DUCTILE DAMAGE ACCUMULATION UNDER CYCLIC DEFORMATION AND MULTIAXIAL STATE OF STRESS CONDITIONS

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ABSTRACT

The possibility to develop reliable predictive tools for the design of components undergoing plastic deformation is connected to the capability to incorporate damage mechanics into the constitutive model of the material. Even though many damage models for ductile failure are available in the literature, none of them, as far as the authors are aware of, is extended to reversal plastic flow occurring under compressive stress states. In this paper the damage model proposed by Bonora (1997) has been reformulated in order to account for compressive loading introducing a new internal variable associated to damage. The model has been implemented in a commercial finite element code and used to predict single element performance under cycling loading and damage accumulation in a round notch tensile bar. Some preliminary experimental results are also presented.

KEYWORDS

Ductile damage, cyclic loading, fatigue, CDM

INTRODUCTION

In the last decades it has been shown that local approaches have a great potential in predicting the occurrence of failure in specimens, components and structures. Today it is well assessed that ductile failure occurs as a result of microvoids nucleation and growth at inclusions. The local approach is based on the assumption that, if the microscopic mechanism of failure is known, the modification of the material constitutive response can be predicted from micromechanical considerations. Consequently, direct transferability from material to structure, without any geometry effect, would be one of the key features. Many theoretical models have been proposed in the literature that can be grouped in two main sets: continuum damage mechanics (CDM) based models and porosity models. Porosity models, derived from the Gurson type model, are based on the modification of the yield function as a result of the increasing porosity with plastic strain. Here, porosity plays the role of a softening variable that progressively implodes the yield surface in order to account for damage effects. CDM models are developed in the framework of continuum mechanics. Here, damage effects are accounted by a thermodynamic variable, D, that reduces material stiffness. Thus, the complete set of constitutive equations for the material undergoing damage is derived. Both approaches suffer major limitations. Porosity models usually require a large number of material parameters, none of which has a physical meaning, that have to be identified using coupled numerical simulation and experiments. On the other side, CDM model performance depends on the assumed form for the dissipation potential from which damage evolution law can be derived by normality rule. All the models proposed in the literature show material dependency, lack of performance under multi-axial state of stress conditions and temperature and strain rate effect is usually neglected. In 1997 Bonora [1] proposed a new non-linear CDM model for ductile failure that overcome material dependency and stress triaxiality effects. The model resulted successful in predicting notched and cracked components response using only information, such as damage parameters, identified in simple uniaxial state of stress condition, [2]. Later, Bonora and Milella [3] extended the damage model in order to incorporate temperature and strain rate effects.

Up to now, very little attention have been given to the mechanics of ductile deformation and damage under compressive state of stresses. This issue becomes very important in order to understand and predict component life under low cycle fatigue regime or under intense dynamic loading in which damage accumulation is related to the bouncing motion of strain waves into the body. Bonora and Newaz [4] demonstrated the possibility to predict low cycle fatigue life at ductile crack growth initiation discussing possible integration scheme for the non-linear damage law. At the moment, as far as the authors are aware of, no attempt to extend CDM model formulation to cyclic loading under variable stress triaxiality loading conditions has been made. In this paper, for the first time, the non-linear damage model proposed by Bonora has been extended to negative stress triaxiality loading condition, based on simple physical considerations, introducing a new internal variable associated to damage D. The model, implemented in form of user subroutine in the finite element code MSC/MARC, has been tested on single FEM element under simple loading conditions such a as normal and shear stress. Successively, it has been applied to round notched bar specimens loaded in tension. At the present time, an extensive experimental program is under investigation. Here, the promising preliminary results are presented and discussed.

NON-LINEAR CDM MODEL FOR DUCTILE FAILURE

Lemaitre [5] firstly defined the CDM framework for plasticity damage. Damage accounts for material progressive loss of load carrying capability due to irreversible microstructural modifications, such as microvoids formation and growth, microcracking, etc. From a physical point of view, damage can be expressed as

$$D_{(n)} = 1 - \frac{A_{eff}^{(n)}}{A_0^{(n)}} \tag{1}$$

where, for a given normal n, $A_0^{(n)}$ is the nominal section area of the RVE and $A_{eff}^{(n)}$ is the effective resisting one reduced by the presence of micro-flaws and their mutual interactions. Even though this definition implies a damage tensor formulation, the assumption of isotropic damage leads to a more effective description where the scalar D can be simply experimentally identified. Additionally, this assumption is not too far from reality as a result of the random shapes and distribution of the included particles and precipitates that trigger plasticity damage initiation and growth. The strain equivalence hypothesis gives the operative definition of damage as:

$$D = 1 - \frac{E_{eff}}{E_0} \tag{2}$$

where E_0 and E_{eff} are the Young's modulus of the undamaged and damaged material, respectively. The complete set of constitutive equation for the damage material can be derived assuming that:
- a damage dissipation potential f_D, similarly to the one used in plasticity theory, exists;
- no coupling between damage and plasticity dissipation potentials exists;
- damage variable, D, is coupled with plastic strain;
-the same set of constitutive equations for the virgin material can be used to describe the damaged

material replacing only the stresses with the effective ones and assigning a state equation for D.

