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Major aspects of Mixed-Mode problems

H.A. Richard

Institute of Applied Mechanics, University of Paderborn
D – 33098 Paderborn, Germany

ABSTRACT

In this paper a survey on Mixed-Mode fracture and fatigue problems is carried out. At first the overlapping of fractures Modes I and II will be considered. For such loading situations both the occurrence of unstable crack growth and the kinking angle of the crack can be predicted by different fracture criterions. For the case of an unstable propagation of a crack under superimposed Mode I and Mode II loading those criteria have proven their reliability for homogenous materials, which has already been shown by various experimental research. If an existing crack is subjected to a Mixed-Mode loading, the direction of crack propagation changes. The kinking angle for such a crack can be predicted very well by the existing criteria. It is notable, that a delayed crack growth can be observed when a kinking of a fatigue crack takes place.
The three-dimensional crack growth in arbitrary structures and components can generally be characterized by the occurrence of all the crack Modes I, II and III. For all these Mixed-Mode loading situations this paper will present the theoretical fundamentals, experimental research and concepts for the prediction of their effects.

KEYWORDS

Mixed-Mode fracture, fatigue crack growth, fracture criteria, prediction concepts, All-fracture-modes-specimen

INTRODUCTION

In various fields of engineering a large number of crack problems can be found, that are caused by Mixed-Mode loading at the crack tip. These loading conditions may occur e.g. as a result of superimposed stresses on construction components, oblique or curved cracks, multiple cracks or cracks in the vicinity of notches.
In the past 2D-problems of crack growth under Mixed-Mode I and II loading conditions have attracted much attention. A number of fracture criteria for predicting the onset of fracture and its direction are well established [1-5]. In particular the CTS-specimen together with the related special loading device [5,6] and detailed theoretical, experimental and computational investigations have contributed to the current state in the field [7-9]. So this problem is now almost understood. In contrast to this, for the more general 3D Mixed-Mode problems only a few approaches are known and there is still a lack of understanding. Indeed in some recent papers [10-12] investigations regarding superimposed Mode I and Mode III can be found, but those results can not be applied to cases of Mode II and III and can especially not be generalised to cases, where all basic fracture modes I, II and III are involved.

TWO-DIMENSIONAL MIXED MODE FRACTURE AND FATIGUE

In 2D or plane Mixed-Mode cases the basic relations can be presented in the form of a fracture curve and a threshold curve in a K_I-K_{II}-diagram (see the grey region in the K-diagram Figure 1). Unstable fracture will occur, when the loading condition at the crack tip, characterised by the stress intensity factors K_I and K_{II} reach any point of the 2D-fracture curve [5,13]. For Mixed-Mode and Mode II fracture the crack will kink off form its initial direction (see Figure 2). For isotropic materials the kinking angle φ_0 is a function of the K_{II}/K_I ratio only (Figure 3).
Besides other criteria [1-4], the K_I-K_{II}-fracture curve can be described by the following relation

$$K_V = \frac{K_I}{2} + \frac{1}{2}\sqrt{K_I^2 + 4(\alpha_1 K_{II})^2} \leq K_{IC} \tag{1}$$

In Eqn. 1 K_V is the effective stress intensity factor and the parameter α_1 is the ratio K_{Ic}/K_{IIc} of the fracture toughness values for pure Mode I and pure Mode II loading. For some materials the value α_1 is determined: AlCuMg1: α_1=1.05; PMMA: α_1=1.08; PVC: α_1=0.93. The well established maximum tangential stress criterion [1] and the maximum energy release criterion [3] can be well approximated by this formula with α_1=1.15. The related kinking angle can be predicted by the following relation[5]

$$\varphi_0 = \pm\left[155{,}5°\frac{|K_{II}|}{|K_I|+|K_{II}|}\right] - 83{,}4°\left[\frac{|K_{II}|}{|K_I|+|K_{II}|}\right]^2 \tag{2}$$

in which φ_0>0 has to be considered for K_{II}<0. A comparison of Eqn. 2 with experimental findings is given in Figure 3 (continuous line: Eqn. 2).

