

## INFLUENCE OF HYDROSTATIC PRESSURE ON MULTIAXIAL FATIGUE OF NOTCHED COMPONENTS

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### ABSTRACT:

Tests have been carried out on smooth and notched specimens submitted to cyclic combined tension and torsion loading. The role of the hydrostatic pressure on multiaxial fatigue has been clarified using a volumetric approach. The average hydrostatic pressure and shear stress in the fatigue process volume have been computed by Finite Element Method. It has been shown an elliptical dependence between the effective shear stress and the hydrostatic pressure.

**KEY WORDS** multiaxial fatigue ; notch; volumetric approach ; hydrostatic pressure

### INTRODUCTION

Hydrostatic pressure has been considered by several authors as an effective parameter on fatigue resistance. The hydrostatic pressure  $\sigma_h$  is defined as the first invariant of the stress tensor. If  $\sigma_1, \sigma_2, \sigma_3$  are the three principal stress, the hydrostatic pressure is given by :

$$\sigma_h = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} \quad (1)$$

Two ways are possible :

- 1) fatigue criterion can be a combination of the second stress invariant  $J_2$  and the hydrostatic pressure according to Sines (1) Crossland (2) and Kakuno and Kawada (3)

$$\sqrt{J_2} + \kappa \sigma_{h,\max} \leq \lambda \quad (2)$$

$$\sqrt{J_2} + \kappa \bar{\sigma}_h \leq \lambda \quad (3)$$

In formulas (2) and (3),  $\kappa$  and  $\lambda$  are material constant,  $\sigma_{h,\max}$  the maximum value of hydrostatic pressure and  $\bar{\sigma}_h$  mean value.

2) fatigue criterion is considered a combination of microscopic shear stress and hydrostatic pressure.

According to Dang Van(4), fatigue crack initiation occurs in critical zones with stress concentrations or near grain plastic sliding which are unfavourably oriented in respect to external loading. Analysing the local stresses and transferring this analysis to a macroscopic scale, Dang Van defined a fatigue initiation criterion at a point and at a time which satisfies the following condition :

$$\tau(t) + A_{DV} \cdot \sigma_h(t) \leq B_{DV} \quad (4)$$

where  $A_{DV}$  and  $B_{DV}$  are material constants. The basic mechanism for fatigue crack initiation is the maximum shearing stress which occurs on the most unfavourably oriented crystallographic plane. The maximal shear stress and the plan of maximal shear stress have to be determined in order to apply this criterion. The influence of hydrostatic pressure increases linearly. The two constant are determined for two particular state of stress : torsion where the fatigue limit is  $\tau_D$  and hydrostatic pressure equal to zero and alternated tension where the hydrostatic pressure  $\sigma_D$  is and the fatigue limit  $\sigma_D$  :

$$\tau + \left( \frac{\tau_D}{\sigma_D} - \frac{1}{2} \right) \cdot \sigma_h = \tau_D \quad (5)$$

Dang Van model is the base of Flavenot and Skalli [5] critical layer criterion. Instead of computing the maximal shear stress and hydrostatic pressure on surface , they propose to compute the average values over a "critical layer" which has the same meaning that the effective distance. Examining a large range of experimental data on steel notched specimens, they found that all data fit the fatigue endurance curve  $\tau = f(\sigma_h)$  determined on smooth specimens. The best value for critical layer is determined by trial and error method and its value is order of material characteristic like grain size. In case of fatigue under combined tension and torsion the relation between shear stress and hydrostatic pressure is given by :

$$\tau + 3 \cdot \frac{2\tau_D - \sigma_D}{2\tau_D \cdot \sigma_D} \cdot \sigma_h = \tau_D \quad (6)$$

An example of Dang Van diagram is given for a steel ( $Re = 312$  MPa) in figure 1

