REFERENCE: ICF100613OR

FREE EDGE AND INTERACTION EFFECTS ON THE STRENGTH OF FINITE-SIZED INTERFACE BONDS

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ABSTRACT

A fracture mechanics based analysis of the strength of a bond between a thin plate and a substrate is presented. The bond edge is regarded as a crack front, which is loaded under combined mode I, II and III conditions. A numerical procedure is used to study crack initiation. Results for bond strength are presented for bonds close to the plate edges and for closely spaced bonds.

KEYWORDS

Bond fracture, interface fracture, three dimensional fracture, fracture criteria, crack shape.

INTRODUCTION

A framework based on linear elastic interface fracture mechanics for studying initiation and propagation of cracks in bonds of arbitrary shape was formulated in Jensen [1]. The work is an extension of previous fracture mechanics based models for predicting the strength of spot welds (*e.g.* Pook [2] and Zhang [3]).

Adhesive bond failure was studied in Reedy and Guess [4] where bond strengths were correlated to the stress intensity factor for the singular stress field at the interface corner as in conventional linear fracture mechanics. The trends in the bond strength dependence on the thickness of the adhesive layer were well captured with this model. Other alternative approaches for studying interface bond failure include methods based on cohesive zone modelling such as Lin *et al.* [5] where these models have been applied to study fracture in weld joints.

The problem analysed is sketched in Fig. 1. A thin plate is bonded to a substrate in a region or several regions of arbitrary shape. The plate is loaded in its plane by a constant normal traction. The bond edge is treated as an interface crack front, and the energy release rate, the decomposition into modes I, II and III and the mixed mode fracture criterion is based on the interfacial fracture mechanics concepts in Jensen *et al.* [6]. The radius of curvature of the crack front is taken to be considerably larger than the thickness, h, of the plate. Results are presented to illustrate the effects of the plate edges and closely spaced bonds on the bond strength predictions.

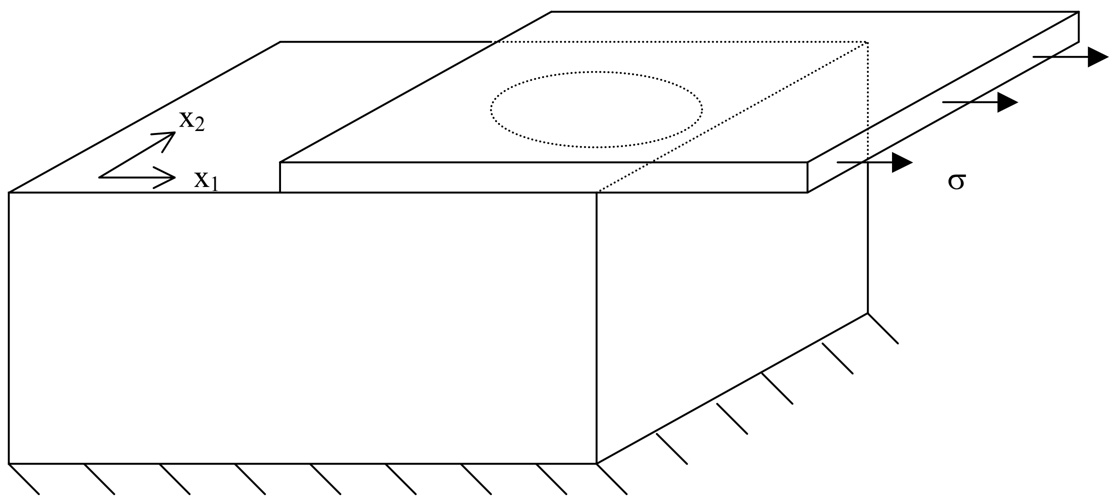

Figure 1: Schematic illustration of two overlapping plates bonded in a finite zone. The thin plate is loaded by a constant normal traction, $\sigma_{11} = \sigma$, as indicated.

