FRACTAL FRACTURE SURFACES AND FLUID DISPLACEMENT PROCESS IN FRACTURED ROCKS

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ABSTRACT

This paper deals with the fractal analysis of the surface roughness of rock fractures and the displacement experiments on these types of surfaces using invasion percolation simulations. Data acquired from natural and synthetic origin (developed under indirect tensional stress) fracture surfaces are subjected to fractal analysis applying different methods. Power spectral density method yields considerably lower fractal dimensions than those obtained by variogram analysis for naturally fractured surfaces. It is observed that the two methods yield more consistent fractal dimension values for synthetically fractured rock samples. The systematic difference between the fractal dimensions of horizontal and vertical profiles indicates that the variogram analysis enables us to asses the anisotropic nature of the fractal behavior. Next, invasion percolation simulations on synthetically generated (fractional Brownian motion) representative fractal fracture surfaces are conducted. It is observed that increasing surface roughness, indicated by increasing surface fractal dimension, causes less efficient displacement. Lower fractal dimension of the surfaces yields more compact cluster of injected fluid.

KEYWORDS

Fractal, fracture surface, invasion percolation, fluid displacement.

INTRODUCTION

In order to assess the transport characteristics of fractured systems, the roughness of fracture surfaces should be characterized quantitatively. For this purpose, first, a measurement technique for mapping the surface should be employed. Then, a quantitative analysis should be performed. The measurement techniques include photographic, the scattering of energy such as neutron and X-ray, laser profilometry, optical and mechanical methods. Develi et al. [1] provided an extensive review of different methods in comparison with their automated scanning device. Data obtained through these measurement methods can be used in the quantification of roughness. Since Mandelbrot *et al.* [2] have observed that the fracture surfaces of metals exhibit fractal characteristics, the fractal concept has been applied in rock fracture surface analysis. Self-affine fractal models were found to be suitable to represent natural rock fracture surfaces [3,4,5]. In this study, a brief description of a system designed to quantify the surface topography by measuring the elevations on the fracture surface was presented. Then, the results of fractal analysis of surface data acquired through this device were summarized. Next, invasion percolation simulations were applied on the synthetically generated self-affine surfaces with different fractal dimensions. In the generation of synthetic representative surfaces, typical fractal dimension values obtained through the measurements were used. Finally, the simulation results were utilized to relate the surface roughness and the fluid transport in fractures.

MEASUREMENT OF THE FRACTURE SURFACE ROUGHNESS

To map the surface roughness, a computer-controlled surface scanning device designed by Develi et al. [1] was used. The device consists of three main parts, each of them capable of moving in three orthogonal directions. The displacement of each part is supplied by three step motors. The scanning is accomplished by a needle that is capable of moving up and down. The needle is fixed at a known distance over the sample and calibrated based on this distance. Once the needle touches the fracture surface, the number of step motor rounds is counted to calculate the traveled distance from the initial point. Then, the needle moves to next pixel by means of other two step motors. When the 55x55 array of pixels (each pixel is 1 mm) have been scanned, the process is terminated. Detailed description of the system can be found in the relevant reference [1]. 3-D representations of natural and synthetic surfaces mapped using this device are shown in Fig. 1. Natural surfaces are the outcrop marble samples cored 55 mm in diameter. Synthetic surfaces were created by Brazilian tests (indirect tensile fractures) on three different marble samples. During the tests, different loading rates between 0.05 and 0.2 kN/s were applied. The fracture surfaces were 55x55 mm. Only the middle 32x32 mm part of the samples was used in fractal analysis.

FRACTAL DIMENSION CALCULATIONS

Methods Applied to Measure Fractal Dimension

The fractal dimension values were calculated using three methods applicable to self-affine fractal sets.

Variogram analysis

The variogram is defined as the mean squared increment of points:

$$\gamma(h) = \frac{1}{2n}\sum_{i=1}^{n}\left[V(x_i) - V(x_{i+h})\right]^2 \quad (1)$$

where h is the lag distance (distance between two successive points), $\gamma(h)$ is variogram at lag distance h, n is the number of pairs at a lag distance h, and $V(x_i)$ is the sample values at location x_i. Fractal distributions are characterized by a variogram model of the following form:

$$\gamma(h) = \gamma_o h^{2H} \quad (2)$$

where H is called the Hurst exponent. H is related to the fractal dimension by $D = 2\text{-}H = 2 - \beta/2$ where β is the slope of lag distance, h vs. variogram, $\gamma(h)$ plot in log-log scale and equal to $2H$.

The 32x32 points taken from the middle portion of 55x55 data set were used for the analysis. The fractal dimension of each profile with 32 points was calculated for both horizontal and vertical directions. Then the 32 fractal dimensions were arithmetically averaged. It was observed that the optimum lag distance yielding the most accurate results is between 4-6 for 32 data points [6] and therefore, maximum length distance was selected to be 5 in the application of Eq. 1.

Roughness-length method
The profile roughness is measured as the root-mean-square value of the residual on a linear trend fitted to the sample points in a window of length w [7]. Then, the root-mean-square roughness is calculated by

$$RMS(w) = \frac{1}{n_w} \sum_{i=1}^{n_w} \sqrt{\frac{1}{m_i - 2} \sum_{j \in w_i} (z_j - z)^2} \tag{3}$$

where n_w is the total number of windows of length w, m_i is the number of points in window w_i, z_j is the residual on the trend and z is the mean residual in window w_i. The fractal dimension can be calculated from $D=2-\beta$, where β is the slope of the log-log plot of the $RMS(w)$ function vs. the window length w. Fractal dimensions were obtained using 20 mm window lengths in 32 mm profile (between 10th and 30th points). Like variogram analysis, fractal dimensions of 32 profiles in both horizontal and vertical directions were calculated and averaged. This method was applied to only natural fracture surfaces.

