POSTER REFERENCE: ICF 100380PR

# Design of experiments for reliability assessment of active mechanical structures

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## ABSTRACT

In recent years, significant activity around the world has been directed toward active control of mechanical structures. The goal of much of this research is to increase the reliability and safety of these structures against dynamic loadings. However, the design of most systems is done under conditions of uncertainty. If the uncertainties are not properly considered in the design of a control strategy, an absolute assurance cannot be given about the performance of the controlled structure. Such assurance can only be given in terms of a probability of failure in satisfying a criterion of performance: it is the reliability. This paper proposes a method to determine active mechanical structures effectiveness while assessing their estimated reliability, using design of experiments. Models joining active mechanical structures theories and those of reliability, and design of experiments method are presented. Numerical examples are provided through a controlled panel subjected to white noise excitation, to illustrate the effectiveness of the method.

## KEYWORDS

Reliability, active control, vibration, design of experiments, white noise, LQG-control.

## INTRODUCTION

Last decade has seen significant activity in the area of active control for mechanical structures against dynamic loading [1-3]. The principle of active control consists in attenuating a noise or a vibration while superimposing, with the initial amplitude, secondary amplitude of the same amplitude, but having an opposite sign. However, the design of most systems is done under conditions of uncertainty. The presence of such uncertainties can degrade control performance and can even lead to structural instabilities. To ensure such a performance, to study the reliability of these systems seemed a measurement necessary [4-7]. In this article, we propose to improve the active reliability of the structures by using the methods of the experimental designs. Initially we point out the various phases of the active systems design, the principle of evaluation of reliability, the methods of the experimental designs. Numerical examples on a panel subjected to an excitation of noise of white, are presented.

## ACTIVE MECHANICAL STRUCTURES THEORY

There has been much research and many developments on active mechanical structures. Various methods incorporate concepts of modern control theory, and the active vibration control using piezoelectric sensors and actuators has drawn attention due to their higher applicability to real structures. Locations of both sensors and actuators have been determined with consideration of controllability, observability and spillover prevention. In the present case, the LQG-control technique is chosen, for the design of the controller [1-3,8-9].

One considers a deadened linear mechanical system disturbed by a white noise W, of which the equation representative is written:

$$M\ddot{q}+H\dot{q}+Kq=Dw \tag{1}$$

The equation of output or observation relating to this system is given by:

$$y=Cq \tag{2}$$

With $E\{w\} = 0$ ; $E\{w(t)w(\underline{t})\} = W\,\delta(t-\underline{t})$ and M, H, K are respectively the matrices of masses, damping, and stiffness. D, C are respectively the matrices of localization of the disturbances, and the exits. $q, \dot{q}, \ddot{q}$ are the vector displacement and its derivative. The equations (1) and (2) give a representation of the state of the following relationships:

$$\begin{cases} \dot{x}=Ax+\underline{D}w \\ y=\underline{C}x \end{cases} \tag{3}$$

With x which indicates the state of the system

The control of the system is done in closed loop, by observation of his state; as shown in figure 1:

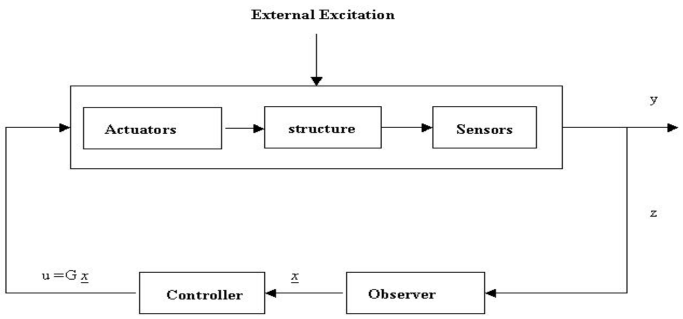


Fig 1. Schematic diagram of active control

The various phases of this technology of design lead to the following equations:
For the mechanical structure:

$$\dot{x}=Ax+\underline{B}u+\underline{D}w \tag{4}$$

$$y=\underline{C}x \tag{5}$$

$$z=\underline{M}x+v \tag{6}$$

For observer

$$\dot{\underline{x}}=A\underline{x}+\underline{B}\,\underline{u}+F(z-\underline{M}\,\underline{x}) \tag{7}$$

For controller

$$\underline{u}=G\underline{x} \tag{8}$$

$$u = \underline{u} + \tilde{n} \qquad (9)$$

With $\underline{M}$ : localization matrix of sensors, $\underline{x}$ : the estimated state, z: the measurement of sensors, F: filter matrix, v: noise of measurement, G: gain matrix, B localization matrix of actuators, u: control vector, $\tilde{\eta}$ : white noise, E $\{\tilde{n}\} = 0$ and E $\{\tilde{n}(t)\ \tilde{n}(\underline{t})\} = \tilde{N}\delta(t - \underline{t})$.