Bonora [1] proposed the following expression for the damage dissipation potential,

$$f_D = \left[\frac{1}{2}\left(-\frac{Y}{S_0}\right)^2 \cdot \frac{S_0}{1-D}\right] \cdot \frac{\left(D_{cr}-D\right)^{\frac{\alpha-1}{\alpha}}}{p^{\frac{2+n}{n}}} \tag{3}$$

where, D_{cr} is the critical value of the damage variable for which ductile failure occurs, S_0 is a material constant and n is the material hardening exponent. α is the damage exponent that determines the shape of the damage evolution curve and is related to the nature of the bound between brittle inclusions and the ductile matrix. Thus, the constitutive equation set for isotropic hardening material is given by:

strain decomposition

$$\dot{\varepsilon}_{ij}^{T} = \dot{\varepsilon}_{ij}^{e} + \dot{\varepsilon}_{ij}^{p} \tag{4}$$

elastic strain rate

$$\dot{\varepsilon}_{ij}^{e} = \frac{1+\nu}{E}\frac{\dot{\sigma}_{ij}}{1-D} - \frac{\nu}{E}\frac{\dot{\sigma}_{kk}}{1-D}\delta_{ij} \tag{5}$$

plastic strain rate

$$\dot{\varepsilon}_{ij}^{p} = \dot{\lambda}\frac{\partial f_p}{\partial \sigma_{ij}} = \dot{\lambda}\frac{3}{2}\frac{s_{ij}}{1-D}\frac{1}{\sigma_{eq}} \tag{6}$$

plastic multiplier

$$\dot{r} = -\dot{\lambda}\frac{\partial f_p}{\partial R} = \dot{\lambda} = \dot{p}(1-D) \tag{7}$$

kinetic law of damage evolution

$$\dot{D} = -\dot{\lambda}\frac{\partial f_D}{\partial Y} = \alpha \cdot \frac{\left(D_{cr}-D_0\right)^{\frac{1}{\alpha}}}{\ln(\varepsilon_f/\varepsilon_{th})} \cdot f\left(\frac{\sigma_H}{\sigma_{eq}}\right) \cdot \left(D_{cr}-D\right)^{\frac{\alpha-1}{\alpha}} \cdot \frac{\dot{p}}{p} \tag{8}$$

Detailed description on the derivation of these Equations can be found elsewhere, [1]. In Equation (9) stress triaxiality effects are accounted by the function $f(\sigma_H/\sigma_{eq})$ defined as,

$$f\left(\frac{\sigma_H}{\sigma_{eq}}\right) = \frac{2}{3}(1+\nu) + 3\cdot(1-2\nu)\cdot\left(\frac{\sigma_H}{\sigma_{eq}}\right)^2 \tag{9}$$

that is derived assuming that ductile damage mechanism is governed by the total elastic strain energy, Lemaitre [5]. Here, $\sigma_H = \sigma_{kk}/3$ is the hydrostatic part of the stress tensor and ν is the Poisson's ratio. The model requires five material parameters in order to be applied. The strain threshold (in uniaxial monotonic loading) ε_{th}, at which damage processes are activated; the theoretical failure strain ε_f, at which ductile failure under completely uniaxial state of stress conditions occurs; the initial amount of damage present in the material, D_0; the critical damage, D_{cr}, at which failure occurs and the damage exponent, α, that control the shape of damage evolution with plastic strain. Experimental procedure for damage parameters identification can be found elsewhere, [6].

