ENVIRONMENTAL EFFECTS ON FRACTURE AND SUB-CRITICAL CRACK GROWTH FOR LIFETIME PREDICTION

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ABSTRACT
In environmentally assisted cracking processes, various parameters in mechanics, chemistry and metallurgy have synergistic effects on fracture and are cross-linked each other. Purely theoretical formulations were developed from slip/oxidation mechanism to deal with the synergy of these parameters for quantitative prediction of crack growth and lifetime of structures in high temperature water environments especially for energy conversion systems. The formulations were derived based on an equation of metal oxidation for cracking and analysis of crack tip strain rate for a growing crack in an elastic/plastic work hardening material, in which crack tip strain redistribution due to crack advance was taken into consideration as a crucial part of cracking processes.

The obtained formulations were solved numerically to get relations between crack growth rate \( \frac{da}{dt} \) and stress intensity factor \( K \) in various situation. The well-known phenomena of existence of threshold stress intensities and plateau growth rates in \( \frac{da}{dt} - K \) diagrams for many materials/environments combinations can be reproduced by the theoretical formulations from crack tip strain rate point, without any assumption on crack tip water chemistry for the threshold or on diffusion-controlled process for the plateau growth behavior. The effects of crack tip solution chemistry and stress state on the crack growth behavior are discussed in the paper.

KEY WORDS:
Theoretical prediction, environmentally assisted cracking, stress corrosion cracking, crack growth rate, threshold stress intensity factor, crack tip strain rate, crack tip solution chemistry

INTRODUCTION
Quantitative evaluation of lifetime of structural components of aged plants and structures is one of the critical issues for safety and reliability evaluation in connection with economics. Such a quantitative capability of life-time prediction needs to be established based on understanding of physical and chemical mechanisms of fracture and sub-critical crack growth where many parameters in mechanics, chemistry and metallurgy have synergistic effects on fracture and are cross-linked each other. In this work, purely theoretical formulations have been developed to deal with the synergy of these parameters for quantitative prediction of crack growth and lifetime of structures in high temperature water environments, especially for energy conversion systems such as light water reactors. Such
environmentally assisted cracking is controlled by various parameters, it is very tedious and difficult to elucidate cracking process and to outline controlling factors quantitatively by only experiments.

**CRACK GROWTH MODEL AND ITS FORMULATION**

Environmentally assisted crack initiation and growth process in many cases especially in high temperature water environments can be described based upon a slip/oxidation mechanism [1]. This mechanism consists of the following three processes,

1. Rupture of protective film by mechanical straining at crack tip or other sites with stress concentration,
2. Enhanced anodic dissolution/oxidation of bare metal, causing crack growth,
3. Covering of bare surface by oxide or other protective film that shows repassivation.

According to this mechanism, the crack growth rate is equivalent to the oxidation rate of the metal at the crack tip.

Quantitative evaluation of crack tip strain rate is one of the most important steps in crack growth rate evaluation. There are two ways. One is to develop based on Gao and Hwang’s work on crack tip plastic strain distribution [2], as approached by Shoji et al [3, 4]. According to Gao and Hwang’s theoretical work and Gerberich et al’s experimental verification [5], crack tip plastic strain distribution along the crack line for a steady growing crack under quasi-static loading in an elastic plastic strain hardening material can be expressed as [2, 5]

\[
\varepsilon_{ct} = \beta \left( \frac{\sigma}{E} \right) - \ln \left( \frac{R_p}{r} \right) \left[ \frac{n}{n-1} \right] \left( \frac{2}{K} \right) + \frac{a}{r_0}
\]

where \( R_p = \lambda \left( K/\sigma_y \right)^2 \) is the plastic zone size, \( \beta \) and \( \lambda \) are dimensionless constants, \( \sigma_y \) and \( E \) are yield strength and elastic modulus, respectively. \( r \) is distance from a growing crack tip, \( K \), stress intensity factor, and \( n \), strain-hardening exponent in power law hardening Ramberg-Osgood stress-strain relations.

