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CRACK INITIATION AND GROWTH IN GRADED MATERIALS DUE TO SLIDING CONTACT

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ABSTRACT

In this article, initiation and subcritical growth of surface cracks in graded materials due to sliding contact is considered. The investigation of the crack initiation process requires the evaluation of tensile cleavage stress on the surface, whereas subcritical crack growth is generally controlled by the stress intensity factors. After a brief introduction, the coupled crack/contact problem for a semi-infinite graded medium loaded by a rigid stamp is outlined, the stress intensity factors are calculated and some sample results are presented.

KEYWORDS

Graded Materials, sliding contact, crack initiation, crack/contact problem, stress intensity factors

INTRODUCTION

Graded materials, also known as *functionally graded materials* (FGMs) are multiphase composites with continuously varying volume fractions and, consequently, thermomechanical properties. Used as coatings and interfacial zones they reduce the stresses resulting from the material property mismatch, increase the bonding strength, improve surface properties and provide protection against adverse thermal and chemical environments. Many of the present and potential applications of graded materials involve contact problems. These are mostly load transfer problems in deformable solids, generally in the presence of friction. In such applications the concept of material property grading appears to be ideally suitable to improve the surface properties and wear resistance of the components that are in contact. From the standpoint of failure mechanics an important aspect of contact problems is surface cracking which is caused by friction forces and which invariably leads to fretting fatigue. In most applications material property grading near the surfaces is used as a substitute for ceramic coatings. Hence, the surface of the composite medium consists of one hundred percent ceramic. As a result the “maximum tensile stress” criterion may be used for crack initiation on the surface. The main objective of this study is to investigate the problem of contact mechanics and the associated fracture phenomenon in graded materials subjected to repeated loading by a rigid stamp. In particular the influence of the coefficient of friction and the material nonhomogeneity parameters on the stress intensity factors is examined. The problem is considered under the assumptions of plane strain and Coulomb friction.

Studies in contact mechanics were originated by Hertz [1]. A thorough description of the underlying solid mechanics problems in homogeneous materials maybe found for example in [2]. Sample results for frictionless contact problems in a semi-infinite graded medium are given in [3]-[5]. The details of the analysis of contact mechanics for elastic solids with graded coatings and extensive results regarding the stress distribution are discussed in [6].

FORMULATION OF SLIDING CONTACT/CRACK PROBLEMS

The general description of a sliding contact/crack problem in a graded medium is shown in Figure 1.

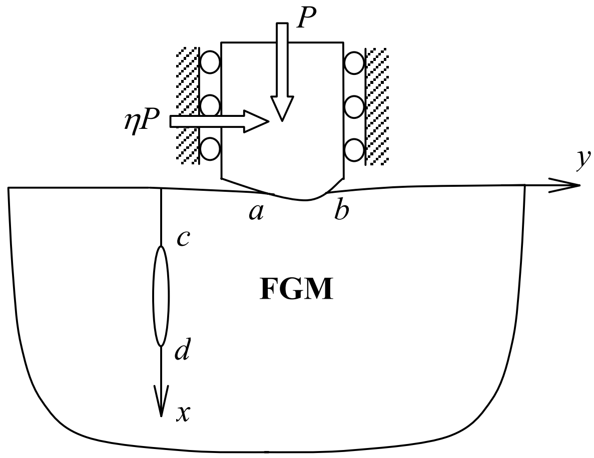

Figure 1: The general description of the crack/contact problem in a graded medium

