MODELING OF DAMAGE EVOLUTION EQUATION OF PZT CERAMICS AND ITS APPLICATION TO CRACK GROWTH ANALYSIS

M. Mizuno [1] , Y. Honda [2] and H. Kato [2]

1 Department of Machine Intelligence and Systems Engineering, Akita Prefectural University, Tsuchiya, Honjo, Akita 015-0055, Japan
2 Department of Mechanical Engineering, Graduate School of Nagoya University, Chikusa-ku, Nagoya, Aichi 464-8603, Japan

ABSTRACT

In order to describe damage of $PbZrO_3$-$PbTiO_3$ (PZT) ceramics, a damage variable based on the continuum damage mechanics is introduced, and an evolution equation of the damage variable for PZT ceramics is formulated by taking into account effects of mechanical and electrical loads on the damage development. On the other hand, the damage variable is introduced into piezoelectric constitutive equations of PZT ceramics by using a modified cubes model; i.e. material constants in the constitutive equations are expressed as a function of the damage variable. Then, a set of the piezoelectric constitutive equations and the evolution equation of damage variable are employed to predict fatigue life under various mechanical and electrical loading conditions, and the validity of the modeling is discussed by comparing the predictions with experimental results. Finally, as an example of applications, the constitutive equations and the damage evolution equation are applied to a crack growth analysis by using a double cantilever beam (DCB) model, and the effects of electric field on the crack growth are discussed.

KEYWORDS

Damage mechanics, Piezoelectric ceramics, Evolution equation of damage variable, Constitutive equation, Modified cubes model, Crack growth analysis, Double cantilever beam model

INTRODUCTION

Piezoelectric ceramics are used in sensors and actuators because of their fast electromechanical response, relatively high power of generating force and small size. In the operation of actuators, the piezoelectric ceramics are subjected to mechanical and electrical loads cyclically, and damage such as cavities and microscopic cracks in the ceramics develops by both loads. Since the damage causes change in mechanical and piezoelectric properties and causes fracture of the ceramics finally, mathematical description of the damage development is necessary to predict fracture of the ceramics as well as to control actuators precisely.

Researches on description of the damage of piezoelectric ceramics in the framework of the continuum damage mechanics have been published, so far. Chuang et al. [1] predicted fatigue life of a 4-point bend

PZT ($PbZrO_3$-$PbTiO_3$) ceramics by a finite element model. Jain and Sirkis [2] modeled the damage by the micromechanics and discussed the effects of the damage on mechanical and piezoelectric properties. On the other hand, Sun and Jiang [3] discussed fatigue crack growth under mechanical and electrical loads and estimated it by the fracture mechanics. Moreover, the effects of electric field on fatigue life and crack growth were discussed by other researchers [4, 5]. However, any damage evolution equation has not been formulated appropriately by taking into account the effects of electric field on the damage.

In the present paper, a damage variable is introduced into piezoelectric constitutive equations by using a modified cubes mdel [6]; i.e. material constants in the constitutive equations are expressed as a function of the damage variable. An evolution equation of the damage variable is formulated by taking into account the effects of electric field on fatigue life. In order to confirm the validity of the formulation, fatigue life is simulated by using the constitutive equations and the evolution equation in comparison with experimental results [7, 8]. Finally, as an application of the constitutive equations and the evolution equation, those equations are applied to a double cantilever beam (DCB) model [9], and the effect of electric field on the crack growth is discussed.

PIEZOELECTRIC CONSTITUTIVE EQUATION AND DAMAGE EVOLUTION EQUATION

Piezoelectric Constitutive Equations by Taking into Account Damage

In general, piezoelectric constitutive equations are given as follows:

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl} - e_{mij}E_m, \tag{1}$$

$$D_i = e_{ikl}\varepsilon_{kl} + \kappa_m E_m, \tag{2}$$

where σ_{ij}, ε_{ij}, E_i and D_i are stress, strain, electric field and electric displacement, respectively, while C_{ijkl}, e_{mij} and κ_m represent elastic moduli, piezoelectric moduli and permittivity, respectively. The material constants are affected by damage such as cavities and microscopic cracks. Thus, if the damage is expressed by a damage variable ω based on the continuum damage mechanics, the material constants are given as a material function of ω:

$$C_{ijkl} = C_{ijkl}(\omega),\ e_{mij} = e_{mij}(\omega),\ \kappa_m = \kappa_m(\omega). \tag{3}$$

In the present paper, the damage is assumed to be isotropic for the sake of simplification.

