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EFFECTS OF T–STRESS AND VOID NUCLEATION ON COHESIVE ZONE MODEL PREDICTIONS FOR DUCTILE FRACTURE

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ABSTRACT

Special interface elements that account for ductile failure by void growth to coalescence are used to analyse crack growth under mode I loading conditions. The tangential stresses inside the interface are determined by requiring compatibility with the surrounding material in the tangential direction. Earlier applications of the model are here extended to consider the effect of a T–stress or effects of void nucleation inside the interface elements.

KEYWORDS

Cracks, voids, fracture, plasticity.

INTRODUCTION

For the analysis of crack growth by a ductile failure mechanism the fracture process zone is represented in terms of special interface elements in the crack plane ahead of the crack. The modified Gurson model is used in the interface elements to represent the nucleation and growth of voids to coalescence. In relation to the finite element mesh the initial width of the interface is taken to be zero, but the traction separation relations represented by the interface are based on assuming an interface width of the order of the void spacing.

The present interface element formulation is not based on assuming that the plastic strains or the stress state inside the interface are identical to the values of these field quantities in the material adjacent to the interface (as in Tvergaard and Hutchinson [1] or Sigmund and Brocks [2]) . Instead, the present model enforces the continuity of the longitudinal strain along the band, across the band interface. Thus, the stiffnesses of the cohesive zone element give a coupling between the displacement normal to the band and those tangential to the band.

Predictions of this cohesive zone model have been presented previously, for growth of pre–existing voids under mode 1 loading conditions (Tvergaard [3]). In the present paper the model is extended to account for the effect of void nucleation inside the interface elements. Furthermore, the model is used to estimate the effect of a T–stress on predicted crack growth resistance curves.

DUCTILE FAILURE MODEL FOR INTERFACE

In the computations the initial width of the special interface element is taken to be zero, as in other cohesive zone calculations [1]. However, the traction–separation properties of the interface element are calculated based on

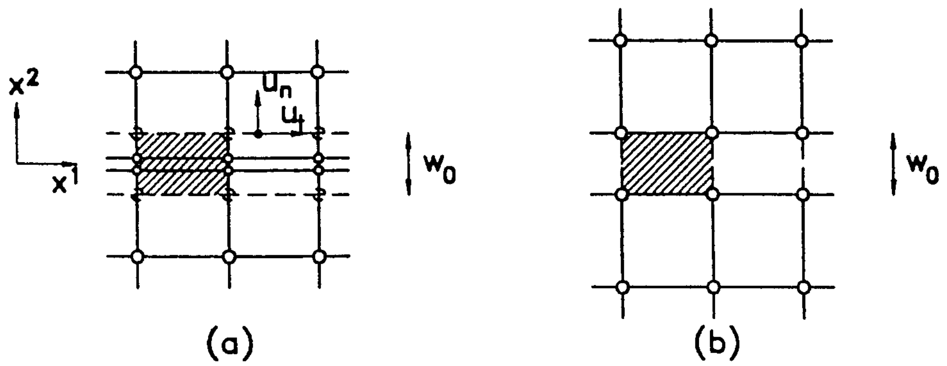

Fig. 1: Interface elements along the crack plane. (a) Shows the artificial overlap between interface elements and surrounding elements. (b) A corresponding configuration with no overlap.

a background element with the non–zero initial width w_0 . Fig. 1a illustrates the finite element mesh near the line of symmetry, with the background interface element sketched in, while Fig. 1b indicates the configuration if this interface element is attached to the surrounding elements as a common element. The displacement components u_n and u_t on the top side of the interface element (Fig. 1a) are required to be compatible with the displacements on the edge of the adjacent finite element. Then with the assumption that the displacements inside the interface element vary linearly through the element width (in the x^2–direction), the displacement gradients inside the element are

$$\frac{\partial u_1}{\partial x^1} = \frac{\partial u_t}{\partial x^1} \quad , \quad \frac{\partial u_2}{\partial x^2} = \frac{2u_n}{w_0} \quad , \quad \frac{\partial u_1}{\partial x^2} = 0 \tag{1}$$

Furthermore, when choosing to replace displacement gradients by their averages through the element width, we find $\partial u_2/\partial x^1 = 0$.