The load is applied through a rigid stamp of arbitrary profile and it is assumed that the conditions of plane strain and Coulomb friction are valid and $h >> d$, $h >> |a|$, $h >> |b|$ where h is the thickness of the medium. Thus, the graded medium may be treated as being semi-infinite. In this study for simplicity it is further assumed that the shear modulus of the medium may be approximated by $\mu(x) = \mu_0 \exp(\gamma x)$ and the effect of the variation of Poisson's ratio ν on such quantities as stress intensity factors is negligible [7]. In the coupled crack/contact problem described in Figure 1 the unknown functions are the crack surface displacements and the contact stresses defined by

$$f_1(x) = \frac{2\mu_0}{\kappa+1}\frac{\partial}{\partial x}\left[v(x,0^+) - v(x,0^-)\right], \qquad c < x < d, \tag{1}$$

$$f_2(x) = \frac{2\mu_0}{\kappa+1}\frac{\partial}{\partial x}\left[u(x,0^+) - u(x,0^-)\right], \qquad c < x < d, \tag{2}$$

$$f_3(y) = \sigma_{xx}(0,y) = \frac{1}{\eta}\sigma_{xy}(0,y), \qquad a < y < b, \tag{3}$$

where η is the coefficient of friction, $\kappa = 3 - 4\nu$ for plane strain and $\kappa = (3-\nu)/(1+\nu)$ for plane stress. The input functions are the crack surface tractions and the stamp profile given by

$$\sigma_{yy}(x,0) = \sigma_{xy}(x,0) = 0, \qquad c < x < d, \tag{4}$$

$$\frac{4\mu_0}{\kappa+1}\frac{\partial}{\partial y}u(0,y) = f(y), \qquad a < y < b. \tag{5}$$

By using the equations of elasticity and the definitions given by 1-5, the mixed boundary value problem described in Figure 1 may be reduced to a system of singular integral equations of the following form:

$$\int_c^d \left[\frac{1}{\pi}\frac{1}{t-x} + k_{11}(x,t)\right] f_1(t)\,dt + \int_a^b k_{13}(x,t)\, f_3(t)\,dt = 0, \quad c < x < d, \tag{6}$$

$$\int_c^d \left[\frac{1}{\pi}\frac{1}{t-x} + k_{22}(x,t)\right] f_2(t)\,dt + \int_a^b k_{23}(x,t)\, f_3(t)\,dt = 0, \quad c < x < d, \tag{7}$$

$$\int_c^d \left[k_{31}(y,t) f_1(t) + k_{32}(y,t) f_2(t)\right] dt + \int_a^b \left[\frac{1}{\pi}\frac{1}{t-y} + k_{33}(y,t)\right] f_3(t)\,dt - \eta\frac{\kappa-1}{\kappa+1} f_3(y) = f(y),$$

$$a < y < b, \tag{8}$$

$$\int_c^d f_1(t)\,dt = 0, \qquad \int_c^d f_2(t)\,dt = 0, \qquad \int_a^b f_3(t)\,dt = -P. \tag{9}$$

From Eqns. 6-8 the singular behavior of the unknown functions f_1, f_2 and f_3 is determined by using a function-theoretic method. The limiting case of $a = c = 0$ is of some theoretical and physical interest. In this case by defining

$$f_1(t) = t^{\alpha}(d-t)^{\delta} g_1(t), \qquad f_2(t) = t^{\alpha}(d-t)^{\delta} g_2(t), \qquad f_3(t) = t^{\alpha}(b-t)^{\beta} g_3(t), \tag{10}$$

the condition of boundedness of $\sigma_{yy}(x,0)$, $\sigma_{xy}(x,0)$, $0 < x < d$ and $f(y)$, $0 < y < b$ would give the following characteristic equations to determine δ, β and α

$$\cot(\pi\delta) = 0, \qquad \cot(\pi\beta) + \eta\frac{\kappa-1}{\kappa+1} = 0, \tag{11}$$

$$\eta\left[4\alpha^2 + 10\alpha + 5 + (\kappa-1)\cos(\pi\alpha) - \kappa(2\alpha+3)\right] + (\kappa+1)\sin(\pi\alpha) = 0. \tag{12}$$

One may note that these results are independent of μ_0 and the material nonhomogeneity constant γ and dependent on η and κ only, meaning that the stress singularities for graded and homogeneous materials are identical. Generally the contact stresses are concentrated toward the trailing end of the stamp. For $c > 0$ it may easily be shown that [6]

$$f_1(x) = (x-c)^{\theta}(d-x)^{\delta} g_1(x), \qquad f_2(x) = (x-c)^{\theta}(d-x)^{\delta} g_2(x),$$

$$\cot(\pi\theta) = 0, \qquad \cot(\pi\delta) = 0, \qquad f_3(y) = (y-a)^{\omega}(b-y)^{\beta} g_3(y),$$

$$\cot(\pi\omega) = \eta\frac{\kappa-1}{\kappa+1} = -\cot(\pi\beta), \quad \omega < 0, \quad \beta < 0. \tag{13}$$