Modified Cubes Model

If an isolated cavity in the material is assumed to be represented by a cube, it is classified into 3-0 type connectivity introduced by Newnham et al. [10]. Since representative material constants of ceramics including cavities are derived as material constants of two-phase composites by combining a series model and a parallel model [6], the material constants are defined as a function of volume fraction of cavity easily.

The material constants in Eqns 1 and 2 in the poling direction of piezoelectric ceramics are given as follows:

$$\overline{C} = (1-a^2)C_0, \tag{4}$$

$$\overline{\kappa} = (1-a^2)\kappa_0 + \frac{a^2\kappa_{air}\kappa_0}{a\kappa_0 + (1-a)\kappa_{air}}, \tag{5}$$

$$\overline{e} = (1-a^2)e_0, \tag{6}$$

where $\overline{C}$, $\overline{\kappa}$ and $\overline{e}$ are representative elastic, dielectric and piezoelectric constants, respectively, while C_0, κ_0 and e_0 are elastic, dielectric and piezoelectric constant of the ceramics without cavity, respectively. κ_{air} represents permittivity of air and a is a dimension of a unit cell in the modified cubes model.

Introduction of Damage Variable

If the damage variable is interpreted as reduction of load-carrying net area caused by cavities [11], the damage variable ω is given by

$$\omega = 1 - \frac{S^*}{S} = 1 - \frac{1-a^2}{1} = a^2, \tag{7}$$

where S and S^* are cross-sectional area and effective load-carrying area of a unit cell in the modified cubes model. Therefore, representative elastic, dielectric and piezoelectric constants in Eqns 4-6 are expressed as a function of damage variable ω as follows:

$$\bar{C} = (1-\omega)C_0, \tag{8}$$

$$\bar{\kappa} = (1-\omega)\kappa_0 + \frac{\omega\kappa_{air}\kappa_0}{\sqrt{\omega}\kappa_0 + (1-\sqrt{\omega})\kappa_{air}}, \tag{9}$$

$$\bar{e} = (1-\omega)e_0. \tag{10}$$

The piezoelectric constitutive equations in the uniaxial state in the poling direction are given as follows:

$$\sigma = \bar{C}(\omega)\varepsilon - \bar{e}(\omega)E, \tag{11}$$

$$D = \bar{e}(\omega)\varepsilon + \bar{\kappa}(\omega)E. \tag{12}$$

Evolution Equation of Damage Variable

In the present paper, the effect of electric field on the damage is assumed to be introduced into the evolution equation of damage variable through internal stress caused by piezoelectric effect [4, 5, 7, 8]. Since the constitutive equations 11 and 12 are not taken into account domain switching, the application of the equations is restricted within lower electric field than coercive field. Since, in general, ceramics tensile strength is lower than compressive and shear strength, reference stress for damage criterion is expressed by linear combination of the maximum principal stress σ^I and the equivalent stress σ^{EQ}. Accordingly, the evolution equation of damage variable is formulated by

$$\frac{d\omega}{dt} = A\left[\frac{\alpha\sigma^I + (1-\alpha)\sigma^{EQ}}{1-\omega}\right]^k, \tag{13}$$

where A, k, α are material constants, and they are determined by different experiments in stress states.

Figure 1 and 2 show simulations of fatigue life of PZT ceramics by using the constitutive equations 8-12 and the damage evolution equation 13 in comparison with experimental results [7, 8].

