

# EFFECT OF THE NOTCH SHAPE AND THE PRESENCE OF A CIRCULAR VOID IN FRONT OF A CIRCULAR NOTCH ON THE FAILURE MODE TRANSITION SPEED IN AN IMPACT LOADED PLATE

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## ABSTRACT

We study dynamic thermomechanical deformations of a prenotched plate impacted on the notched side by a cylindrical projectile of diameter equal to the distance between the notches, and made of the same microporous thermoviscoplastic material as the plate. For the ratio of the semimajor to the semiminor axes of the notch tip equal to 0.4, 1.0, 2.0 and 10.0, the impact speed at which the failure mode transitions from the brittle to the ductile is computed. For a circular notch-tip of radius  $r_0$ , we delineate the effect on the failure-mode initiation of the presence of a hole of radius  $r_0$  and situated directly ahead of the notch tip at distances of  $3r_0$ ,  $4r_0$ ,  $5r_0$  and  $6r_0$ .

## 1 INTRODUCTION

Kalthoff (2000) and Kalthoff and Winker (1987) have experimentally studied dynamic deformations of a rectangular high strength steel plate containing two notches, with circular notch tips, parallel to the top edge of the plate and extending to the middle of the plate. They found that, with an increase in the impact speed, the failure mode changed from a brittle failure in the form of a crack propagating at about  $70^\circ$  to the notch-axis to a ductile failure in the form of a shear band emanating essentially parallel to the axis of the notch. The failure mode transition speed was found to depend upon the notch tip radius  $r_0$ . Needleman and Tvergaard (1995), Zhou et al. (1996), Batra and Nechitailo (1997), Batra and Gummalla (2000), Batra and Ravisankar (2000) and Batra and Jaber (2001) have numerically studied the problem. Whereas Batra and Ravisankar analyzed the three-dimensional problem, other investigations scrutinized plane strain deformations of the plate. These works employed different thermoviscoplastic relations and obtained qualitatively similar results. The maximum principal tensile stress was found to occur at a point a little above the upper surface of the notch tip, and the shear band initiated from a point on the lower surface of the notch tip, which is closer to the impacted surface of the plate. Only Batra and Jaber (2001) computed the failure mode transition speed and found it to depend upon the thermoviscoplastic relation employed to characterize the material response. Here we explore the effect on the failure mode transition speed of the notch tip shape and the presence of a circular hole ahead of a circular notch tip.

## 2 FORMULATION OF THE PROBLEM

A schematic sketch of the problem studied is shown in Fig. 1. A rectangular plate with two parallel notches extending nearly to the vertical centroidal axis is struck at normal incidence on the notched side by a cylindrical projectile made of the same material as the plate. The diameter of the projectile equals the distance between the

notches, and its speed equals  $v_0$ . The plate is assumed to be made of a homogeneous and isotropic microporous thermoviscoplastic material, and it rests on a flat rigid smooth surface prior to being impacted. Batra and Ravisankar (2000) analyzed three-dimensional deformations of the plate and the projectile and found that the deformations of the central 75% of the thickness of the material closely resembled those obtained by the two-dimensional plane strain analysis. Here we do not study deformations of the projectile, replace its action on the plate by prescribing the normal component of velocity of the plate particles on the impacted surface, and presume that a plane strain state of deformation prevails in the plate. All of the contact surfaces are taken to be smooth, and the bounding surfaces including those of the notches thermally insulated. However, heat conduction is considered in the plate. Noting that the bottom edge of the plate is far away from the notch, we assume that deformations of the prenotched plate are symmetrical about the horizontal centroidal axis, and examine deformations of only the upper half of the plate. Thus at a point on the plane of symmetry, the vertical component of displacement, the tangential traction and the normal component of the heat flux are set equal to zero.