Power spectral density analysis
The fractal dimension of 2-D data set can be calculated from the slope of a log-log plot of power $S(k)$ vs. wavenumber k. The relationship between the power, $S(k)$, and the wavenumber, k, is given as [3]:

$$S(k)\ \alpha\, k^{-\beta} \tag{4}$$

where β is the slope of the log-log plot. The fractal dimension, D, is related to the slope as follows:

$$D=(8+\beta) / 2 \tag{5}$$

Results

Results obtained by the above methods are summarized below. The power spectral measurement directly yields fractal dimension for 2-D data set. Whereas other two methods give the fractal dimension of 1-D set and these fractal dimensions were extended to represent 2-D data set by $D_{surface} = D_{profile} + 1$.

For natural fracture surfaces:

- The power spectral method yielded fractal dimensions between 1.70 and 2.25. They were expected to be greater than 2 to be in fractal regime but more than half of the samples yielded values less than 2.
- The fractal dimensions obtained through variogram analysis varied between 2.10 and 2.33 for vertical and 2.25 and 2.62 for horizontal profiles (for maximum lag distance of 5). Roughness length measurement fractal dimensions were between 2.03 and 2.52 for vertical and 2.03 and 2.63 for horizontal direction profiles. In general two methods were observed as consistent. Also, the fractal dimensions of the profiles in horizontal direction were generally greater than vertical ones.

For synthetic fracture surfaces:

- The power spectral method yielded fractal dimensions between 2.00 and 2.50. They were expected to be greater than 2 to be in fractal regime and all the values fell into this interval unlike natural fracture surfaces. The fractal dimensions of upper and lower surfaces were not consistent for any loading rate.
- The fractal dimensions obtained through variogram analysis varied between 2.21 and 2.52 for vertical and 2.43 and 2.59 for horizontal profiles (for maximum lag distance of 5). The fractal dimensions of upper and lower surfaces were consistent at any loading rate. As similar to the natural surfaces, dimensions in horizontal direction were generally higher than the dimensions in vertical direction.

The difference between the fractal dimensions of horizontal and vertical profiles indicates the anisotropic nature of the fracture surfaces. Variogram analysis and roughness length measurement were, therefore, found as suitable methods to assess this feature of the surfaces. Detailed analysis of the fracture surfaces [4-6] and selection and application criteria of the methods [6,8,9] can be found in previous publications. The results gave an idea of the typical range of the fractal dimensions. Based on these observations, computer generated fracture surfaces were created for invasion percolation simulations.

GENERATION OF SYNTHETIC SURFACES

Mid-point displacement and successive random addition algorithm introduced by Voss [10] was used to generate 2-D self-affine fractional Brownian motion (fBm) data. The method relies on the displacements of mid points using a value of independent Gaussian random variable. This process produces 2-D self-affine distributions, which have a fractal dimension $D=3-H$. Fig. 2 illustrates 3-D representation of fBm surfaces generated using the same random number seed with different fractal dimensions. As can be visually observed, increasing fractal dimension yields more tortuous surfaces. As the fractal dimension decreases and approaches Euclidean dimension of 2, smoother surfaces are obtained. Invasion percolation simulations were performed on these lattices to qualitatively analyze the displacement patterns between two fracture surfaces.

INVASION PERCOLATION SIMULATIONS

Invasion percolation generates structures that are similar to patterns obtained during displacement of one fluid with another in porous media [11]. The simulation algorithm is as follows: The invasion starts from the bottom of the lattice and the smallest pixel neighboring the invasion front is occupied first. Both sides are close to flow and once the front reaches the top of the lattice, the process is stopped. In all simulations no trapping rule was applied.

Fig. 3 demonstrates the fractal distribution of aperture divided into 100x100 grids (Fig. 3-a) and the corresponding displacement patterns (Fig. 3-b). It was observed that the aperture between two fracture surfaces yields the same fractal dimension with fracture surfaces if the aperture is defined as the difference between the pixel values of upper and lower fracture surfaces. The invasion percolation simulations represent the displacement through fracture aperture. Note that no contact exists between two walls and all points are open to flow. As the fractal dimension decreases, smoother surfaces are obtained (Fig. 3-a). Typical fractal dimension for both natural and synthetic surfaces (measured by variogram analysis and roughness length measurement) of fracture surfaces lies between 2 and 2.5. Therefore, one has to bring attention to the patterns obtained for the lattices with fractal dimensions lower than 2.5. An extreme and unusual value of fractal dimension (D=2.9) for fracture surfaces was also included to compare the displacement behavior. More compact clusters are obtained with decreasing fractal dimension of fracture surface or aperture. However, no typical shape of the invasion cluster is expected as also observed by Babadagli [12]. Thus, the shape of the displacement pattern is determined by the local heterogeneity which is a random characteristic of the fracture surface. Typically, the displacement patterns follow the same trend for all surfaces with different fractal dimensions but the displacement efficiency increases with an increase in the fractal dimension of lattice.

CONCLUSIONS

The fractal dimensions obtained by power spectral density, variogram analysis and roughness length measurement methods indicated that the naturally and synthetically developed fracture surfaces represent fractal feature with the dimension values between 2 and 2.5. The only exception for this was the fractal dimension values of power spectral method for natural fracture surfaces. The results obtained by the variogram analysis were consistent with the ones obtained by the roughness length measurement method. The systematic difference between the fractal dimensions of horizontal and vertical profiles enables us to asses the anisotropic nature of the fractal behavior. Invasion percolation simulations on synthetically generated fractal lattices revealed that more compact clusters are obtained as the fractal dimension of the surfaces decreases. This implies that the displacement efficiency in the fracture increases with decreasing fractal dimension of the fracture surfaces.

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