.5cm} (7)

According to Mai and Cotterell [11], $u_f$ can be expressed in the following form:

$$u_f = \delta_c + k \ L$$  \hspace{0.5cm} (8)

Our results are in quite good agreement with such an expression as shown in Figure 5.
Replacing $P_{\text{max}}$ and $u_f$ according to eqn.4. and eqn.8. respectively, we obtain:

$$\frac{dW_f}{dL} = \gamma_p \sigma_y B (\delta_c + 2kL)$$

(9)

In another hand eqn.1. and eqn.2. give:

$$\frac{dW_f}{dL} = B [w_c + 2\beta w_{\text{pl}} L]$$

(10)

Identifying eqn.9. and eqn.10. allows to analytically express $w_c$ and $\beta w_{\text{pl}}$. Indeed, these terms can be written as:

$$w_c = \gamma_p \sigma_y \delta_c$$

(11) and

$$\beta w_{\text{pl}} = \gamma_p \sigma_y k$$

(12)

Eqn.11. is identical to that introduced by Cotterell and Reddel [5] but they established it using a different way.

Finally, replacing the different parameters by their own numerical values, we obtain $w_c = 13.2 \pm 4.2 \text{kJ}/\text{m}^2$ and $\beta w_{\text{pl}} = 2.8 \pm 0.1 \text{kJ}/\text{m}^3$. These values are in a quite good agreement with those above estimated indicating that the intrinsic relations give a reasonable estimation of these parameters.

**CONCLUSION**

In this study, The EW F approach has been successfully applied to investigate fracture properties of plasticized silicon acrylic polymer films. The following main results can be reminded:

- A systematic analysis of the maximal net-stress in the ligament is necessary to point out the stress state transition and therefore to validate the lower bound in terms of ligament lengths required to establish the $w_c$ value. Indeed, this lower bound seems to be material nature dependent.
- The analytical expressions of $w_c$ and $\beta w_{\text{pl}}$ we have proposed lead to values which are in good agreement with those obtained when using a linear fit of $w_c$ data. Nevertheless, the parameters values which are required to compute these two terms, have to be connected to other mechanical and/or physical properties.
NOMENCLATURE

\( W_f \) : Total work of fracture
\( w_f \) : Specific total work of fracture
\( W_e \) : Essential work of fracture, expended in the FPZ
\( w_e \) : Specific essential work of fracture
\( W_{pl} \) : Non-essential work of fracture, dissipated by plastic deformation in the outer region
\( w_{pl} \) : Specific non-essential work of fracture
\( B \) : Thickness specimen
\( P \) : Applied load
\( P_{\text{max}} \) : Maximum recorded load
\( u \) : Total displacement
\( u_f \) : Total displacement corresponding to the complete failure
\( \beta \) : Shape factor concerning the outer plastic zone
\( L \) : Ligament length
\( a \) : Crack length
\( w \) : Specimen width
\( r_p \) : Yielded zone radius
\( P \) : Plastic-constraint factor
\( \sigma_y \) : Yield stress
\( \sigma_n \) : Net-stress
\( \gamma(L) \) : Shape factor concerning the load-displacement curves
\( \delta_C \) : Critical Crack tip opening displacement at initiation
\( k \) : Opening angle of the elaboration zone such as defined by Mai and Cotterell [11]

REFERENCES