Note that for $\eta > 0$ the stress singularity at $y = a$ is greater than that at $y = b$. The powers of singularity at a, b, c and d for $c > 0$, $a > 0$ as well as at 0 for $a = c = 0$ are shown in Figure 2 as functions of the friction coefficient η. From the standpoint of cracking $\eta > 0$ is the physically meaningful case for which α is real and, for high values of η, can be greater than the corresponding uncracked value ω. This unusual result given by Eqn. 12 has also been verified independently by using Mellin transforms.

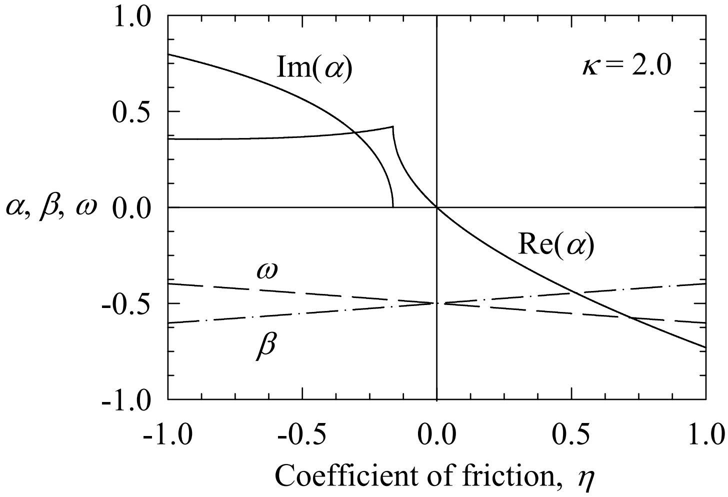

Figure 2: Variation of exponents α, ω and β with friction coefficient η

From Eqns. 7-9 it may be observed that the characteristic roots $\delta,\ \theta,\ \beta,\ \omega$ and α are multiple valued. The particular values of these exponents within the acceptable range $(-1,\ +1)$ are determined from physical considerations. For the crack $\delta = -1/2$ and $\theta = -1/2$ for $c > 0$ and $\theta = 0$ for $c = 0$. For $a = c = 0$ and $\eta > 0$ the dominant (and acceptable) root of Eqn. 12 is real and $\alpha < 0$. In the general stamp problem at an end point a (or b) ω (or β) is positive if the contact is smooth and negative if the stamp has a sharp corner [6].

Once the exponents $\delta,\ \theta,\ \beta,\ \omega$ and α are determined the weight functions $w_i(r)$ and the form of the solution of the integral equations may be obtained by normalizing the intervals $a < t < b$ and $c < t < d$ to $-1 < r < 1$ and by expressing the unknown functions as (see Eqns. 1-3 and 6-8)

$$f_i(t) = w_i(r)\sum_0^\infty C_{in} P_n^{(\delta,\theta)}(r) \quad i = 1,\ 2,\ \ w_1(r) = w_2(r) = (1-r)^\delta (1+r)^\theta,$$

$$f_3(t) = w_3(r)\sum_0^\infty C_{3n} P_n^{(\beta,\omega)}(r) \qquad w_3(r) = (1-r)^\beta (1+r)^\omega, \tag{14}$$

where P_n are the Jacobi polynomials associated with the weight functions w_j and C_{in} are unknown coefficients ($j = 1,\ 2,\ 3$). In the general form given by Eqn. 14 it is assumed that $c > 0$ (and $-\infty < a < \infty$). In the special cases of ($c = a = 0$) and ($c = 0,\ a > 0$) we have ($\theta = \alpha,\ \omega = \alpha$) and ($\theta = 0$), respectively. The integral equations are solved by truncating the series in Eqn. 14 and by using a suitable numerical method. After solving the integral equations, the quantities of physical interest, namely the stress intensity factors and the in-plane stress on the surface $\sigma_{yy}(0,y)$ may be obtained from