The thermomechanical deformations of the plate are governed by the balance of mass, linear momentum, moment of momentum and internal energy. The hypoelastic material of the plate is modeled by a linear relation between the Jaumann derivative of the Cauchy stress tensor  $\boldsymbol{\sigma}$  and the elastic part of the strain-rate tensor  $\mathbf{D}$  with Young's and the bulk moduli decreasing affinely with an increase in the porosity. The material obeys Gurson's (1977) yield criterion as modified by Tvergaard and Needleman (1984), the associated flow rule, Fourier's law of heat conduction with the thermal conductivity decreasing affinely as the porosity increases, and the Bodner-Partom (1975) thermoviscoplastic relation

$$\begin{aligned}\dot{\varepsilon}^p &= D_0 \exp \left[ -\frac{1}{2} \left( \frac{K^2}{3\sigma_e^2} \right)^n \right], \\ K &= K_1 - (K_1 - K_0) \exp(-mW_p), \quad n = \frac{\hat{a}}{\theta} + \hat{b}, \\ W_p &= \int_0^t \text{tr}(\boldsymbol{\sigma} \mathbf{D}^p) dt, \quad \varepsilon^p = \int_0^t \left[ \frac{2}{3} \text{tr}(\mathbf{D}^p \mathbf{D}^p) \right]^{1/2} dt.\end{aligned}\tag{1}$$

Parameters  $D_0$ ,  $n$ ,  $K_1$ ,  $K_0$ ,  $m$ ,  $\hat{a}$  and  $\hat{b}$  characterize the material of the plate,  $\theta$  is the absolute temperature, and  $\mathbf{D}^p$  is the plastic part of the strain rate tensor. The material parameters were assigned the following values: Mass density = 7860 kg/m<sup>3</sup>, specific heat = 473 J/kg<sup>o</sup>C, thermal conductivity = 49.73 W/m<sup>2</sup>°C, Young's modulus = 208 GPa, Poisson's ratio = 0.3, coeff. of thermal expansion = 11.5 × 10<sup>-6</sup>/°C,  $\hat{a}$  = 1200 K,  $\hat{b}$  = 0,  $K_1$  = 2950 MPa,  $K_0$  = 2937 MPa,  $m$  = 3510 MPa,  $D_0$  = 1.732 × 10<sup>8</sup>/s, yield stress  $\sigma_0$  in a quasistatic simple tension test = 702 MPa. Values of material parameters in the yield criterion and the evolution equation for the porosity are the same as those given in Batra and Jaber (2001). Note that in the Bodner-Partom relation, the plastic strain rate is positive even when the effective stress is less than the pertinent yield stress of the material.

The plate is taken to be initially at rest, stress free, and at a uniform temperature. The normal velocity,  $v_1$  imposed on the impacted surface is given by

$$v_1/v_0 = \begin{cases} 0.3t, & 0 \leq t \leq 2\mu s, \\ 0.525 + 0.0375t, & 2 \leq t \leq 10\mu s, \\ 0.9, & 10 \leq t \leq t_s, \\ 0, & t > t_s, \end{cases}\tag{2}$$

where  $t_s$  equals the time when the projectile separates from the plate. The expressions (2) for  $v_1$  are derived by fitting straight lines to the data of Batra and Ravisankar (2000).

### 3 COMPUTATION AND DISCUSSION OF RESULTS

The aforesaid problem is solved numerically by using an in-house developed finite element code. It employs constant strain triangular elements, lumped mass matrix obtained by the row sum technique, and the subroutine LSODE (Livermore Solver for Ordinary Differential Equations) for integrating the stiff set of coupled nonlinear ordinary differential equations. A reasonably fine mesh was employed in the region around the notch tip and the hole, and a coarse mesh elsewhere. Results presented herein have been computed with a fixed mesh.

A brittle failure is assumed to initiate at a point if the maximum principal tensile stress there exceeds  $2.34\sigma_0$ . The ductile failure in the form of an adiabatic shear band ensues at a point as soon as the effective stress there has dropped to 90% of its maximum value at that point and the material point is deforming plastically. The computation of the numerical solution is continued till an element has been very severely distorted. Once a shear band has initiated, the time step size continues to decrease rapidly. The subroutine LSODE adjusts the time step adaptively in order to compute the solution within the prescribed accuracy. We note that no failure criterion has been implemented in the code. Neither the failed elements are eliminated from the mesh nor values of their elastic moduli are reduced. However, elastic moduli decrease because of the increase in the porosity, and the radius of the yield surface decreases because of the rise in the temperature and the porosity.

