CRACK PATH DEVELOPMENT IN MATERIALS WITH NONLINEAR PROCESS ZONE AND COMPOSITES

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ABSTRACT

Crack path development under mixed mode loading is an important and still not completely understood aspect of fracture process. A detailed understanding of this process could have several valuable benefits for failure prediction and for design of tougher materials using a crack path deflection toughening mechanism. The methods of linear fracture mechanics may give qualitative results, but do not accurately predict the crack path development, especially when the crack path is simulated for a certain distance due to a cumulative error. To improve the modeling process, nonlinear aspects of fracture process were introduced into consideration to determine the crack path direction criterion under more realistic conditions. A numerical procedure for solving this nonlinear problem has been introduced. A case illustrating the basic aspects of the solution method and the results is described using a deflecting line plastic zone as an example. The method is applicable to any nonlinear relationship within the deflecting process zone and is particularly useful for crack path analysis in composites.

KEYWORDS

Fracture, crack path, failure in composites, crack growth in nonlinear materials, crack growth in heterogeneous materials.

INTRODUCTION

The objectives of the presented investigation are aimed at determining the effect of nonlinear material behavior within the crack tip process zone on the crack path formation direction. The influence of the crack path deflection effect on the overall material toughness could be significant, as demonstrated by Rubinstein [1] for brittle materials. However, the current crack path prediction methods are not adequate for cumulative curvilinear crack path predictions. Generally, local crack path deflection in brittle materials is determined by one of three criteria: (*a*) maximal energy release rate, (*b*) maximal local K_I acting at the crack tip, or (c) zero local K_{II} acting at the crack tip. Most commonly used is criterion (*c*). Although physically all these criteria are similar, and one would expect them to predict the same crack path direction, there is a difference. For example [1], in case of a straight crack loaded under pure Mode II loading conditions, the resulting predictions would vary from a possible deflection in the direction of 75^o (criterion *a*) to 83^o (criterion *c*) depending on what criterion is used. For formation of a single crack kink the difference may be considered to be insignificant, but for continuous crack path curving and kinking over a finite distance, the difference will be noticeable. In a controlled experiment [2] with an accurate numerical analysis [3, 4], an attempt was made to determine which of the three mentioned criteria actually controls the crack path development. It was found [4] that none of the mentioned criteria could be selected as the determinative one, and that the crack path actually does not follow any of them. Buzzard, Gross and Srawley [5] demonstrated that under certain conditions, the crack in fact could propagate under Mode II loading without changing direction. These observations led to assumption [4], that a nonlinear process zone, which is present in all considered cases [1, 5, 6, 7, 8], plays a significant role in crack path development even if in some cases the size of the process zone may appear to be insignificant on a large scale. Even in seemingly brittle materials, as observed by Chudnovsky et al. [1], a nonlinear process zone is present on microscale. The present investigation attempts to include the influence of the nonlinear process zone on crack path development. The nonlinear effects are of particular importance in the analysis of crack path development in composite materials, especially composite systems which exhibit linear elastic behavior everywhere and a special crack opening displacement - local traction relationship within the process zone, usually identified as a bridging zone.

The modeling principles of the crack path deflection mechanism presented here are aimed at application to materials which exhibit nonlinear behavior within the crack tip process zone while remaining elastic in the surrounding region. To include nonlinear material behavior within the crack tip process zone in the model, a proper computational procedure has been developed. Typically, the process zone in these materials is small as compared to the crack size; thus, some cases may be studied using small-scale analysis principles. A numerical procedure based on the integral equation technique has been developed for solving the resulting nonlinear problem. Several critical aspects typical for these nonlinear problems will be discussed. Specifically, the scaling problem, nonlinear zone size determination techniques, the numerical stability of the possible solution procedures, and ill-posed numerical schemes practiced for these types of problems will be discussed. Several solutions for kinked cracks under mixed mode loading will be presented and compared for different material properties exhibited within the process zone. The developed methods will be especially useful for failure modeling and crack path prediction in heterogeneous materials such as reinforced ceramics, concrete, and some metal alloys, and for development of smart materials.

The method developed in conjunction with this investigation is capable of handing various conditions within the process zone. However, as a representative example, primary attention in this report is given to the case of a process zone consisting of an inclined, under angle θ, rectilinear segment of a plastically deformed zone, Figure 1. The case of $\theta = 0$ is the well-known Dugdale plastic zone model [9]. Several interesting aspects of this nonlinear problem could be learned and understood based on this example. As is often the case in nonlinear numerical analysis, this problem may be easily misled into an ill-posed numerical scheme. This aspect of the analysis will be discussed in the following section.

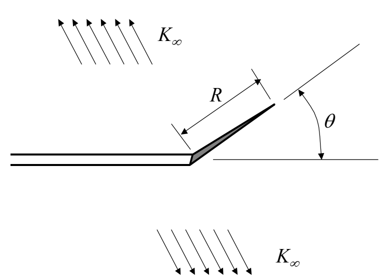

Figure 1. Considered problem

SOLUTION SCHEME

The considered problem has a specified remote load K_∞, which represents a mixed mode loading. The condition on the inclined segment, in this special case, states the yielding condition,

$$\sigma_{\theta\theta} = \sigma_y. \quad (1)$$

The inclination angle, θ, and the length of the inclined plastically deformed segment, R, are unknown variables. This simple fact is sometimes neglected and only the inclination angle is treated as an unknown variable, especially when a curvilinear crack path is simulated. Most typically for development of a direct numerical scheme, one needs to specify the length of the inclined segment or the inclination angle. Several examples could be presented wherein a finite element scheme was set with a specified length of inclined segment. In fact, the development of a direct computational scheme for this highly nonlinear problem could be complicated by convergence problems and mesh size dependency. To avoid that, the following indirect scheme is proposed, which allows the problem to be treated as a linear problem at every significant computational step.