$$k_i(d) = -\lim_{x\to d} \exp(\pi\alpha)\sqrt{2(d-x)}\, f_i(x), \qquad k_i(c) = \lim_{x\to c} \exp(\pi\alpha)\sqrt{2(x-c)}\, f_i(x) \tag{15}$$

$$\sigma_{yy}(0,y) = \sum_1^2 \int_c^d h_j(y,t) f_j(t)\,dt + \int_a^b h_3(y,t) f_3(t)\,dt, \tag{16}$$

where k_1 and k_2 are the modes I and II stress intensity factors and h_1, h_2 and h_3 are known kernels associated with the in-plane stress component $\sigma_{yy}(0,y)$. From the standpoint of crack initiation the critical point on the surface is the trailing end ($y = a$, Figure 1) of the contact region where the cleavage stress $\sigma_{\theta\theta}(a,\theta)$ is positive and may be obtained from

$$\sigma_{\theta\theta}(a,\theta) = \sigma_{xx} \sin^2(\theta) + \sigma_{yy} \cos^2(\theta) + \sigma_{xy} \sin(\theta)\cos(\theta) \tag{17}$$

In the notation of Figure 1, from (17) it may be shown that $\theta_{cr} = 0$, and $\sigma_{\theta\theta cr} = \sigma_{yy}(0,a)$.

SAMPLE RESULTS

Some sample results giving the modes I and II stress intensity factors for a graded medium containing a surface crack of length d and subjected to a sliding rigid flat stamp are shown in Figure 3. (Figure 1, $c = 0$). The stiffness variation of the medium is given by $\mu(x) = \mu_0 \exp(\gamma x)$. On the top row of Figure 3 the full lines are obtained from the FGM solution for $\gamma d = 0.0001$, whereas the closed circles are given by the corresponding homogeneous medium. The results clearly show the strong influence of the stamp location a / d and the material inhomogeneity parameter γd on the stress intensity factors. Note that, as formulated the problem is one of mixed-mode. Consequently, crack growth would be curved. Also, in the absence of additional in-plane tension, k_1 could be negative, implying crack closure which can be treated in a straightforward fashion.