EXTENSION TO REVERSAL PLASTIC FLOW

Ductile damage formulations available in the literature always address tensile loading configuration, since it is well known that positive stress triaxiality enlarges nucleating voids in the material microstructure. The possible effect on damage variable due to compressive loading is usually neglected in the theoretical formulations. The major consequence of this limitation is that the damage variable, D, has to be associated to the total effective accumulated plastic strain, usually indicated with p, that plays the role of the associated internal variable. In the literature, little attention is given to the effects on the material constants, or constitutive response, due to plastic deformation under pure compressive loading. This knowledge is critical in order to develop a predicting model capable to account for plastic strain reversal as for low cycle fatigue. Few attempts based on the cyclic accumulation of damage, or its associated variable, always resulted in predicted very short lives as a consequence of the fact that p usually accumulates quickly. Porosity models, such as the Gurson model, are incapable to predict material failure since porosity effects are fully recovered during compressive loading resulting in a unrealistic healing-material behavior, [7]. These premises clearly indicate that additional hypotheses must be formulated in order to describe properly material behavior under compressive stress states. If ductile damage can be imputed to the formation and growth of microcavities that have the effect to reduce the net resisting area, and consequently material stiffness, thus the following scenarios can be speculated.

Scenario a). The material is initially stress-free and it is assumed that no strain history has modified its status from the one of “virgin material”. Let us assume to start to load a material reference volume element, RVE, under pure compressive uniaxial state of stress avoiding any buckling phenomena. In this configuration, microvoids cannot nucleate since the ductile matrix is compressed around the brittle inclusions. If the local stress in the particle overcomes the particle strength, the particle itself can eventually break. This kind of damage should not affect material stiffness since no reduction of the net resisting area is occurred. The only effect that we would expect is probably an anticipated microvoids nucleation, due to an early void opening since the particle is broken, when the stress state is reversed in tension, (i.e. a lower strain threshold value). Even though an irreversible process such as particle breaking will eventually occur under compressive loads, the stiffness should remain unaffected indicating no damage in compression.

Scenario b). Let say that the virgin RVE is initially loaded in tension until some amount of damage. Then, the load is reversed in compression developing additional plastic strain. In this case, during the unloading from positive stress-state to zero, microvoids can close controlled by the large ductile matrix volume movement, (here, potential buckling of microcavities is neglected). Voids implode back to the particle from which they have nucleated. Void closure can eventually close to the zero displacement condition. During this phase the net resisting area is restored and the stiffness should be the same as for the virgin material. Continuing in the compressive ramp the stiffness, once again, should remain unaffected. Further compressive loads, will eventually breaks some particles, but no effects are expected on E. A new reload in tension would see both the opening of the previously grown voids plus the opening of the new ones nucleated at the broken particles. However, at this stage it can be assumed that compressive damage does not modify damage developed under positive stress states.

It follows that ductile damage can accumulate under positive stress state only, while total plastic strain will accumulate under both. Consequently, the associate damaged variable has to be a redefined as an “active accumulated plastic strain” $\boldsymbol{p}^{+}$, i.e. the plastic strain that accumulates if and only if, the actual stress triaxiality is greater than zero. Similarly, the damage effect on material stiffness will also be activated if and only if the current stress triaxiality is positive.

According to this, the damage model proposed by Bonora can be modified in terms of active damage $\boldsymbol{D}^{+}$ and active plastic strain $\boldsymbol{p}^{+}$ as follows:

$$\dot{D}^{+} = -\dot{\lambda}\frac{\partial f_D}{\partial Y} = \alpha \cdot \frac{(D_{cr}-D_0)^{\frac{1}{\alpha}}}{\ln(\varepsilon_f/\varepsilon_{th})} \cdot f\left(\frac{\sigma_H}{\sigma_{eq}}\right) \cdot \left(D_{cr}-D^{+}\right)^{\frac{\alpha-1}{\alpha}} \cdot \frac{\dot{p}^{+}}{p^{+}} \tag{10}$$

$$\dot{p}^{+} = \frac{\dot{\lambda}}{(1-D^{+})} H\left\langle f\left(\sigma_m/\sigma_{eq}\right)\right\rangle \tag{11}$$

$$\tilde{E} = E(1-D^{+})H\left\langle f\left(\sigma_m/\sigma_{eq}\right)\right\rangle \tag{12}$$

where

$$H\left\langle f\left(s_m / s_{eq}\right)\right\rangle = \begin{cases} 0 & s_m / s_{eq} < 0 \\ 1 & s_m / s_{eq} \geq 0 \end{cases} \tag{13}$$

FINITE ELEMENT ANALYSES

Damage model performance under cycling loading has been firstly checked on a single axisymmetric 4 node element. The element size is 1.0 x 1.0 mm. The material is a 22NiMoCr37 steel of German production for which damage parameters were previously determined by Bonora *et al.* (1998). A cyclic imposed displacement with zero mean value δ_m=0 and amplitude δ_a=0.2 mm has been applied until element failure. Response under isotropic and kinematic hardening is given in figure 1 together with the displacement evolution with time. Here, time is a fictitious variable since viscoplastic and time dependent behaviors have been neglected.