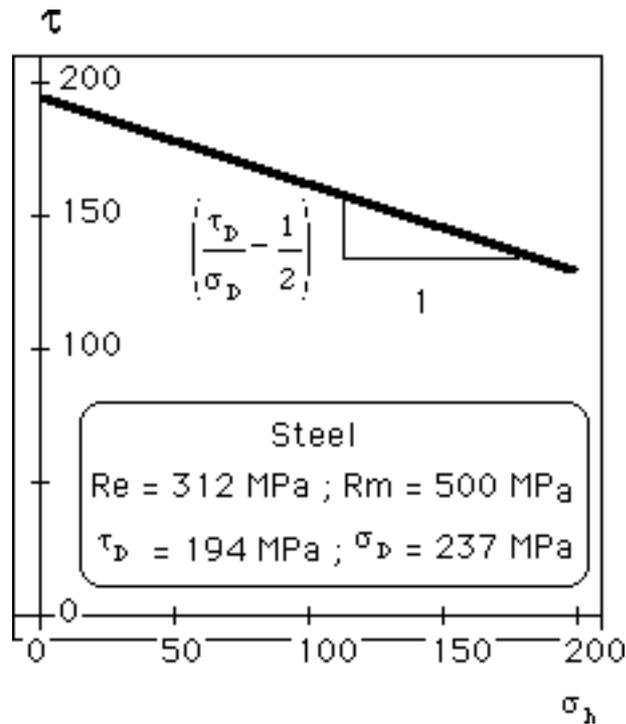


Figure 8.14 : Example of Dang Van diagram for a steel.

## PRINCIPLE OF THE VOLUMETRIC METHOD FOR PREDICTING FATIGUE LIFE DURATION

The volumetric approach is an alternative and innovative way of modelling the fatigue failure process emanating from notches. The assumption made in this approach is that the fatigue failure needs a physical volume to occur. Its extent from the notch tip is called the effective distance.

The fatigue process volume is the high stressed region. The choice of the effective distance was made by trial and error method. Verifications on different materials and specimen geometry's [6] have shown that the limit between region II and III and corresponds to the minimum of relative stress gradient. The fatigue process volume is assumed to be cylindrical with a diameter called  $X_{ef}$ . This assumption is based on an analogy with the notch plastic zone which is practically cylindrical. The effective stress range according to the volumetric approach plays the essential role in the fatigue process. It is now defined as the average of the weighted stresses in this volume. This weight stress depends on the relative stress gradient in order to take into account the loading mode and the scale effect in fatigue.

This weighed stress is defined as :

$$\sigma_{ij}^* = \sigma_{ij} \cdot \phi(r, \chi) \quad (7)$$

where  $\phi(r, \chi)$  are the weight function which depends the distance  $r$  and the relative stress gradient  $\chi$ . The weight function is assumed to be unit at notch tip and at point of maximum stress. For this reason, the choice of the weight function has the following linear form :

$$\phi(r, \chi) = 1 - r \cdot \chi \quad (8)$$

The effective stress  $\sigma_{ef}$  is defined as the average value of the weighted stress in the fatigue process volume . During fatigue propagation, crack path is always perpendicular to the maximal principal

stress. In tension or bending this stress is conventionally  $\sigma_{yy}$  stress. We can write in bidimensional case :

$$\sigma_{ef} = \frac{1}{X_{ef}} \int_0^{X_{ef}} \sigma_{ij}^* \cdot dr \quad (9)$$

The validity of volumetric method can be checked by the fact that the effective stress range versus number of cycles to failure coincide with the fatigue reference curve get from smooth specimens. The principle of volumetric method have been applied to tests performed in tension- compression figure 2a and torsion figure 2b on notched specimens (notch radius 0.4mm) made with a low strength steel (yield stress  $Re = 312$  MPa). Experimental Wöhler curve on notch and smooth specimens are presented with the computing data (full square dots). We can notice the good agreement between the prediction of the volumetric method and the fatigue reference curve (smooth specimens curve).