SIMPLIFIED ANALYSIS OF CRACK GROWTH

As an application of the constitutive equations 8-12 and the damage evolution equation 13, a simplified analysis of crack growth in steady state is performed by using a double cantilever beam (DCB) model [9].

Formulation of DCB Model

In the DCB model, a plate with a crack is divided into three zones as shown in Figure 3; i.e. elastic zone Ω_e, fractured zone Ω_F and damaging zone Ω_D. The elastic zone is treated as a cantilever beam in which elastic shear deformation is dominant, while a crack is included in the fractured zone where any deformation is not considered. The constitutive equations and the damage evolution equation are applied

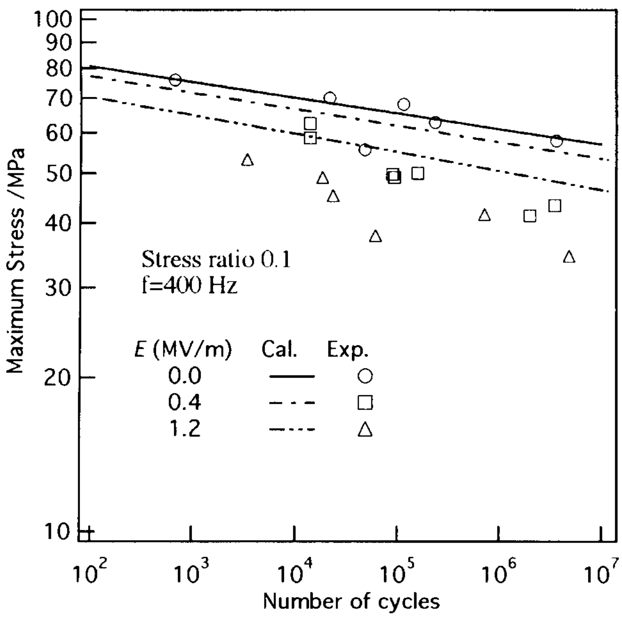

Figure 1: Fatigue life under DC electric field

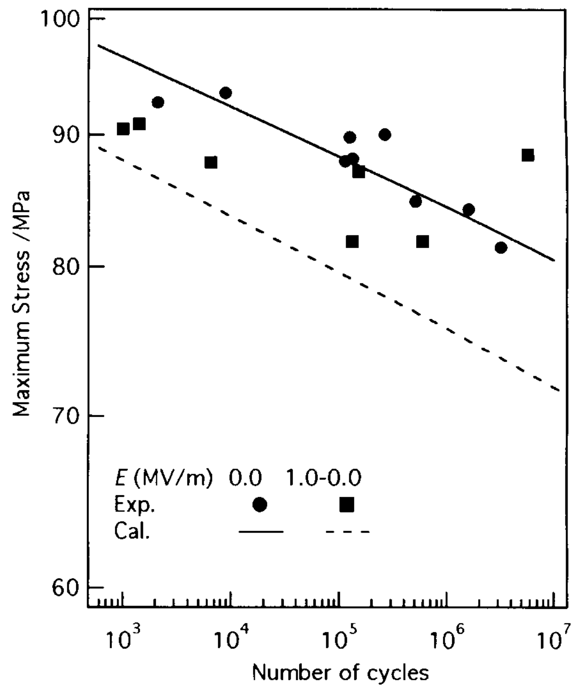

Figure 2: Fatigue life under AC electric field

to the damaging zone, in which stress state is assumed to be uniaxial. In the model, poling direction coincides with axis y in the coordinate system $o-xy$, and electric field in the same direction as the poling direction is positive.