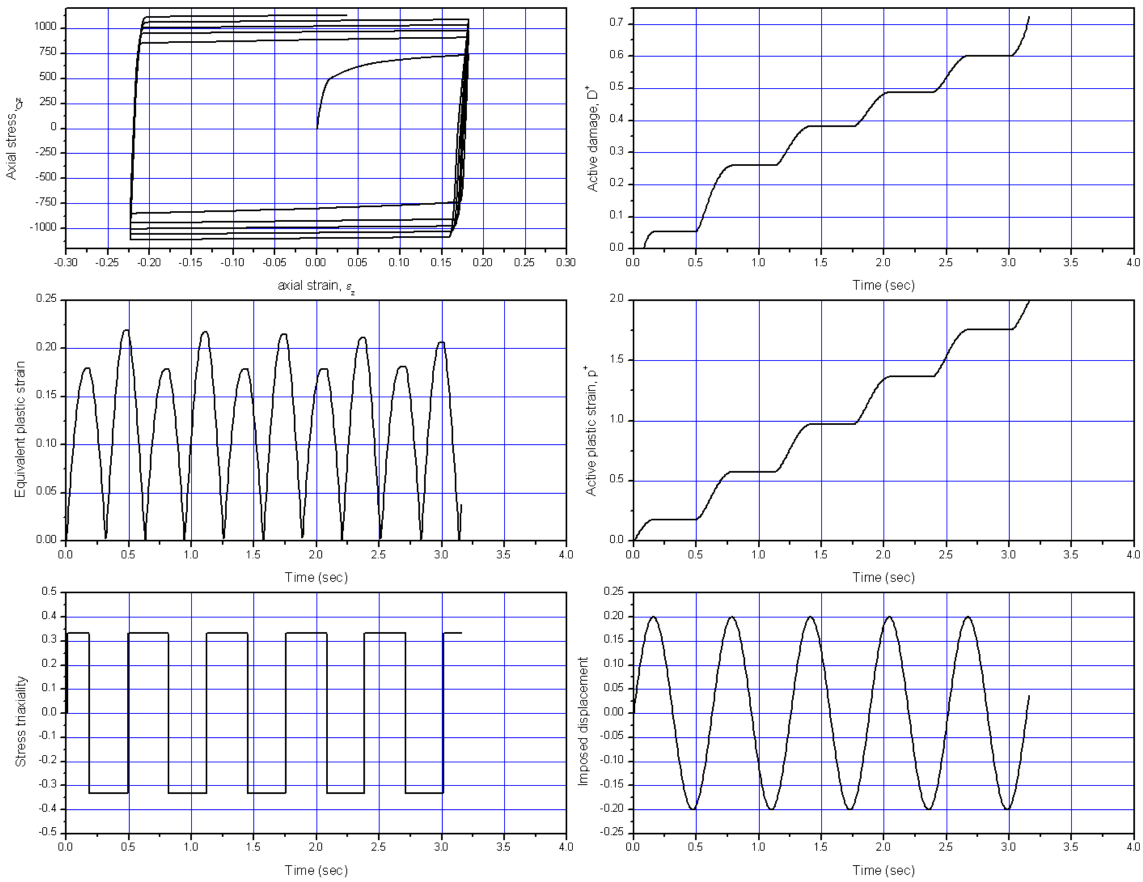

Figure 1 – Single element response under cyclic loading: a) axial stress vs axial strain; b) equivalent plastic strain vs time; c) stress triaxiality vs time; d) active damage vs time; e) active strain vs time; f) imposed displacement vs time

In figure 1 it is shown how the effective plastic strain, together with active damage, accumulates only when stress triaxiality is positive. In this case, where the imposed nominal strain amplitude is 20%, failure is expected to occur after 5 cycles. Subsequently, the model has been used to investigate damage evolution in round notched tensile bar (R = 2mm) under reversal plastic flow loading conditions, figure 2a. At the present time a single test has been performed on SA 537 steel. An initial monotonic ramp up to 0.2 mm has been applied followed by sinusoidal cycling with an amplitude of 0.125 mm. The cycling frequency was 0.0125 Hz. Local deformation field across the notch has been monitored using an extensometer with a gage length of 10 mm. In figure 2b, the comparison between the finite element results and the experimental data is given in term of applied load versus displacement at the gauge. It is important to note that the FEM model incorporating damage is capable to reproduce the load cycle in the near notch region pretty well if kinematic hardening is used. Isotropic hardening results in a very narrow hysteresis loop. The major difference is given by the macroscopic ratcheting. A posteriori it has been found that this phenomenon can occur in SA537 steel under very low strain rate since at this deformation rate viscoplastic behavior becomes manifested. Experimental test has been stopped after 20 cycles. According to finite element prediction only a limited amount of damage should be generated in the specimen minimum section without failure in this cycling. In figure 3 the deformed mesh showing damage contours after 7 cycles is given as a sample.