From these displacement gradients it is possible to determine the current metric tensor G_{ij} , the Lagrangian strain tensor η_{ij} and its increment $\dot{\eta}_{ij}$ in any point on the middle surface, $x^2 = 0$, of the interface element. Thus the evolution of stresses and of damage inside the interface can be calculated from a set of constitutive relations. These constitutive relations are based on the modified Gurson model [4,5] , which makes use of an approximate yield condition for a porous solid

$$\phi = \frac{\sigma_e^2}{\sigma_M^2} + 2q_1 f^* \cosh\left[\frac{\sigma_k^k}{2\sigma_M}\right] - 1 - \left(q_1 f^*\right)^2 = 0 \tag{2}$$

where $\sigma_e = (3s_{ij}s^{ij}/2)^{1/2}$ and $s^{ij} = \sigma^{ij} - G^{ij}\sigma_k^k/3$. The void volume fraction is f , and $f^*(f)$ is a function that approximately describes final failure by void coalescence. The change of the void volume fraction during an increment of deformation is taken to be given by

$$\dot{f} = (1 - f)G^{ij}\dot{\eta}_{ij}^P + \mathcal{A}\dot{\sigma}_M + \mathcal{B}\left(\dot{\sigma}_k^k\right)/3 \tag{3}$$

where the first term results from the growth of existing voids, and the two last terms model the increment due to nucleation [6] .

The numerical solutions are obtained by a linear incremental finite element method, based on the incremental principle of virtual work, with the material outside the interface represented by finite strain J_2 flow theory. The details of the computational method are given in [3] and shall not be repeated here. As described in [3] , an extra parameter r_0 is introduced, to be able to reduce the tangential components of the nodal forces calculated by the interface element procedure described above. Thus, if $\left(P_t\right)_{elm}$ denotes the interface element contribution to the value of the nodal force component tangential to the interface, as calculated by the interface element procedure, the tangential component P_t actually applied in the solution is taken to be given by

$$P_t = r_0\left(P_t\right)_{elm} \tag{4}$$

Here, $r_0 = 0$ will be used, as this gives the best representation of an initially sharp crack (see further discussion in [3]). The value of the initial width w_0 of the strip (Fig. 1) is non–zero, representing approximately 0.7 times the initial void spacing. In the small–scale yielding solution for crack growth, the value of the J–integral is calculated on a number of contours around the crack–tip to check agreement with the prescribed amplitude K of the edge displacements (i.e. $K^2 = JE/(1 - \nu^2)$), and very good agreement is found. For the presentation of the results reference values for the J–integral and for the corresponding size of the plastic zone are defined

$$J_0 = \sigma_y w_0 \ , \ \ R_0 = \frac{1}{3\pi}\left(\frac{K_0}{\sigma_y}\right)^2 \tag{5}$$

where the value K_0 corresponds to J_0 .

According to the small strain linear elastic solution the in–plane stress components near the crack–tip are of the form

$$\sigma_{\alpha\beta} = \frac{K}{\sqrt{2\pi r}} f_{\alpha\beta}(\theta) + T\delta_{1\alpha}\delta_{1\beta} \tag{6}$$

where $(r \ , \ \theta)$ are polar coordinates, δ_{ij} is Kronecker's delta and T is a non–singular stress term, acting parallel to the crack plane. In the present analyses the T–stress is applied first, together with the corresponding transverse stress $\sigma_{33} = \nu T$ under plane strain conditions.

RESULTS

The elastic–plastic material parameters in the first case analysed are specified by $\sigma_y/E = 0.002$ and $\nu = 0.3$ with the power hardening exponent $N = 0.2$. Furthermore, in the porous ductile material model the initial void volume fraction is $f_I = 0.01$, and other parameter values in the model are $q_1 = 1.25$, $f_C = 0.15$ and $f_F = 0.25$. Fig. 2 shows predicted crack growth resistance curves for various values of the T–stress according to (6) . It is well knows from other fracture models, e.g. [7] , that positive values of T tend to have little effect on the fracture toughness, while negative values of T tend to increase the toughness. Therefore, Fig. 2 uses the curve for $T = 0$ as reference. The other three curves show that also the present interface model for ductile fracture predicts increasing fracture toughness for increasing negative values of T.