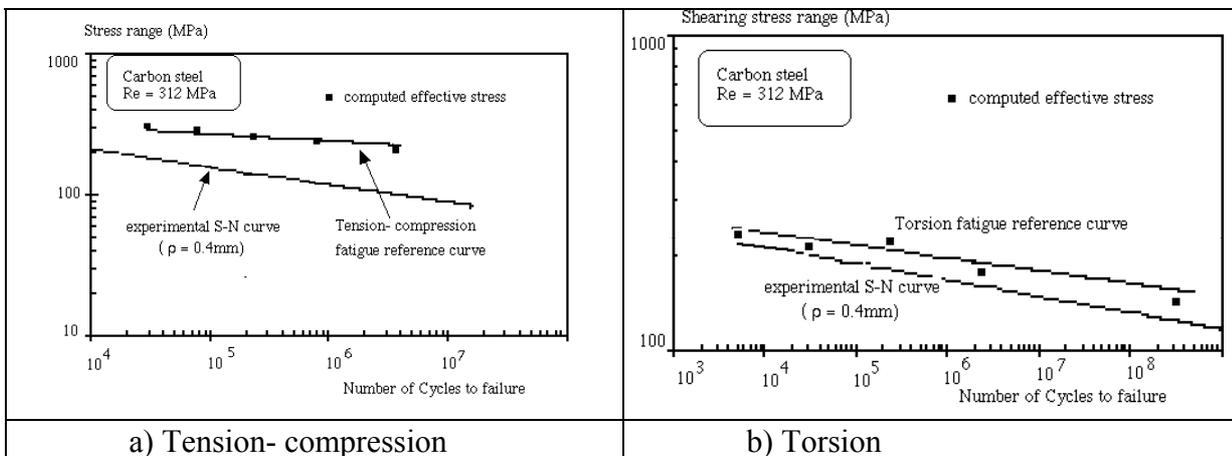


Figure 2 : Experimental Wöhler curve on notch and smooth specimens presented with the computing data get from the volumetric method.

### INFLUENCE OF HYDROSTATIC PRESSURE ON FATIGUE LIFE DURATION OF COMBINED TENSION + TORSION FATIGUE TESTS

Influence of hydrostatic pressure on fatigue life duration is examined on combined tension + torsion tests. The ratio between shear and normal stress gross stress is 2. Specimen are smooth and notched specimens described in figure 8.8.

The material used is a low carbon steel with yield stress  $Re = 312$  MPa and ultimate stress  $Rm = 500$  MPa. The notched specimens are of two types with different notch radius  $\rho = 0.2$  and  $\rho = 0.4$  mm. Basquin's coefficient for Wöhler curves have been determined and reported in table 1.

Specimen	$\sigma'_f$ (MPa)	b	$R^2$
	718	-0.066	0.9345
$\rho = 0.2$ mm	911	-0.1335	0.813
$\rho = 0.4$ mm	1035	-0.1526	0.8974

Table 1

By Finite Element Method and using elastoplastic behaviour of material, maximal principal stress  $\sigma_1$ , maximal shear stress  $\tau$ , hydrostatic pressure  $\sigma_h$  and relative stress gradient  $\chi$  were computed.

Example of such computing is given in figures 2 for (a) a smooth specimen loaded in tension and a notched specimen ( $\rho = 0.4\text{mm}$ ) loaded with combined tension + torsion.