Based upon this strain distribution expression, crack tip strain rate can be derived and expressed as follows by considering strain increment due to both, change in distance ‘r’ from crack tip with crack advance and change in \( K \) caused by load change and/or crack growth [3, 4], even though these situations are beyond the conditions which Gao and Hwang were assumed to derive Eq. (1) because of the experimental verification made beyond the theoretical assumption.

\[
\frac{d \varepsilon_{ct}}{dt} = \beta \left( \frac{\sigma}{E} \right) - \frac{n}{n-1} \left[ \ln \left( \frac{\lambda \cdot K^2}{\sigma_y^2 r_0} \right) \right] \left( \frac{2}{K} \right) + \frac{a}{r_0}
\]

where, \( \dot{K} = dK/dt \), \( \dot{a} = da/dt \), \( r_0 \) is distance ahead of the crack tip where strain rate is defined.

Another way to evaluate crack tip strain rate is based on J. R. Rice et al’s work on crack tip opening displacement rate for a growing crack under plain-strain condition in elastic-perfectly plastic solids (no strain-hardening) [6]. The crack opening displacement rate can be converted to crack tip strain rate by assuming a length parameter such as the total width of the active flow bands. This length parameter was also assumed to be the same as \( r \) in the calculation [7]. The validity of these assumptions, however, has not been verified.

Currently, Gao and Hwang’s expression is the best available for evaluation of crack tip strain rate, as has been experimentally verified by Gerberich et al [5], and this expression takes into consideration the effects of strain hardening that is important for engineering materials, although it may have some limitations as it was developed for steady crack growth under quasi-static loading. More work on this issue should be focused by, for example, FEM in the future.

Based upon Eq. (2) and slip/oxidation mechanism, theoretical crack growth rate can be derived as follows for plane strain condition, with details described in Ref. [1, 3, 4]
\[
\frac{da}{dt} = \frac{M \cdot i_0}{z_p F (1 - m) \varepsilon_f} \left( \frac{t_0}{r_0} \right)^n \left[ \beta \sigma_I \frac{n}{E} \left( -1 + \ln \left( \frac{\lambda \cdot K}{r_0} \right) \right)^{n-1} \left( 2 \frac{K}{K} + \frac{\dot{a}}{r_0} \right) \right]^n
\]  

(3)

where \( r_0 \) is a characteristic distance for which a strain rate is defined and can be assumed as a material property depending on the microstructure such as grain size.

Similar equation for crack growth rate can be derived for plane stress condition as shown below:

\[
\frac{da}{dt} = \frac{M_i_0}{z_p F (1 - m) \varepsilon_f} \left( \frac{t_0}{r_0} \right)^n \left[ \beta \sigma_I \frac{n}{E} \left( -1 + \ln \left( \frac{\lambda \cdot K}{r} \right) \right)^{n-1} \left( \frac{K}{K} + \frac{\dot{a}}{r} \right) \right]^n
\]  

(4)

These crack growth rate formulas are unique expressions for environmentally assisted crack growth rate as a function of stress intensity factor by combining a mechanism of crack growth and mechanics of crack tip stress/strain field. These equations also clearly show the synergistic effects of material parameters, mechanical properties and electrochemical properties. It is important to note here that the crack tip strain rate is a function not only of loading rate but also of crack growth rate for growing crack. Hence, as a typical case, crack tip strain rate does exist even under a constant load or a constant displacement as long as crack grows.

**NUMERICAL SOLUTION AND RESULTS**

**General trend of crack tip strain rate as a function of K**

Eq. (2) describes the crack tip strain rate as a function of \( K, \frac{dK}{dt}, \frac{da}{dt}, \) and mechanical properties. Eq. (2) can be solved numerically and some results are shown in Figs. 1 and 2 as examples. These plots demonstrate the effects of yield strength and crack growth rate upon the relationship between the crack tip strain rate and \( K \). The crack tip strain rate strongly depends on \( K \) in a relatively low \( K \) range, whereas less dependence can be seen at the higher \( K \) values. As expected from Eq. (2), the crack tip strain rate shows a threshold depending on the yield strength and characteristic distance, \( r_0 \). Smaller yield strength tends to give a lower value of threshold \( K \), which is shown in Fig. 1. In the vicinity of a threshold, the crack tip strain rate decreases rapidly with decreasing \( K \) to threshold stress intensity of \( K_{th} \). Below \( K_{th} \), the value in the logarithmic term of Eq. (2) becomes less than unity and the rate becomes a complex number. The physical meaning in such solutions is not clear at this moment.