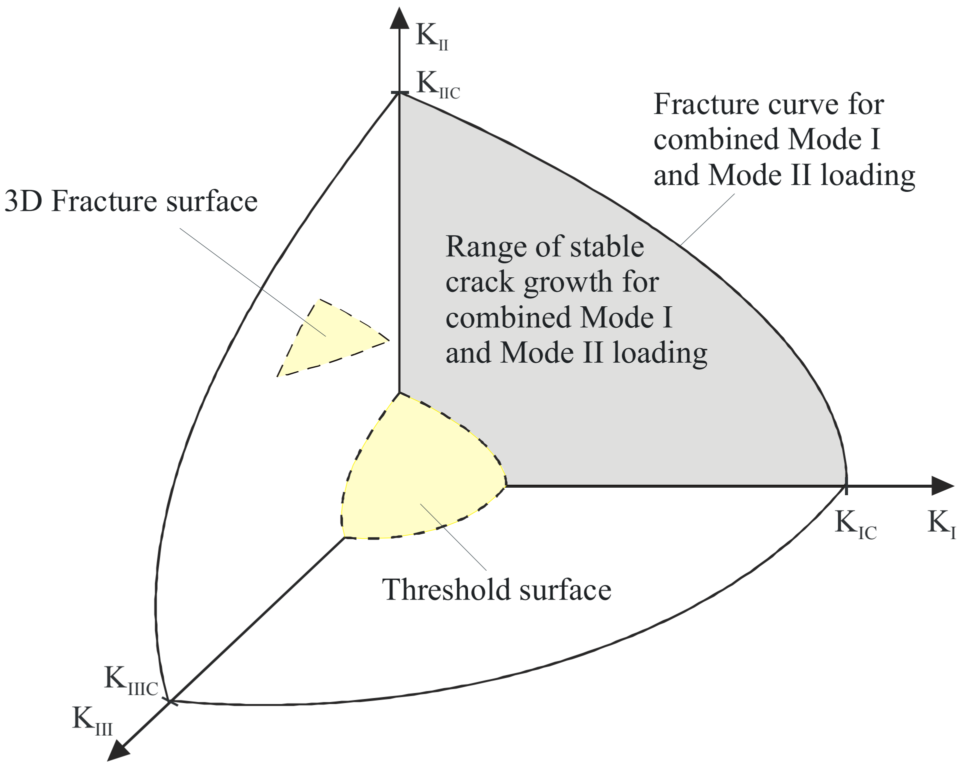

Figure 1: K-diagram for Mixed-Mode I+II+III loading

For plane Mixed-Mode cases fatigue crack growth (stable crack growth) can only develop, if the crack tip loading conditions are characterised by a point lying between the threshold curve and the fracture curve (Figure 1). In that case the effective cyclic stress intensity factor

$$\Delta K_V = \frac{\Delta K_I}{2} + \frac{1}{2}\sqrt{\Delta K_I^2 + 6\Delta K_{II}^2} \tag{3}$$

is characterising the stable crack growth, which means that the crack growth rate da/dN is a function of ΔK_V.