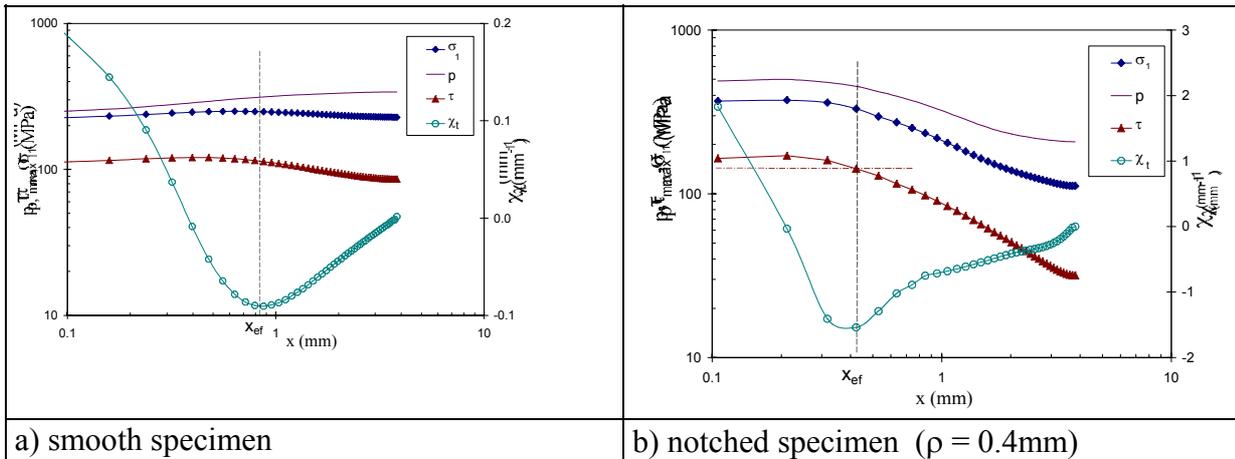


Figure 3 : maximal principal stress  $\sigma_1$ , maximal shear stress  $\tau$ , hydrostatic pressure  $\sigma_h$  and relative stress gradient  $\chi$  in a bilogarithmic diagram.

We can notice a higher relative stress gradient in presence of torsion loading. In order to take the influence of the stress gradient the effective maximal shear stress and hydrostatic pressure are computed; These quantities are defined as the average values of maximum shear stress and hydrostatic pressure over the effective distance which is defined as the distance of minimum relative stress gradient.

The effective shear stress range and hydrostatic are defined as :

$$\Delta\tau_{ef} = \frac{1}{X_{ef}} \cdot \int_0^{X_{ef}} \Delta\tau_{max} \cdot dx \quad p_{ef} = \frac{1}{X_{ef}} \cdot \int_0^{X_{ef}} p(x) \cdot dx$$

### AN ELLIPTICAL CRITERION FOR THE INFLUENCE HYDROSTATIC PRESSURE OF FATIGUE LIFE DURATION

It is assumed in Dang Van and Flavenot and Skalli's models that the influence of hydrostatic pressure is linear. Plotting the effective maximal shear stress  $\tau_{ef}$  versus effective  $\sigma_{h,ef}$  for the previous experimental results, and for different life duration, we can notice that the assumption of linear dependence is not well satisfied. In Figure 14, Dang Van's model is applied for different fatigue life durations. The maximal shear stress  $\tau_{DV}$  and hydrostatic pressure values  $p_{DV}$  according to Dang Van's model are obtained from elasto-plastic finite element computing and chosen as illustrated in Figure 4. It has been found that the correlation coefficients obtained by least square method are very low.

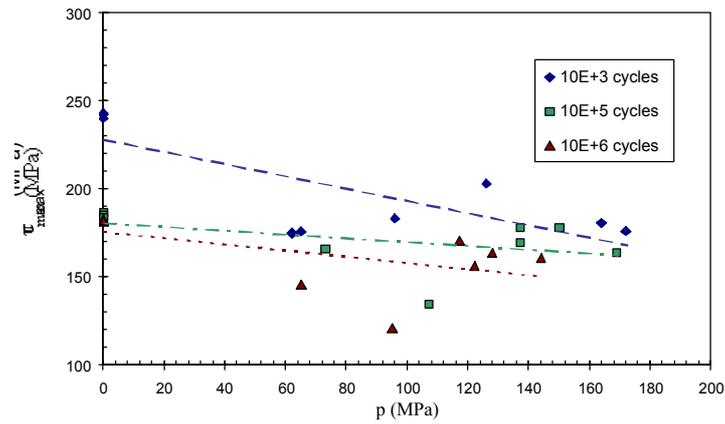


Fig. 4. Application of Dang Van's model for different fatigue life duration

A better representation can be obtained if an elliptical dependence is used (Figure 5). We can see that the size parameter of such elliptical representation is not constant but depends on fatigue life duration (figure 6).