According to the equations of (5) to (10) we obtain for the global system, the following equations:

$$\dot{x} = \mathcal{A}x + \mathcal{D}w \qquad (10)$$

$$y = Cx \qquad (11)$$

The covariance of output is expressed by:

$$Y = \underline{\mathbf{C}}\chi\underline{\mathbf{C}}^T \qquad (12)$$

With $\chi$ the covariance of the state solution of the equation of Lyapounov

$$\mathcal{X}\mathcal{A}^T + \mathcal{A}\mathcal{X} + \mathcal{D}\mathcal{W}\mathcal{D}^T = 0 \qquad (13)$$

**RELIABILITY ANALYSIS**

This paper proposes to study the robustness of the laws of controls in the presence of uncertainties around the parameters of design of an active mechanical system. These uncertainties influence the stability and the performance of the controlled system. In the literature, a great importance is attached to the evaluation of the reliability of these types of systems [4-6]. Let us consider X = (X1, X2... Xn) the vector of the characteristic variables of the studied system. The first stage towards the evaluation of its reliability is to establish the functional relation binding these variables. This noted function G (X) is called function of state or function of absolute limit [10].

$$G(X) = G(X_1, X_2 \ldots X_n) \qquad (14)$$

This function makes it possible to distinguish two fields: the field of safety or reliability called S (Sure) with G (X) > 0, and the field of failure called F (Failure) with G (X)<0. G (X)=0 is the surface of absolute limit. The reliability of the structure, noted PF is the probability that vector X is in an unfavorable position with the structure.

$$P_f = P[G(X) \leq 0] \qquad (15)$$

$$P_f = \underset{G(X) \leq 0}{\iiint \ldots \int} f_{X1, X2 \ldots Xn}(x_1, x_2 \ldots x_n) dx_1 dx_2 \ldots dx_n \qquad (16)$$

$f_{X1,X2,\ldots Xn}(x_1,x_2,\ldots x_n)dx_1\ldots dx_n$ indicate the density of joined probability of the basic variables. Two methods are used to solve the integral (16): techniques of simulations of Monte Carlo or methods of approximations (FORM/SORM) [7]. Theoretical complexity for the installation of this type of method around the active systems led us to apply an experimental step. This step is based on the theory of the experimental designs.

**DESIGN OF EXPERIMENTS METHOD**

The method of the experimental designs helps in the study of the behavior of a complex system. It showed its effectiveness in different field from mechanics.
This method proposes a total model of behavior at the base of a series of experiments. This model is valid in the field of definition of the experiments. The number of experiments is depending on the number of the studied factors and the number of the methods chosen for each one. Each experiment of the plan brings information on the behavior of the studied system. The total analysis associated each experimental design is single. This analysis shows the influence of each factor on the total behavior of the answer (fig. 2). Concerning the study presented in this paper, it is reasonable to adopt a numerical step of experimentation in order to limit the number of experiments to be led to the laboratory.

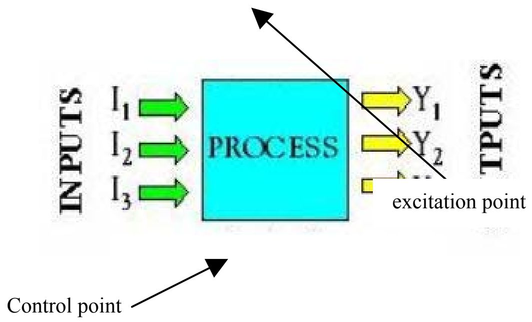


Fig 2 Process of Design of Experiments

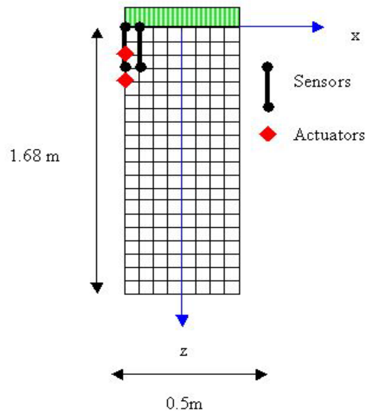


Fig 3 Configurations end dimensions of the Panel

This paper presents the study of the active control of a vibrating panel which undergoes a white noise excitation (fig 3). The study is carried out with the method of the experimental designs. The whole of the undertaken experiments is numerical. Calculations are done with a software for modeling of structures developed under MATLAB.
Concerning the application presented in this paper the objective is to determine the conditions of control of the standard deviation ($\sigma_y$) of clearance at the studied point. This standard deviation must be lower than 10E-3m (covariance=1E-6).
It is about clearance according to the direction perpendicular to the plan of figure 3 (y). However, in the presence of the observer and of the regulator $\sigma_y$ is 1.51E-3 (covariance=2.27E-6).
The objective of the experimental step presented in this paper is to determine the adjustments of the factors of design to control the dispersion of the clearance measured with the node indicated on figure 3. This adjustment is carried out in a configuration given concerning the positioning of the sensors and the actuators.
The experimental design carried out relates to three factors. These factors and their level are presented in table 1. In this case the number of experiments is 27. Table 2 presents the plan used as well as the results obtained.