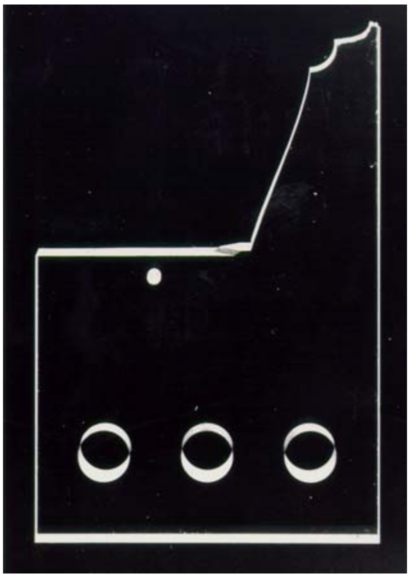

Figure 2: CTS-specimen broken under pure Mode II loading

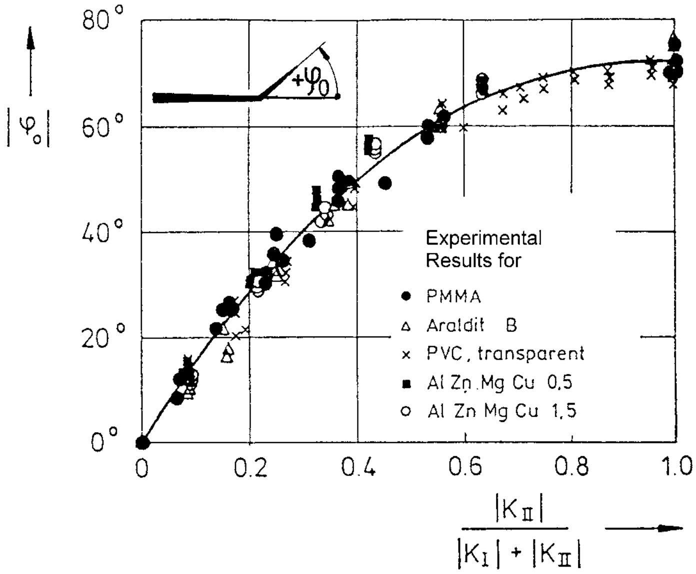

Figure 3: Measured crack deflection angels for Mixed-Mode-I+II loading

Also under fatigue loading conditions the kinking of the crack as described above can be observed. After the kinking the ratio $\Delta K_{II}/\Delta K_I$ tends towards 0, which means, that the crack propagates after the kinking without any abrupt change of direction as long as no further loading change occurs. The process of kinking results in a notable retardation of the crack growth, which can be seen in Figure 4.

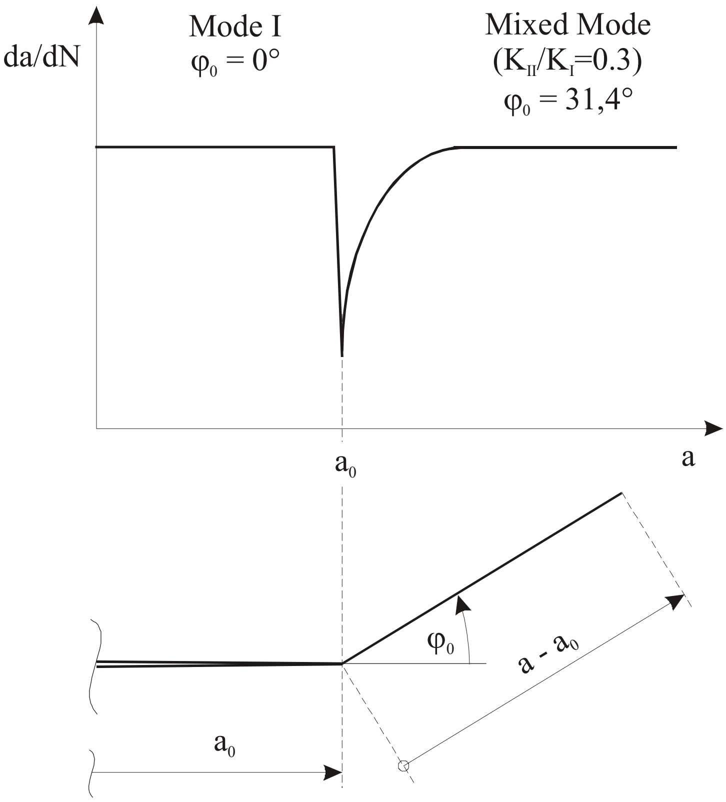

Figure 4: Retardation of crack growth rate during the kinking process after a Mode I to Mixed Mode load change

THREE-DIMESIONAL MIXED MODE FRACTURE AND FATIGUE

In 3D-Mixed-Mode cases, where all basic fracture Modes I, II and III are involved, the situation can be explained by the aid of Figure 1.
Unstable crack growth will occur, if a local loading condition along the crack front reaches a point of the fracture surface, which expands between the fracture limit curves for Mode I and II, Mode I and III and Mode II and III. This fracture surface can be described by the following relation [14]

$$\left(\frac{K_I}{K_{IC}}\right)^u + \left(\frac{K_{II}}{K_{IIC}}\right)^v + \left(\frac{K_{III}}{K_{IIIC}}\right)^w = 1 . \tag{4}$$

With u=1, v=w=2 and $\alpha_1=K_{Ic}/K_{IIc}$ and $\alpha_2=K_{Ic}/K_{IIIc}$ by Eqn. 4 a fracture criterion

$$K_V = \frac{K_I}{2} + \frac{1}{2}\sqrt{K_I^2 + 4(\alpha_1 K_{II})^2 + 4(\alpha_2 K_{III})^2} \le K_{IC} \tag{5}$$

can be established, which covers also most of the general cases, in which all fundamental fracture modes are superimposed. Whereas for plane Mixed-Mode problems the fracture limit curve is well established with $\alpha_1=1.155$, the value to be taken for α_2 has not yet been fixed. In three-dimensional Mixed-Mode cases fatigue crack propagation can be observed, if the local crack tip loading conditions, characterised by the stress intensity factors K_I, K_{II} and K_{III} can be found in between the treshold-surface and fracture surface in the K-diagram (Figure 1).

