DYNAMIC STRESS INTENSITY FACTOR FOR UNSTEADY
RAPID CRACK PROPAGATION

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ABSTRACT

Dynamic crack propagation in PMMA was studied using the method of caustics in combination with a Cranz-Schardin type high-speed camera. Four different types of specimen geometries were employed to achieve the crack acceleration, deceleration and re-acceleration process in one fracture event. Dynamic stress intensity factor $K_{id}$ and crack velocity $\dot{a}$ were evaluated in the course of crack propagation to obtain the relationship between $K_{id}$ and $\dot{a}$. The effect of crack acceleration and deceleration on the $K_{id}$-$\dot{a}$ relations was examined.

KEY WORDS

Dynamic crack propagation, stress intensity factor, crack velocity, crack acceleration, caustic method, high-speed photography, PMMA

INTRODUCTION

Dynamic crack propagation in brittle materials has been investigated using many experimental techniques. Optical methods such as photoelasticity [1-4] and the method of caustics [5-9] have been widely employed for evaluating the state of dynamic stress field around a propagating crack tip, i.e. dynamic stress intensity factor $K_{id}$. Crack velocity $\dot{a}$ was also estimated in crack propagation to correlate with $K_{id}$. Many experimental studies have been made on the relation between $K_{id}$ and $\dot{a}$, however, different experiments derived significantly different types of $K_{id}$-$\dot{a}$ relationships. Much discussion has taken place on the applicability of the methods employed for $K_{id}$ evaluation, the definition of the crack tip stresses, the influence of specimen geometries and loading conditions and so forth. In the case of brittle fracture, $\dot{a}$ generally changes with time, resulting in crack acceleration or deceleration according to the stress distribution in the specimen. The crack acceleration is an important parameter to understand the behavior of dynamic crack propagation, however, quantitative discussion on the effect of the crack acceleration and deceleration on $K_{id}$ has been limited.

The purpose of the present work was to study this problem in more detail using the method of caustics in combination with a Cranz-Schardin type high-speed camera[10,11]. Four different types of specimen geometries were employed so that cracks could undergo acceleration, deceleration and re-acceleration stages in one fracture process. Dynamic stress intensity factor $K_{id}$ and crack velocity $\dot{a}$ were evaluated in the course of crack propagation. The $K_{id}$-$\dot{a}$ relations were determined for the stages of acceleration, deceleration and/or re-acceleration. Attention was focussed particularly on the effect of the crack acceleration and deceleration on the $K_{id}$-$\dot{a}$ relations.

EXPERIMENTAL PROCEDURE

Specimen geometries used in this experiment are illustrated in Fig. 1, where (a) represents a single-edge-notched (SEN) specimen, (b) a uniaxially pin-loaded specimen, (c) a biaxially pin-loaded specimen and (d) a SEN with two circular holes specimen. These four types of specimens were selected to obtain the different behaviors of dynamic crack propagation. The
specimens were fabricated from a 5mm-thick sheet of PMMA (Acrylite S-001). A sharp precrack was generated by momentum-controlled chisel-impact into a pre-machined saw-cut on the specimen edge.

All specimens were tested under a displacement controlled condition using a tensile testing machine. Tests were performed at room temperature and at a constant crosshead speed of 1mm/min. The dynamic crack propagation was photographed using a Cranz-Schardin type high-speed camera with 30 sparks and a maximum frequency of $1.42 \times 10^8$ frames/sec [10,11]. This camera permitted a simultaneous record of two images with different focal distances. One focal distance was selected for specimen-focussed images and the other for caustic images.

**EVALUATION OF $K_{id}$ AND CRACK VELOCITY $\dot{a}$**

Figure 2 shows examples of high-speed photographs taken with a SEN specimen, where series (a) represents the specimen-focussed images and (b) the corresponding caustic patterns. As seen, size of the caustic changed with growing crack length. The stress intensity factor $K_{id}$ was determined from the following equation:

$$K_{id} = (2\sqrt{2\pi / \theta_0 dc \eta \theta_{10}^2}) (\phi / 3.17)^{3/2}$$  (1)

where $\phi$ is the caustic diameter at a crack tip, $z_0$ is the distance between the specimen and the image plane, $d$ is the specimen thickness and $\eta$ is a convergency factor for incident light rays [9].

The values of $K_{id}$ and crack length $a$ obtained for the four specimens are shown in Figs. 3-6 as a function of time $t$. The $K_{id}$ variations for the specimens were different. In the SEN specimen under uniform tensile loading, $K_{id}$ increased in the initial stage of crack propagation and gradually approached a constant value. The uniaxially pin-loaded specimen exhibited $K_{id}$ increasing and decreasing behavior. In both the biaxially pin-loaded and SEN (Holes) specimens, there existed three stages of recognizable $K_{id}$ increasing, decreasing and re-increasing regions.