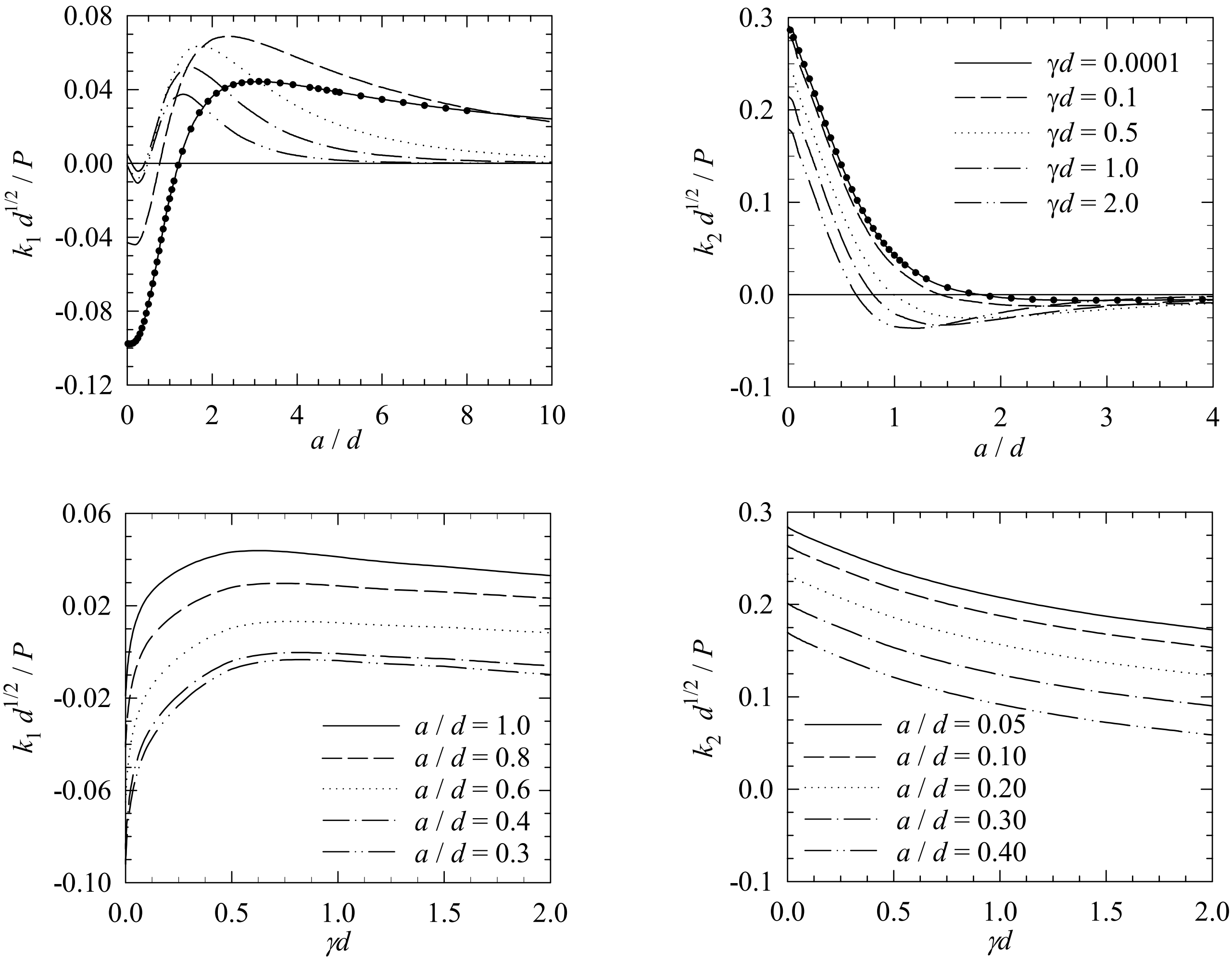

Figure 3: Stress intensity factors in an FGM half-plane with a surface crack loaded by a rigid stamp (Figure 1, $c = 0$), $\kappa = 2$, $\eta = 0.4$, $(b - a)/d = 0.1$.

For a graded medium in the absence of a crack and loaded by a sliding flat stamp the normal component of the contact stress $\sigma_{xx}(0,y)$, $a < y < b$, and the in-plane surface stress $\sigma_{yy}(0,y)$, $-\infty < y < \infty$, are given in Figure 4 for various values of γ and for $\eta = 0$ and for $\eta = 0.4$. Note that for $\eta = 0$ the stress distribution is symmetric, whereas for $\eta > 0$ $|\omega| > |\beta|$ and the stresses are concentrated near the trailing end $y = a$. Also, at $y = a$ $\sigma_{yy}(0,y)$ has a singularity of the order $(a-y)^{\alpha}$, implying that $y = a$, is a likely location of crack initiation.