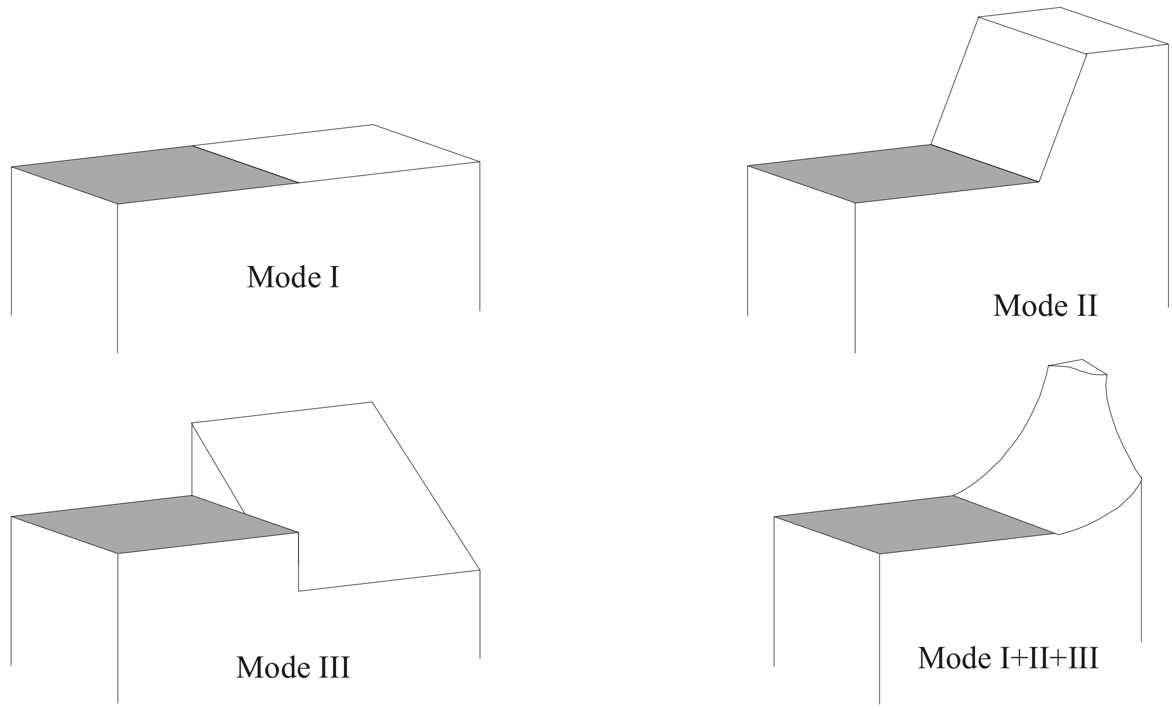

Figure 5: Principle orientation of the fracture surfaces for different fracture modes

The prediction of fracture surfaces, that describe stable and unstable crack growth (see Figure 5) is widely open. Theoretical approaches e.g. in [2,11] need to be experimentally verified [15]. This is also true for the recently published new criterion [16], which allows a 3D-Finite Element simulation of arbitrary crack problems [17,18].

For experimental investigations of crack growth and fatigue under different Mixed-Mode loading conditions several types of specimens are available [1,5,10]. But in fact none of theses specimens covers neither the full range of all basic fractures modes nor all combinations thereof. These requirements are fulfilled so far only by the AFM-specimen [14,15] in combination with the special loading device (Figure 6). After an optimisation of the AFM-specimen (Figure 7) many fatigue and fracture experiments are necessary to verify the theoretical concepts and criteria and the results of the numerical studies already performed as well as to investigate the so far unsolved Mixed-Mode problems.

Figure 6: Loading device for the AFM-specimen developed for the superposition of Mode-I+II+III

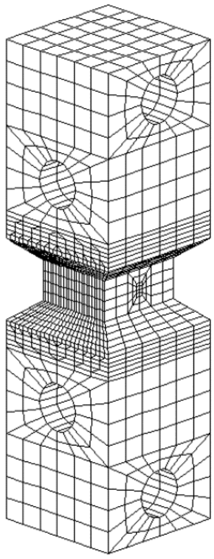

Figure 7: Finite Element model for a modified AFM-specimen

CONCLUSIONS

In this paper major aspects of Mixed-Mode crack problems are described, that are of especial relevance for industrial components and structures. It becomes apparent, that two-dimensional fracture problems in isotropic materials are widely solved. In contrast to this, further investigations of fatigue crack propagation problems especially in regions near the threshold value have to be carried out.
For three-dimensional fracture and fatigue cases many further experimental and numerical studies have to performed.

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