[Fig. 1 Effect of yield strength on the crack tip strain rate for \( n = 5, \frac{da}{dt} = 10^{-9} \text{ m/s} \) and \( \frac{dK}{dt} = 0 \).]

[Fig. 2 Effect of crack growth rate on the crack tip strain rate for \( \sigma_y = 500 \text{ MPa}, n = 10, \) and \( \frac{dK}{dt} = 0 \).]
Significance of crack growth rate in crack tip strain rate evaluation under constant K/load tests

The effect of crack growth rate on the crack tip strain rate is shown in Fig. 2, which shows the crack tip strain rate as a function of stress intensity under static constant K test such as \( \frac{dK}{dt} = 0 \). Under constant K or load mode, a growing crack brings about the crack tip strain rate that shows a significant K-dependence. The behavior of the crack growth rate as a function of stress intensity factor is similar to that of crack tip strain rate with the same variable of K, showing a threshold behavior and a plateau crack growth region at the prevailing stress intensity level.

Sensitivity analysis of crack growth formulation

Eq. (3) can be used in estimating the crack growth rate in plane strain under dynamic and static loading conditions, which can be expressed in terms of \( \frac{dK}{dt} \). In addition, important mechanical and electrochemical parameters affecting the cracking processes are included in Eq. (3). The last term in Eq. (3) in curly brackets, for example, stands for the strain rate, which has an interaction with parameter of “m”. The parameter, m is determined by the film formation kinetics and the crack tip solution chemistry.

Fig. 3 Evaluation of crack growth rate as a function of K under RLT and constant K

Table 1 Summary of parameters used in the calculation of crack growth rate plotted in Fig. 3

<table>
<thead>
<tr>
<th>Case #</th>
<th>m</th>
<th>n</th>
<th>( \sigma_y ) (MPa)</th>
<th>( \frac{dK}{dt} ) (MPa(^{1/2})/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5</td>
<td>3</td>
<td>1000</td>
<td>( 1 \times 10^{-5} ) (RLT)</td>
</tr>
<tr>
<td>2</td>
<td>0.5</td>
<td>3</td>
<td>500</td>
<td>( 1 \times 10^{-5} ) (RLT)</td>
</tr>
<tr>
<td>3</td>
<td>0.5</td>
<td>3</td>
<td>200</td>
<td>( 1 \times 10^{-5} ) (RLT)</td>
</tr>
<tr>
<td>4</td>
<td>0.5</td>
<td>3</td>
<td>200</td>
<td>0 (Constant K)</td>
</tr>
<tr>
<td>5</td>
<td>0.8</td>
<td>10</td>
<td>200</td>
<td>0 (Constant K)</td>
</tr>
<tr>
<td>6</td>
<td>0.8</td>
<td>3</td>
<td>800</td>
<td>0 (Constant K)</td>
</tr>
<tr>
<td>7</td>
<td>0.8</td>
<td>10</td>
<td>800</td>
<td>0 (Constant K)</td>
</tr>
</tbody>
</table>

Note: RLT: Rising Load Test. All other parameters used for calculation are as follows:

\( \beta: 5.08, \lambda: 0.5, M: 55.85, z: 2.67, p: 0.00786 \text{ g/mm}^2, F: 96500 \text{ C}, \varepsilon_f: 0.001, r_o: 0.03 \text{ mm}, t_o: 0.3 \text{ s}, i_o: 0.005 \text{ A/mm}^2, E: 210 \text{ GPa} \)

Figure 3 shows the effect of yield strength, strain hardening exponent “n”, and repassivation kinetics parameter “m” on the crack growth rate as a function of K for given loading conditions, which are listed in Table 1. The profiles of crack growth rate as a function of K are close to the behavior of crack tip strain rate, both of which show a threshold and plateau region.