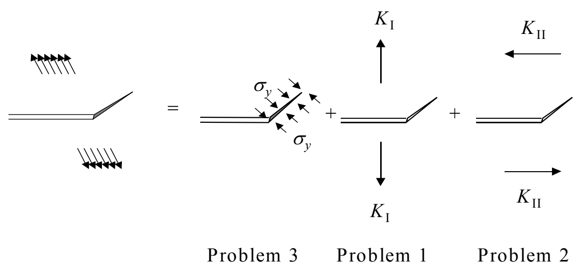

Figure 2. Superposition scheme.

The considered problem, as specified in Figure 1, can be represented as a superposition of three problems illustrated in Figure 2. Problem 3 has loading only on the kinked segment; Problem 1 is a problem of a kinked crack under Mode I loading; and Problem 2 is a problem of a kinked crack under Mode II loading. Problem 3 results in two local stress intensity factors, K_I^3 and K_{II}^3 at the tip of the kink Both of them must be compensated for by the local stress intensity factors of Problems 1 and 2, K_I^1, K_{II}^1 and K_I^2, K_{II}^2. The relationship between these three sets gives a set of two equations:

$$\begin{aligned} K_I^1 + K_I^2 + K_I^3 &= 0 \\ K_{II}^1 + K_{II}^2 + K_{II}^3 &= 0 \end{aligned} \quad (2)$$

The inclination angle, θ, determines the relationships between the applied load components K_I, K_{II}, and the corresponding local values of the stress intensity factors as

$$\begin{aligned} K_I^i &= f_I^i(\theta) K_i \\ K_{II}^i &= f_{II}^i(\theta) K_i \end{aligned} \quad (3)$$

i here runs values 1 and 2, and on the right hand side of (3) this index corresponds to the loading Mode. The functions $f^i{}_I(\theta)$ and $f^i{}_{II}(\theta)$ can be generated using a scheme given by Rubinstein [1], or used directly from the data given in [1].

Setting the length of the kink to a unit of length, substituting equations (3) into equations (2), one obtains a linear system for determination of the remote load that will generate a unit length plastic kink inclined at a specified angle θ, if solution of problem 3 is known. Thus, the following solution scheme can be employed using the inverse order for a unit length kink and a unit yield stress: (a) set a value for the inclination angle; (b) solve problem 3 and determine numerically the values of K_I^3 and K_{II}^3; (c) solve system (2) with (3) for K_I, K_{II}, using the given functions $f^i{}_I(\theta)$ and $f^i{}_{II}(\theta)$. The result is the load required to produce a unit length plastic kink at a specified angle, Figure 3. To obtain a general solution, these data in Figure 3 can be rescaled using the following relationship for the absolute value of the remote stress intensity factor, (4).

$$K_\infty = K_\infty(R=1, \sigma_y = 1)\sigma_y \sqrt{R} \tag{4}$$

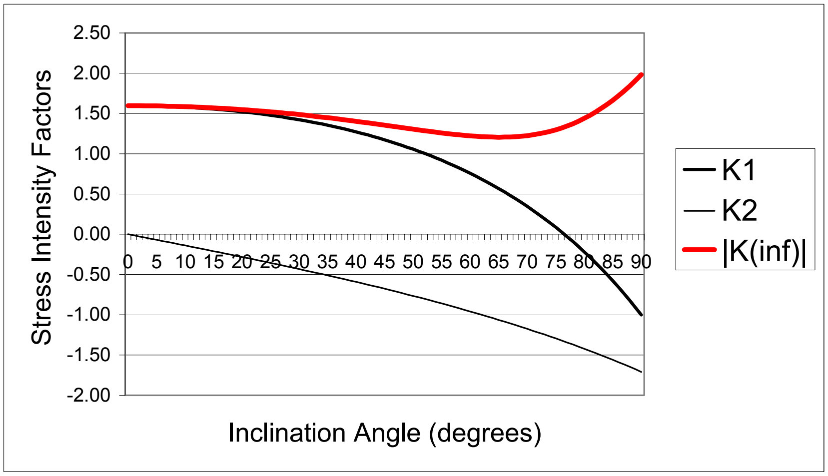

Figure 3. Applied load generating a unit-length inclined plastic zone (σ_y=1)

Relationship (4) uses the results of the scaling analysis for the described nonlinear case. This relationship is not a general one. Using the data in Figure 3, or the corresponding parametric form of that data and (4), one can determine the inclination angle and the length of the inclined plastic zone for an arbitrary loading. It is also convenient to use the loading stress intensity factor in a complex variable form. For the described case, the relationship between the loading phase and the inclination angle does not depend on the loading magnitude [10]. After one numerically generates the function representing this relationship, the inclination angle can be directly determined; then, using the data in Figure 3 and relationship (4), one obtains the length of the plastic kink.

CONCLUSIONS

A computational scheme for the analysis of crack kinking in materials with a nonlinear process zone has been developed. An example of the inclined Dugdale zone has been considered to illustrate the computational procedure. Further details of this procedure and examples could be found in [10].

ACKNOWLEDGMENT

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