In Fig. 3 the level of strain hardening is lower, $N = 0.1$, but otherwise all material parameters are identical to those considered in the previous figure. It is seen that also here the negative T–stresses give a significant increase of the fracture toughness, in fact a relatively much larger increase than that found in Fig. 2. This in qualitative agreement with predictions of a more standard cohesive zone model [7] , where it was also found that the T–stress effect is stronger for the more low hardening material.

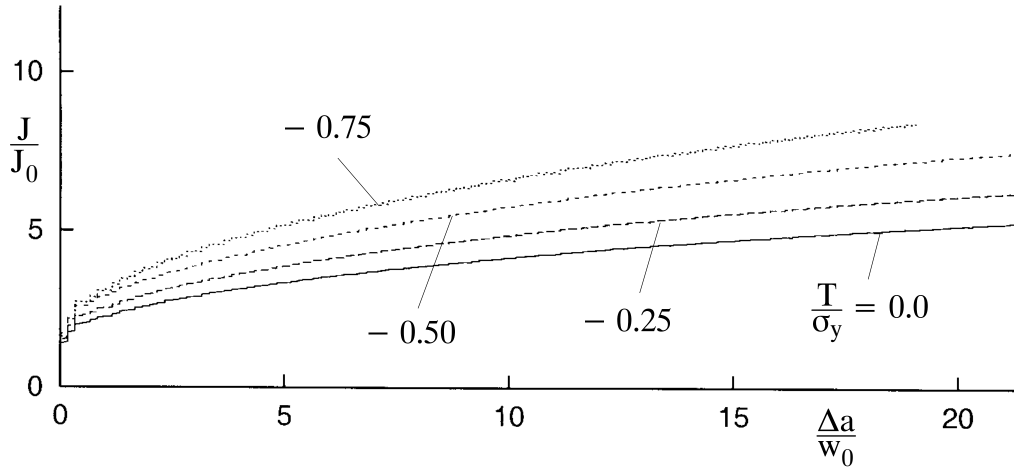

Fig. 2: Crack growth resistance curves for various values of the T–stress, when $f_I = 0.01$, $N = 0.2$ and $r_o = 0$.

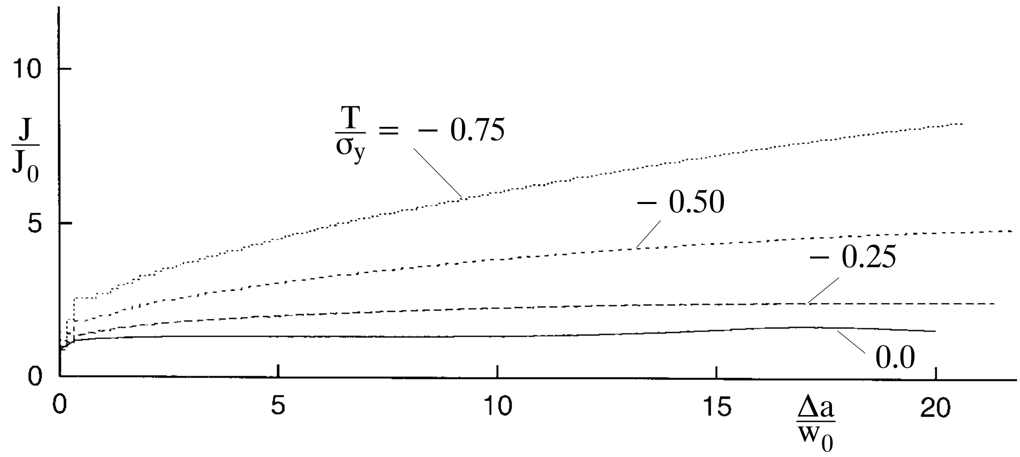

Fig. 3: Crack growth resistance curves for various values of the T–stress, when $f_I = 0.01$, $N = 0.1$ and $r_o = 0$.