FRACTURE MECHANICS

The edge of the bond zone is regarded as an interface crack front, which is subject to combined mode I, II and III loading. For the plate thickness, h, smaller than the extent of the bond zone, the energy release rate, G, and the mode I, II and III contributions to G can be calculated by the coupling of an inner, fracture mechanics based solution close to the crack tip with an outer solution for the stress state in the plate.

The relations between the combined mode I/II and the mode III energy release rates and the normal stress, σ_{nn}, and shear stress, σ_{nt}, in the plate along the crack front are given by [6]

$$G = G_{I/II} + G_{III} \quad , \quad G_{I/II} = \frac{1-\nu^2}{2E} h\sigma_{nn}^2 \quad , \quad G_{III} = \frac{1+\nu}{E} h\sigma_{nt}^2 \tag{1}$$

where E and ν are the Young's modulus and the Poisson's ratio for the plate, respectively.

A family of interface fracture criteria formulated in [6] for non-oscillating singular crack tip fields is applied here in the form

$$G_I + \lambda_2 G_{II} + \lambda_3 G_{III} = G_{1c} \tag{2}$$

where λ_2 and λ_3 denote parameters between 0 and 1 adjusting the relative contributions of mode II and III to the fracture criterion, and G_{1c} is the mode I fracture toughness of the bond. For $\lambda_2 = \lambda_3 = 1$ the fracture criterion is the Griffith criterion, while for $\lambda_2 = \lambda_3 = 0$ the fracture criterion is independent of mode II and III. The criterion (2) has been applied to thin film debonding problems in *e.g.* [6] and Jensen and Thouless [7]. It contains as a special case the fracture criterion applied for spot welds in Radaj [8] and [3]. The fracture criterion captures the mixed mode dependence of interface fracture toughness due to, for instance, plastic deformation at the crack tip or rough crack faces contacting under mode II and III dominant loading conditions.

For the present problem, which probably represents the simplest case possible since the stress state in the plate is planar so that the ratio between the mode I and mode II energy release rate along the crack front is constant, (1) and (2) reduces to

$$F \equiv \sigma_{nn}^2 + k\sigma_{nt}^2 = \sigma_c^2 \tag{3}$$

The ratio between the combined mode I/II energy release rate and the mode III energy release rate changes as the shear to normal stress ratio varies along the crack front. Denoting by ω the angle, which has been tabulated as a function of the elastic mismatch between the plate and the substrate in Suo and Hutchinson [9], then the parameters k and σ_c are given by

$$k = \frac{2\lambda_3}{(1-\nu)\left(1+(\lambda_2-1)\sin^2\omega\right)}$$
$$\sigma_c = \left(\frac{2EG_{1c}}{(1-\nu^2)h\left(1+(\lambda_2-1)\sin^2\omega\right)}\right)^{1/2} \tag{4}$$

In the following, it is assumed that $\nu = 1/3$ so that the usual mode independent Griffith fracture criterion corresponds to $k = 3$, while $k = 0$ corresponds to a fracture criterion independent of mode 3. In [6] a value close to $k = 1$ gave best agreement with experimental results for a polyamide/glass system, while the criterion applied in [8] and [3] for spot welded sheet metal corresponded to a value close to $k = 2$.

NUMERICAL RESULTS FOR BOND STRENGTH

In the calculations below, the bond shape (sketched in Fig. 1) is taken to be circular with diameter d, and its centre is located at the centre of a square plate of in-plane dimension αd with α specified below. The plate is loaded by a constant normal traction, σ, in the x_1-direction along a side parallel to the x_2-axis, and it is free along the other side parallel to the x_2-axis. The plate sides parallel to the x_1-axis are both free in the calculations below or symmetry conditions are specified on both sides so that the interaction between bonds in an infinite periodic array of identical bonds is modelled. Symmetry with respect to the x_1-axis is assumed and the results below are presented for the lower half of the crack front, only.