For the four shapes of the notch tip considered herein, Fig. 2 depicts the dependence of the times of initiation of the brittle and the ductile failures upon the impact speed. For the blunt notch tip with  $a/b = 0.4$ , where  $a$  and  $b$  equal the semimajor and the semiminor axes of the elliptical notch tip, the two failure modes occur essentially simultaneously;  $b = 0.15\text{mm}$ . The failure mode transitions from brittle to ductile at an impact speed of about  $19\text{m/s}$  for the circular notch tip, and the ductile failure mode precedes the brittle failure mode for elliptical notch tips with  $a/b = 2$  and  $10$ . It provides an explanation for the different failure modes observed by Kalthoff (2000) and Zhou et al. (1996) in their experiments on very similar steels. Since the notches were cut by different techniques, the notch shapes may not have been the same in the two experiments. However, when a circular hole of radius  $r_0 = 0.15\text{ mm}$  was located ahead of the circular notch tip of radius  $r_0$  with the distance,  $d$ , between the centers of the circular notch tip and the hole equal to  $3r_0$ ,  $4r_0$ ,  $5r_0$  and  $6r_0$ , the brittle failure always preceded the ductile failure. Whereas the brittle failure initiated from the point,  $P$ , on the lower surface of the circular hole that is closer to the notch tip, the ductile failure originated from a point either on the lower surface of the notch tip or on the upper surface of the hole. For the distance of  $3r_0$  between the centers of the hole and the notch tip, the line joining point  $P$  to the center of the hole made an angle of  $9^\circ$  counterclockwise with the axis of the notch, this angle equaled  $18^\circ$  for the other three cases. Figure 3 illustrates the dependence of the times of initiation of the brittle and the ductile failures upon the impact speed for the four locations of the circular hole.

## 4 CONCLUSIONS

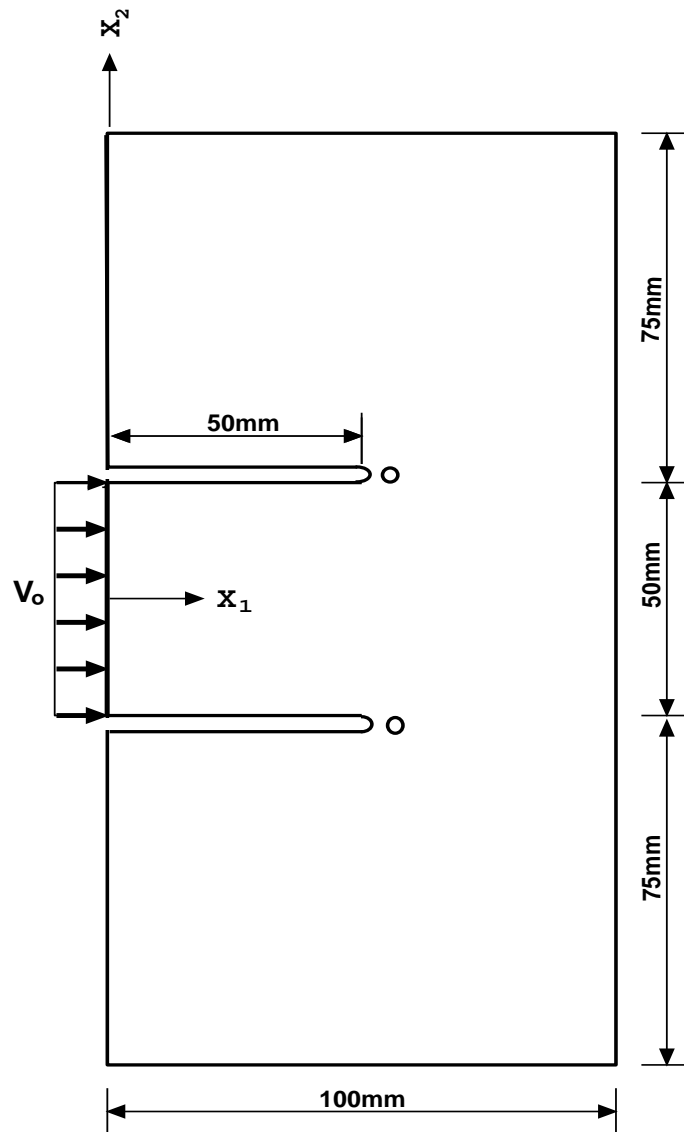
For the impact loaded prenotched plate with the blunt and the circular notch tips considered herein, the failure mode transitions from the brittle to the ductile with an increase in the impact speed. However, when a circular hole of radius equal to the radius of the circular notch tip is located ahead of the notch, the brittle failure always occurs first and it initiates from a point on the lower surface of the circular hole.