Furthermore, for the sake of the simplification, crack growth in steady state at constant rate v and a concentrated load W at $x=0$ are considered. Then, if the Galilean transformation, which transforms the crack tip to the origin $z=0$ in the new coordinate system $o'-zy$, is introduced, Eqn 13 and the relation between displacement and stress in the elastic zone are given as follows:

$$-v\frac{d\omega}{dz} = A\left[\frac{\sigma}{1-\omega}\right]^k, \qquad (14)$$

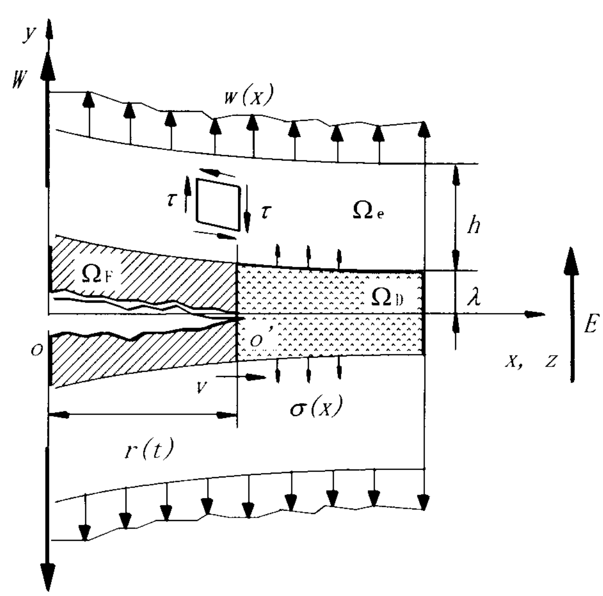

Figure 3: Double cantilever beam model

$$G_0 h\lambda \frac{d^2\varepsilon}{dz^2} = \sigma, \tag{15}$$

where G_0, h and λ are the shear modulus, width of the elastic zone and width of damaging zone, respectively.

Boundary Condition

In the present paper, $w(x) = 0$ since a concentrated load W at $x = 0$ are considered, and considering the fracture condition at the crack tip, $z = 0$, boundary conditions are represented as follows:

$$\frac{d\varepsilon}{dz}(0) = -\frac{W}{G_0 h\lambda}, \tag{16}$$

$$\omega(0) = 1, \tag{17}$$

$$\varepsilon(0) = \varepsilon_R, \tag{18}$$

where ε_R is fracture strain.

On the other hand, since the shearing force at infinity, $z = \infty$, may vanish and the strain generates due to piezoelectric effect by electric field, boundary conditions at $z = \infty$ are depicted as follows:

$$\frac{d\varepsilon}{dz}(\infty) = 0, \tag{19}$$

$$\varepsilon(\infty) = \frac{e_0}{C_0} E. \tag{20}$$

Eqns 11, 14 and 15 are solved under initial conditions 16-18, then, distribution of stress, strain and damage variable in front of the crack tip in the steady state of crack growth can be obtained. In the system of differential equations, since the crack growth rate v and the fracture strain ε_R are unknown, v and ε_R are determined so that the boundary conditions 19 and 20 are satisfied.

Results of Analysis

In the present paper, crack growth of PZT ($PbZrO_3$-$PbTiO_3$) ceramics is analyzed and following material constants are employed:

$$C_0 = 16.3\ \text{N/m}^2,\ \ e_0 = 7.1\ \text{C/m}^2,\ \ \kappa_0 = 34.0\times 10^{-10}\ \text{C/Vm},\ G_0 = 6.0\times 10^{10}\ \text{N/m}^2,$$
$$A = 7.4\times 10^{-103},\ \ k = 51,\ \ h = 0.05\ \text{m},\ \ \lambda = 0.005\ \text{m}. \tag{21}$$

Figures 4 and 5 show distributions of stress σ and strain ε in front of the crack tip under various electric fields E, and Figure 6 shows the effect of electric field on the crack growth rate v.

CONCLUDING REMARKS

In the present paper, damage such as cavities and microscopic crack in PZT ceramics was represented by the damage variable based on the continuum damage mechanics, and introduction of the damage variable into the piezoelectric constitutive equation and formulation of evolution equation of damage variable were performed. Then, as an application of those equations, they were applied to the DCB model to analyze the crack growth in the steady state, and the effect of electric field on the crack growth was discussed.