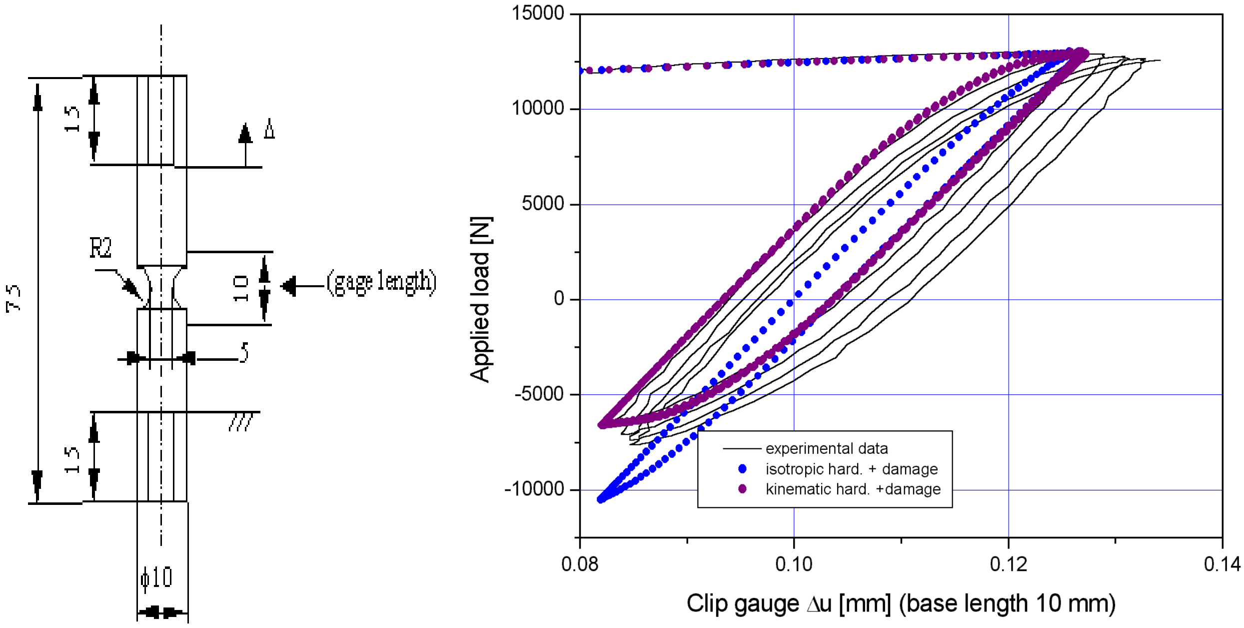

Figure 2a – RNB(T) specimen geometry and dimensions 2b- cyclic response: comparison between FEM results and experimental data

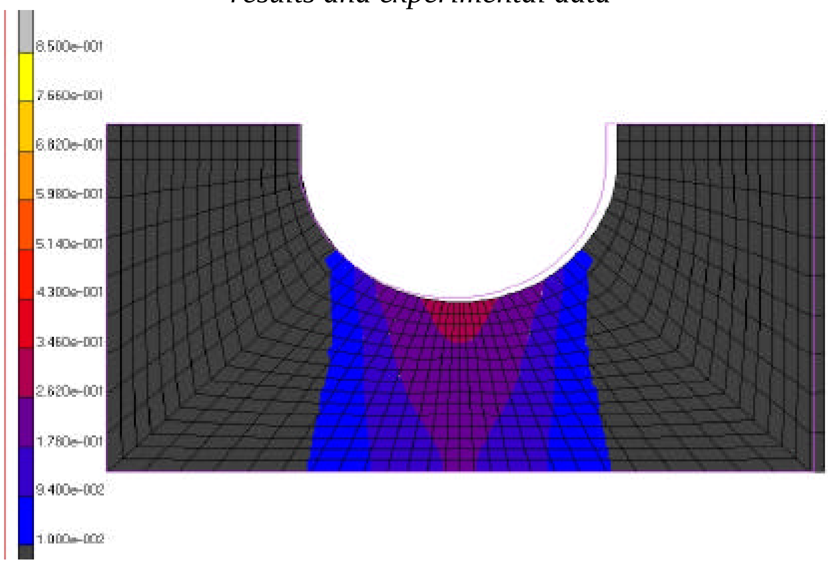

Figure 3 – Damage contours along the minimum section after 7 cycles.

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