The stress state in the plate is solved numerically by the finite element method using typically 288 x 50 planar elements each consisting of two linear displacement triangles. The mesh is graduated with decreasing element sizes towards the interface crack for the best possible accuracy along the crack front.

In Fig. 2, the variation along the bond edge of F defined in (3) is shown in the case $k = 2$ with k defined in (4). The plate edges parallel to the x_1-axis are free in these calculations. By the fracture criterion (3), the most critical location along the bond edge, $\Theta = \Theta_c$, is at the peak value of F, which is denoted F_p so that $F(\Theta_c) = F_p$. The value of the applied traction, $\sigma = \sigma_0$, required to initiate crack propagation is

$$\sigma_0 = \frac{\sigma_c}{\sqrt{F_p}} \tag{5}$$

with σ_c given in (4). The angle Θ and the aspect ratio, α, defined as the ratio between the in-plane dimensions of the square plate and the bond diameter, are indicated in the inset in Fig. 2.

By Fig. 2 it is seen that F_p is significantly increased due to the presence of the free edge as in particular this increases the shear stress along the crack front close to the free edges. The applied traction is scaled by the aspect ratio, α, in Fig. 2 so that the resulting force acting on the bond is independent of the size of the plate. By (5) an increase in the peak value of F is equivalent to a decrease in the bond strength. Furthermore by (3) in the case $k = 2$ it is seen that the fracture criterion is more sensitive to shear stresses along the crack front than to normal stresses.

Calculations similar to those in Fig. 2 are presented in Fig. 3 but now with a value $k = 1$ so shear stresses contribute relatively less to the fracture criterion. It can be seen, as one would expect, that the strength of the bond in cases where the fracture criterion (3) applies with $k = 1$ is higher than bonds with

Figure 2: Variation along the bond edge of the function F defined in (3) for k = 2. The peak value, F_p, gives via (5) the bond strength and the location, Θ_c, along the bond edge for initial crack propagation.

Figure 3: Variation along the bond edge of the function F in the case k = 1.

k = 2. Furthermore, as the plate dimensions are decreased the most critical location along the crack front, Θ_c, decreases from 180^o to 148^o for α = 1.5. For k = 2 the critical angle increases as the plate dimensions increase relative to the bond diameter from 115^o when α = 8 to 135^o when α = 1.5.

Figure 4: Variation along the bond edge of the function F in the case k = 1 and with periodic boundary conditions.

In Fig. 4 results are presented for k = 1 for cases where the boundary conditions along the plate edges parallel to the x_1-axis are changed from being free to being periodic in order to study the interaction effects resulting from closely spaced bonds. By comparison of Figs. 3 and 4 the interaction of bonds reduces the strength more than the effect of free edges. It is emphasised that the present procedure for calculating bond strength assumes that the radius of curvature of the crack front and the distance from the crack front to the plate edges must be large compared to the plate thickness. Also effects of large-scale plastic deformation and crack face contact have not been included in the analysis.

CONCLUSION

Significant effects of edges on the strength of finite bonds within plates have been demonstrated by numerical examples. The stresses along the crack front are enhanced due to the presence of the plate edge. There is a tendency for periodic boundary conditions to raise the stress levels more than free boundaries. Especially the shear stresses are increased due to periodic boundary conditions, and dependent on the factor k in the fracture criterion (3), this will affect the strength of the bonds significantly. Calculations carried out for $k = 2$, not shown here, indicate a larger decrease in bond strength due to periodic boundary conditions than the results for $k = 1$ (compare Figs. 3 and 4).

Results for crack propagation following crack initiation under quasi-static conditions have been obtained in [1] based on a crack growth criterion. This issue has not been dealt with in the present work. It was shown in [1], however, that the ultimate strength of the bond might be significantly higher than initial strength where crack growth is initiated. Such effects could also be expected to play a role for the present problems.

ACKNOWLEDGEMENTS – Support from the the EFP-2001 research programme on scale effects in design with composites is gratefully acknowledged.

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