Since it is found that concentration of stress and electric field around the crack tip causes domain switching which generates internal stress and affects the damage development, it is necessary to take into account the domain switching in the constitutive equations and the damage evolution equation and to consider the distribution of electric field around the crack tip in the analysis of crack growth. In order to perform coupling analysis of stress-electric-damage field, the local approach based on the continuum damage mechanics by using a finite element method is useful.

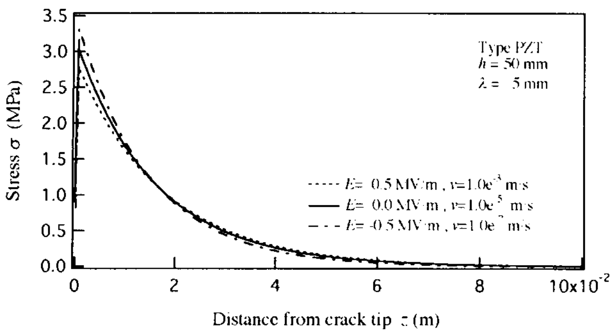

Figure 4: Distribution of stress in front of crack tip

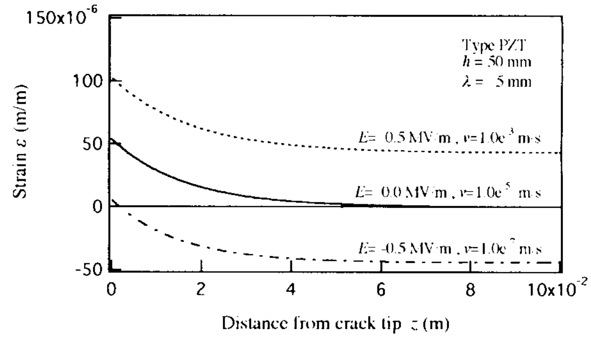

Figure 5: Distribution of strain in front of crack tip

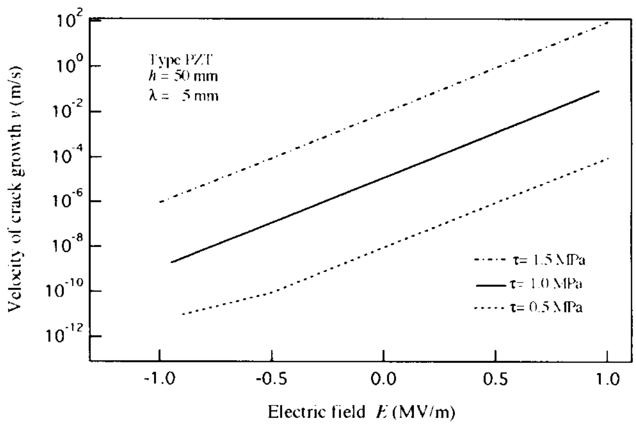

Figure 6: Effect of electric field on crack growth rate

REFERENCES

1. Chuang, T., Wang, Z., Hill, M. and White, G. (1996) *Fract. Mech. Ceramics* 12, 135.
2. Jain, A. K. and Sirkis, J. S. (1995) *SPIE* 2443, 374.
3. Sun, C. T. and Jiang, L. Z. (1997) *SPIE* 3040, 129.
4. McHenry, K. D. and Koepke, B. G. (1978) *Fract. Mech. Ceramics* 5, 337.
5. Wang, D., Fotinich, Y. and Carman, G. P. (1998) *J. Appl. Phys.* 83, 5342.
6. Banno, H. (1987) *Ceramic Bulletin* 66, 1332.
7. H. Makino and N. Kamiya (1994) *Jpn. J. Appl. Phys.* 33, 5323.
8. H. Makino and N. Kamiya (1998) *Jpn. J. Appl. Phys.* 37, 5301.
9. S. Murakami, K. Hayakawa and M. Mizuno (1993) *Eur. J. Mech., A/Solids* 12, 249.
10. R. E. Newnham, D. P. Skinner and L. E. Cross (1978) *Mater. Res. Bull.* 13, 525.
11. S. Murakami (1988) *J. Appl. Mech.* 110, 280.