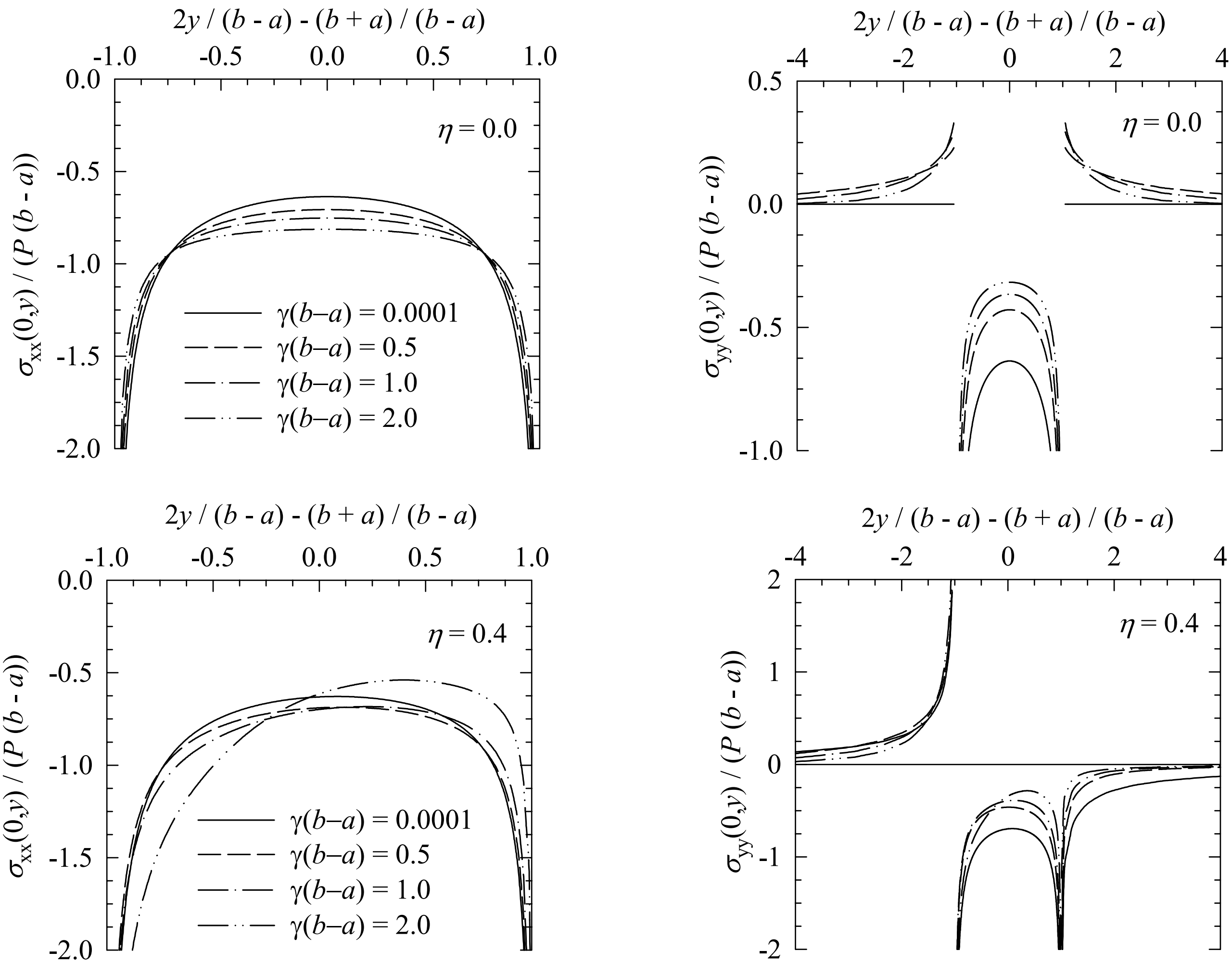

Figure 4: The contact stress $\sigma_{xx}(0,y)$ and the in-plane stress $\sigma_{yy}(0,y)$ on the surface of a graded medium loaded by a flat stamp.

REFERENCES

1. Hertz, H. (1882) *J. Reine und Augewandte Mathematik* 92, 156.
2. Johnson, K.L. (1985). Contact Mechanics. Cambridge University Press, Cambridge, UK.
3. Bakirtas, I. (1980) *Int. J. Engng. Sci.* 18, 597.
4. Giannakopoulos, A.E. and Suresh, S. (1997) *Int. J. Solids Structures.* 34, 2393.
5. Suresh, S., Giannakopoulos, A.E. and Alcala, J. (1997) *Acta Mater.* 45, 1307.
6. Guler, M.A. (2001). Ph.D. Dissertation, Lehigh University, Bethlehem, PA, USA.
7. Delale, F. and Erdogan, F. (1983) *ASME, J. Appl. Mech.* 50, 609.