The general trend of crack growth response to yield strength is similar to that obtained from the calculation of crack tip strain rate. Higher yield strength results in higher crack growth rate at a given K level. While higher yield strength presents a higher threshold K value in crack propagation, the strain hardening exponent “n” does not affect the threshold K, which can be expected from Eq. (3). The repassivation kinetics parameter “m” has a strong impact on the crack growth behavior. Generally a lower value of “m”, corresponding to slower repassivation kinetics, gives higher crack propagation.

The effect of yield strength on crack growth rate is significant irrespective of repassivation kinetics, which can be verified by comparing Case #1, 2 and 3 for m=0.5 and also Case #5 and 7 for m=0.8. This fact can be rationalized under the condition that all parameters are kept the same as shown above.

The crack growth behavior also is determined by the repassivation kinetic parameter “m”, which is the slope of the current transient curve for a metal surface exposed to aqueous environment. Comparison of Case # 1 to 3 (m=0.5) and Case # 5 to 7 (m=0.8) illustrates the significance of the repassivation
behavior. At the higher value of m, the crack growth rate is several orders of magnitude lower than that at lower m. Besides, comparison of Case #3 and #4 presents the effect of loading rate on crack growth rate. In this particular case, no significant difference could be found between Rising Load Test (RLT, dK/dt=1×10^{-5} MPa m^{1/2}/s) and Constant K Test.

The effect of strain hardening exponent “n” on crack growth rate can be seen from comparing Case #6 vs. 7 where n values are 3 and 10, respectively. Higher n value, corresponding to less strain hardening, gives rise to lower crack growth. Such a fact can be important to understand the behavior of irradiation-assisted stress corrosion cracking (IASC) in that mechanical properties of materials such as yield strength and strain hardening exponent change due to the evolution of microstructure and grain boundary chemistry under irradiation.

**DISCUSSION**

**Evaluation of crack tip solution chemistry and its implication for crack growth response**

Figure 4 shows a clear evidence of how the crack tip solution chemistry controls the crack growth. Cracking completely comes to an arrest due to the removal of crack tip solution by microsampling. Microsampling extracts pre-existing aggressive crack tip solution containing anions, and the crack tip is replenished with bulk solution containing high dissolved oxygen. Consequently, the potential gradient and pH become in equilibrium. It takes time for the depletion of dissolved oxygen inside the crack and for the creation of potential gradient, which later causes the accumulation of anions at the crack tip. Once anions accumulate up to sufficient concentration, the quiescent crack might be activated again.

![Fig. 4 Variation of crack growth behavior with microsampling for low alloy steel SQV2A (0.021%S)](image)

**Effects of stress state on crack growth behavior**

Figure 5 shows the numerical results of crack growth rate that is calculated in plane strain and stress condition for three values of “m”, 0.4, 0.5 and 0.6 [4]. In this calculation, the equations of (3) and (4) were used and parameters were kept the same as those listed in Table 1. It is clearly seen that under plane stress condition the crack growth rate in terms of K is higher than under plane strain one. Also, crack growth takes place at lower K level under plane stress condition. This trend can be understood from the fact that smaller plastic constraint at the crack tip provides a higher crack tip strain rate for a given K. Such a state becomes important for the plant lifetime prediction where defects are found as part-through cracks in most cases and the stress condition in near surface region comes close to a plane stress condition. Also, plastic constraint at crack front for small cracks would be small in plane stress because the stress state is close to uniaxial stress state. Very small cracks can cause high strain rate, which might be higher than that in plane stress. However, this situation becomes complicated when a strain concentration is taken into account. In order to explain this important problem, it is required to
analyze the strain distribution for a growing crack, particularly part-through cracks using 3D finite element method (FEM).

Fig. 5 Effects of stress state and repassivation kinetic parameter upon crack growth rate [4]

CONCLUSIONS
Based on slip-oxidation mechanism and crack tip mechanics, purely theoretical formulations have been developed to elucidate environmentally assisted cracking process and to outline controlling factors. The well-known phenomena of existence of threshold stress intensities and plateau growth rates in $da/dt$-$K$ diagrams for many materials/environments combinations can be reproduced by the theoretical formulations from crack tip strain rate point.

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