Table 1

| Factors | Level | | | Units |
|---|---|---|---|---|
| | 1 | 2 | 3 | |
| A: width of the Panel | 0.5 | 0.55 | 0.6 | m |
| B: Young Modulus | 7.1981E+10 | 7.2E+10 | 7.2019E+10 | N/m² |
| C: Thickness of the Panel | 1.8 | 2.0 | 2.2 | Mm |

The statistical analysis of the results for the experimental design confirms the importance of factors A, and C. At the same time, it reveals a strong presence of an interaction between these two factors. The nonlinear character of the results is with the results observed when A is on level 1 and C on level 1. This particular case is a configuration not wished for in design. Figure 4 shows the behavior of the factor A, B, C like that of the interaction AC.

| Number of test | A width of the Panel | B Young Modulus | C Thick of the Panel | Y | $\sigma_y$ |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 7,10E-06 | 2,67E-03 |
| 2 | 1 | 1 | 2 | 3,83E-06 | 1,96E-03 |
| 3 | 1 | 1 | 3 | 3,43E-06 | 1,85E-03 |
| 4 | 1 | 2 | 1 | 7,04E-06 | 2,65E-03 |

| Factors | variability | % | d.o.f | variance | experimental Fischer |
|---|---|---|---|---|---|
| A | 1,71E-11 | 33,63 | 2 | 8,56E-12 | 5177,56 |

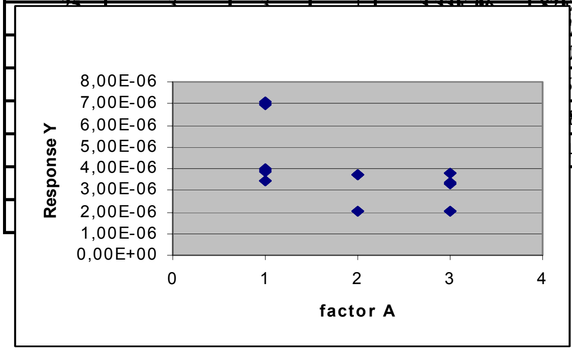


Table 2

Table 3 : Variance analysis

Fig. 4-1: effect of A

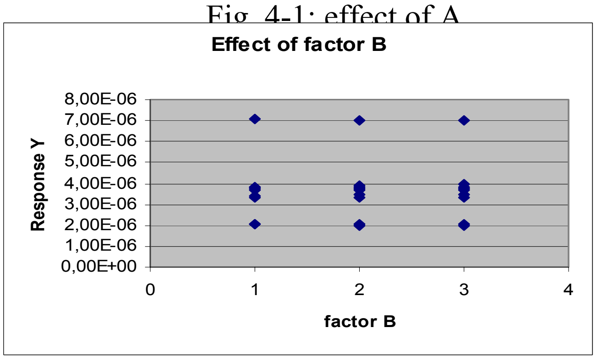


Fig. 4-3: effect of B

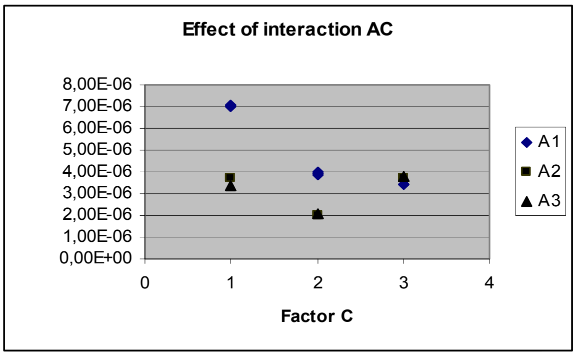


Fig. 4-3: effect of C

Fig. 4-4: effect of AC

The mathematical modeling suggested by the method of the experimental designs shows that it is possible to obtain levels of answers closer to the desired objective ($\sigma_y$ < 1.51E-3 thus a covariance < 2.27E-6). Indeed, the strong influence of the interaction AC disturbs the level of the results.
Thus two configurations were retained to carry out experiments complementary with A to the level 0.575mm, B on level 7.210E+10 N/m2 and C on the levels 19mm and 21mm. The two results obtained are: with C with 19mm $\sigma_y$ = 1.42E-3 (covariance=2.019E-6) and for C to 21 mm $\sigma_y$ = 1.49E-3 (covariance = 2.227E-6).
Thus the configuration which consists in locking A on the level 0.575mm, B on the level 7.210e+10 N/m2 and C on the level 19mm lead to an interesting technological solution. Indeed it is about a thin plate with an optimized width and an average Young modulus.

The mathematical model resulting from the method of the experimental design shows in this configuration that the standard deviation of the clearance of the studied point lies between [1.4E-3 and 1.44E-3] with a confidence of 95%. Thus probability $P_f$ (probability of failure) is in this case is 5%.

## CONCLUSION

The evaluation of the reliability of an active structure is difficult to obtain according to steps employed today for the passive mechanical structures. Indeed, the use of the theoretical methods is not easy. The methodology of the experimental designs A made it possible to explore various configurations of design and to retain the best without the obligation to change the strategy of control adopted with the design. It remains to explore the possibility of controlling the parameters of manufacture of this type of structure (roughness, surface treatment...) and their interactions with the active character of the structure.

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