The effect of void nucleation is not easily accounted for in interface elements or those of [2] , where the dependence on stress triaxiality outside the interface is based on fitting with various cell model studies. However, in the present interface elements nucleation criteria can be specified in the usual manner for Gurson model elements, as the stress and strain fields in the interface elements are fully specified by the conditions of equilibrium and compatibility with neighbouring elements specified above. Nucleation is here taken to be plastic strain controlled, as modelled in terms of (3) by taking $\mathcal{A} > 0$ and $\mathcal{B} = 0$. The dependence of $\mathcal{A}$ on the effective plastic strain is specified such that nucleation follows a normal distribution [8] , with the volume fraction f_N of void nucleating particles, the mean strain for nucleation ϵ_N , and the corresponding standard deviation s.

Fig. 4 shows two examples of crack growth resistance curves predicted with void nucleation in the interface. Both curves use the parameter values $f_N = 0.04$ and $s = 0.1$, so the only difference is that the lower curve has a smaller mean strain for nucleation, $\epsilon_N = 0.1$, than the value $\epsilon_N = 0.2$ used for the upper curve. It is

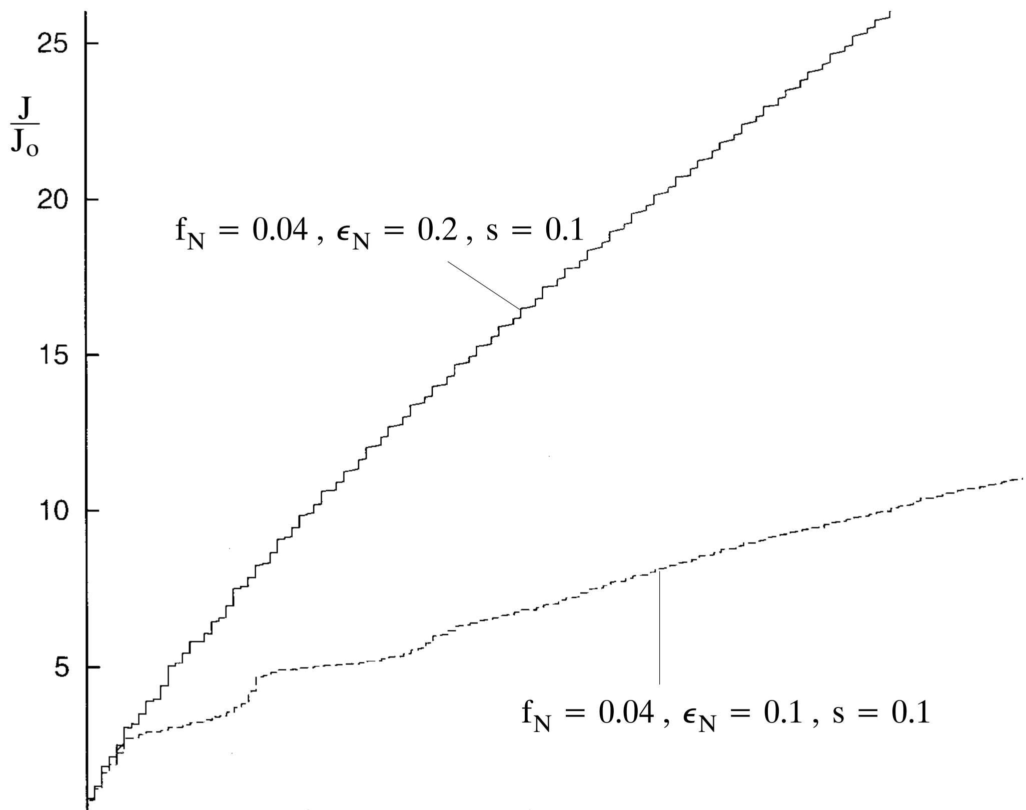

Fig. 4: Crack growth resistance curves for cases with plastic strain controlled nucleation in the interface elements, when $f_I = 0.0$, $N = 0.1$, $r_o = 0$ and $T = 0$.

seen that the material with no initial voids but with void nucleation can show a significantly higher crack growth resistance than that found in Fig. 3 for a material with voids present initially.

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