To minimize data scattering in the evaluation of fracture parameters, a data-fitting procedure which was proposed in a previous work [9] was employed; obtained values of $K_{id}$ and $a$ were expressed as ninth order polynomial of $t$ based on the least-squares method so that they

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Fig. 1. Specimen geometries and loading methods
Fig. 2. Example of dynamic crack propagation in a SEN specimen. (a) specimen-focussed images, (b) caustic patterns

Fig. 3. Time variations of $K_{ID}$ and crack length $a$ for a SEN specimen

Fig. 4. Time variations of $K_{ID}$ and crack length $a$ for a uniaxially pin-loaded specimen

Fig. 5. Time variations of $K_{ID}$ and crack length $a$ for a biaxially pin-loaded specimen

Fig. 6. Time variations of $K_{ID}$ and crack length $a$ for a SEN with holes specimen
closely fitted their observed values (see Figs. 3-6). Crack velocity \( \dot{a} \) was determined from the first time derivatives of the fitted curve \( a(t) \). If \( \dot{a} \) was determined simply from the first time derivative of increment \( \Delta a \) which was obtained from successive pictures taken on a film, large scatter in \( \dot{a} \) was inevitably caused primarily by errors arising from visual identification of the crack tip position on film. Thus, the data-fitting procedure employed enabled us to determine crack velocity accurately.

**RELATIONSHIPS BETWEEN \( K_{IP} \) AND \( \dot{a} \)**

Figures 7-10 show values of \( K_{IP} \) and \( \dot{a} \) as a function of crack length \( a \). There are several interesting points in their relations. First, the change in \( \dot{a} \) was qualitatively in accord with the one in \( K_{IP} \). Second, \( \dot{a} \) rose earlier than \( K_{IP} \) associated with \( a \). Finally, \( K_{IP} \) for a constant \( \dot{a} \) was larger when the crack was decelerated than when it was accelerated (see Figs. 8-10). Similar results were also obtained by the authors for epoxy and Homalite-100 specimens [12-15].

To study the effect of the crack acceleration and deceleration, the values of \( K_{IP} \) were expressed as a function of \( \dot{a} \). Figures 11-14 show \( K_{IP}(\dot{a}) \) curves, where arrows indicate the direction of progress of the fracture. The open circles represent the acceleration-free points (\( \dot{a}=0 \)) obtained from the maximum and minimum velocity positions. There are several interesting points in the \( K_{IP}-\dot{a} \) relations. First, \( K_{IP}(\dot{a}) \) for the SEN specimen only exhibited the increasing process so that it gradually approached a constant value. Second, distinct crack
acceleration ($\ddot{a}>0$) and deceleration ($\ddot{a}<0$) can be seen in the uniaxially pin-loaded specimen. Finally, in both the biaxially pin-loaded and SEN (Holes) specimens, three stages of distinct crack acceleration ($\ddot{a}>0$), deceleration ($\ddot{a}<0$) and re-acceleration ($\ddot{a}>0$) can be seen in one fracture process. Although $K_{ID}(\ddot{a})$ is shown to increase with $\ddot{a}$, it should be noted that their relation was not unique. For a constant $\ddot{a}$, the decelerating crack had a larger value of $K_{ID}$ than the accelerating or re-accelerating one. Such was also the case with other specimens tested.

The authors have suggested that $K_{ID}$ was expressed as two parametric functions of $\ddot{a}$ and $\ddot{\ddot{a}}$, i.e. $K_{ID}(\ddot{a}, \ddot{\ddot{a}})$, and that $K_{ID}(\ddot{a}, \ddot{a}=constant)$ was uniquely related to $\ddot{a}$ [9,12,13]. This was examined using the obtained results. Figure 15 shows the $K_{ID}(\ddot{a})$ curves determined for the four specimens. The dotted curve of $K_{ID}(\ddot{a}, \ddot{\ddot{a}}=0)$ connecting the acceleration free points can separate the acceleration ($\ddot{a}>0$) and deceleration ($\ddot{a}<0$) area in the $K_{ID}-\ddot{a}$ diagram. As seen, $K_{ID}(\ddot{a})$ for a constant $\ddot{a}$ had a larger value when the crack was decelerated than when it was accelerated, i.e. $K_{ID}(\ddot{a}, \ddot{\ddot{a}}<0) > K_{ID}(\ddot{a}, \ddot{\ddot{a}}>0)$. It should be noted that $K_{ID}(\ddot{a}, \ddot{a}=0)$ can be uniquely related to $\ddot{a}$ as suggested in previous studies [9,13]. Hence, this clearly appears to indicate that $K_{ID}(\ddot{a}, \ddot{\ddot{a}}=0)$ can be the material property, while the time variations of $K_{ID}$ and $\ddot{a}$ were strongly influenced by specimen geometries and loading methods as shown in Figs 7-10.

CONCLUSIONS

Dynamic crack propagation in PMMA was studied using the method of caustics and a Cranze-Schardin high-speed camera. Four different types of specimen geometries were employed to
achieve the crack acceleration, deceleration and re-acceleration process in one fracture event. Dynamic stress intensity factor $K_{ID}$ and crack velocity $\dot{a}$ were evaluated, and the following findings were obtained:

1. The variations of $K_{ID}$ and $\dot{a}$ were strongly influenced by the specimen geometries and loading methods.
2. $\dot{a}$ change was qualitatively in accord with the one in $K_{ID}$.
3. $K_{ID}$ for a constant $\dot{a}$ was larger when the crack was decelerated than when it was accelerated or re-accelerated.
4. $K_{ID}$ for acceleration-free can be uniquely related to $\dot{a}$.

Fig. 15. $K_{IM}-\dot{a}$ curves for the four different types of PMMA specimens

REFERENCES