**Acknowledgment:** This work was partially supported by the ONR grant N00014-98-0300 to Virginia Polytechnic Institute and State University with Dr. Y. D. S. Rajapakse as the cognizant program manager.

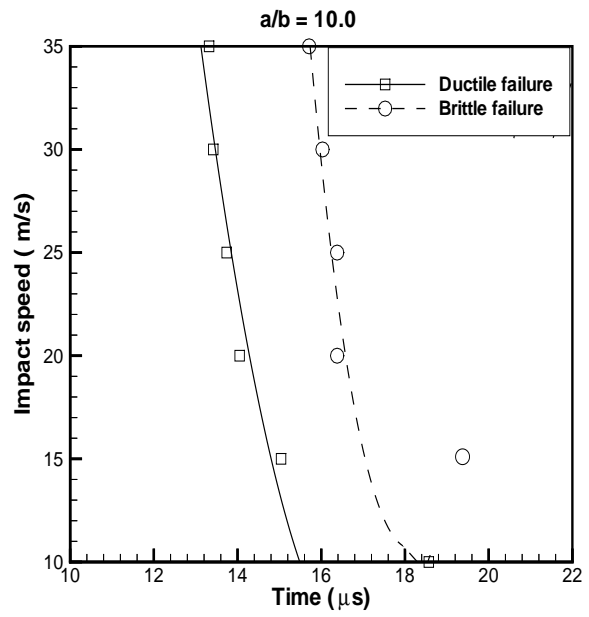
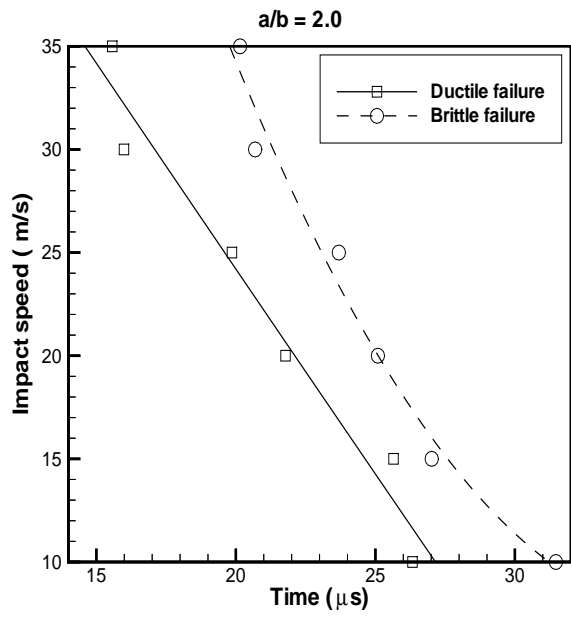
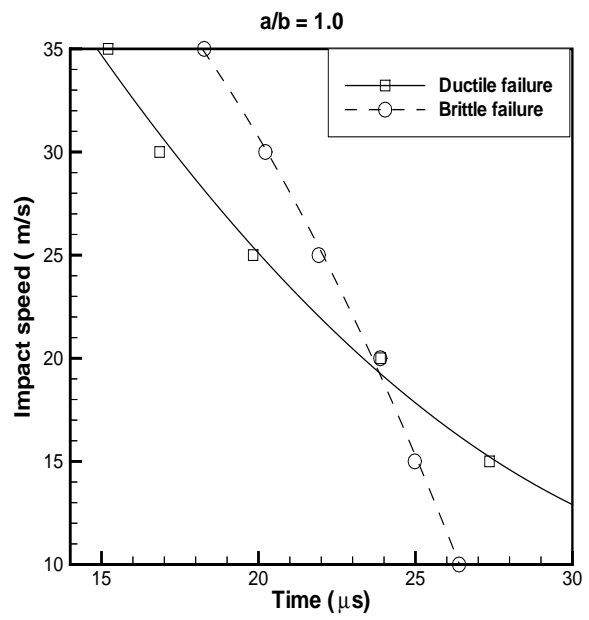
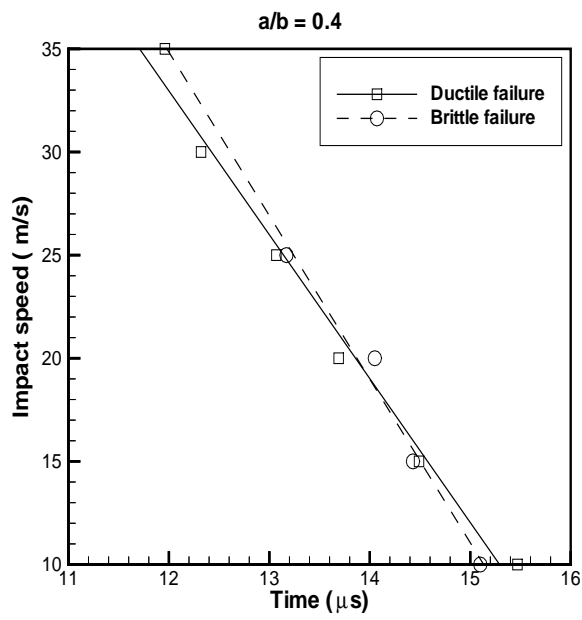
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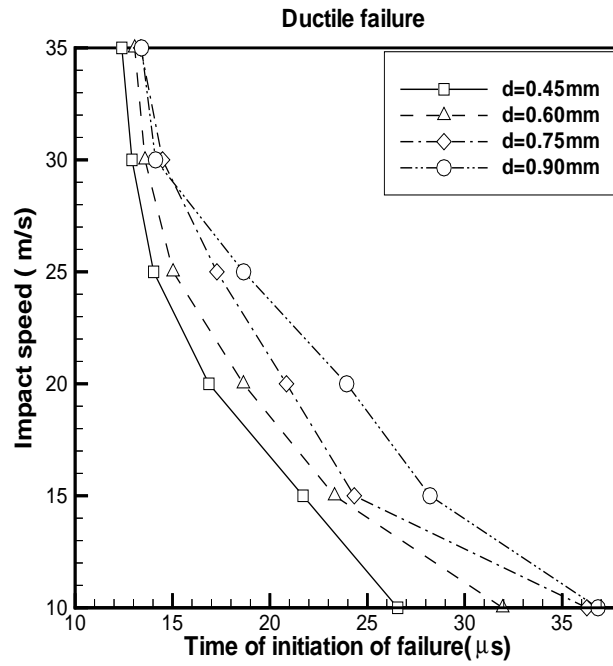