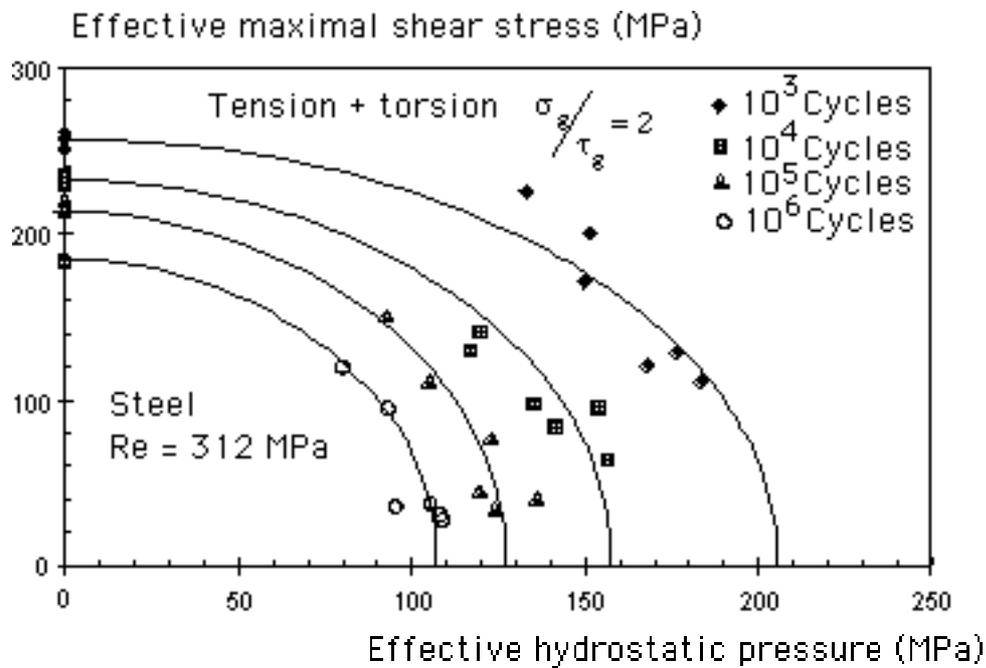


Figure 8.16 : Elliptical representation of the dependence of effective maximal shear stress  $\tau_{ef}$  versus effective  $\sigma_{h,ef}$ .

## CONCLUSION

In the above study and the experimental results it is shown that the fatigue failure of parts subjected to combined loading, as well as the fatigue under simple loading, has a volumetric character. It depends both on the shear stress and hydrostatic pressure on the most affected material volume.

From the point of view of the state of stress, the torsion loading is the easiest loading mode. The other loading modes can be considered as superposition of hydrostatic pressure to a corresponding simple torsion with the same field of shear stress, here including simple uniaxial loading. From

this fact, the use of two reference curves corresponding to alternated tension-compression and torsion, is quite obvious.

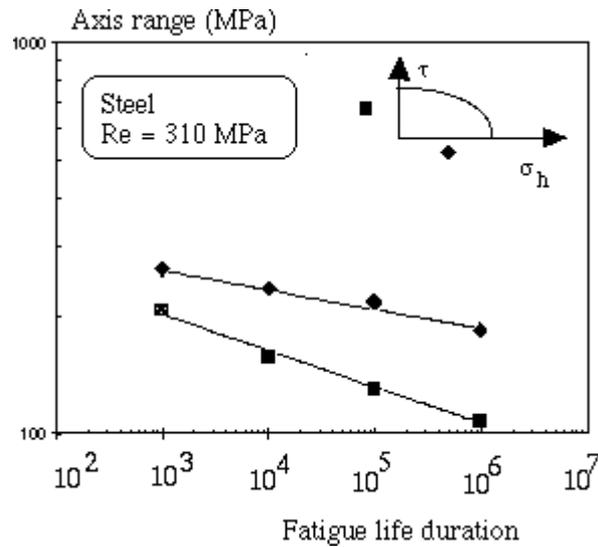


Figure 6 Ellipse axis size versus number of cycles.

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