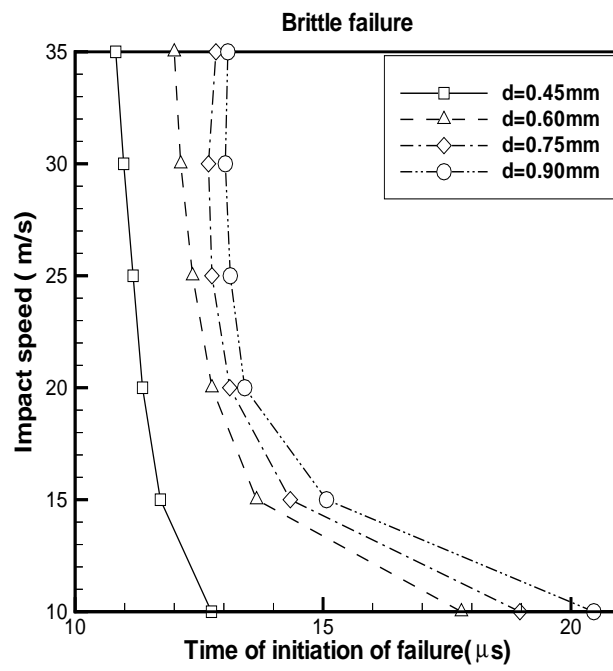
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**Figure 1:** A schematic sketch of the problem studied.



**Figure 2:** For the four shapes of the notch-tip, the dependence of the times of initiation of the brittle and the ductile failures on the impact speed.



**Figure 3:** For the four locations of the circular hole, the dependence of the times of initiation of the brittle and the ductile failures on the impact speed.