CRACK GROWTH RESISTANCE OF METAL FOAMS

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ABSTRACT

This study investigates the crack growth initiation and subsequent resistance in ductile, open cell foams. We consider a macroscopic crack in a cellular structure in mode I loading and under small scale yielding conditions. The elasto-plastic response of the cell wall material is described by a bilinear stress-strain relation and fracture of the cell walls is characterised by a fracture energy per unit area. Suitable normalised problem parameters are identified in a dimensional analysis. Crack growth is simulated numerically by removing elements from a finite element model. In this way, evolving plastic zones and macroscopic K-resistance curves are calculated. Specifically, the dependence of the macroscopic fracture properties upon the parameters of the cell wall material is addressed. The results are compared with analytic estimates which are derived on the basis of simple considerations. The results include the findings that the toughness of the foam scales linearly with the fracture strength of the cell walls and quadratically with the relative density of the foam.

KEYWORDS

ductile fracture, crack resistance, R-curve, honeycombs, foams

INTRODUCTION

Metallic foams have unique property profiles and are considered for a number of engineering applications such as e.g. cores of sandwich constructions. An increasing use of such materials calls for an investigation of their failure mechanisms. Previous studies of the fracture properties of cellular materials seem to have been limited to the case of *brittle* base materials and to the prediction of a critical stress intensity factor. For example, Gibson & Ashby [1] derive an estimate for K_c by calculating the bending moment in a cell wall adjacent to the macroscopic crack tip from the asymptotic singular stress field of linear elasticity-theory. The local stresses associated with bending of the cell walls can then be expressed in terms of the applied K, and the fracture strength of the cell wall determines K_c. Similar approaches have been followed by Choi & Lakes [2] and Chen & Huang [3], where the

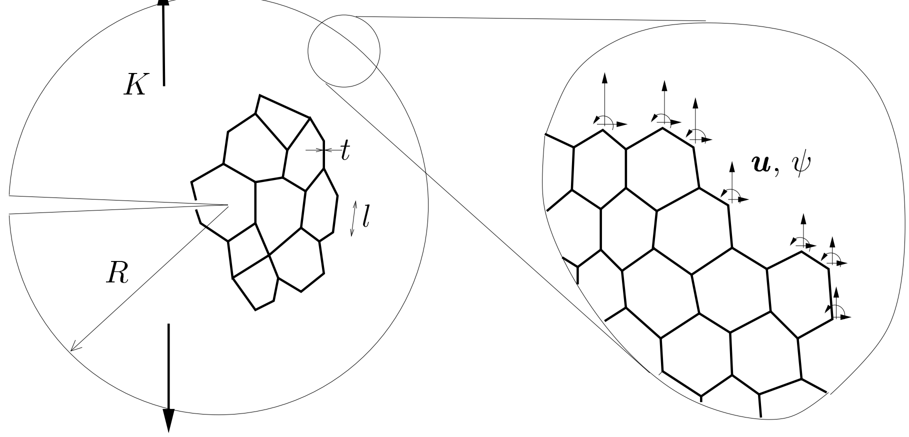

Figure 1: Cellular structure with a macroscopic crack under small-scale yielding conditions

latter used the crack tip asymptotics of a micro polar continuum theory. The authors are unaware of any micro-mechanical studies on the influence of the cell walls' ductility upon the macroscopic fracture properties of a cellular solid; this is the objective of the present paper. We consider two-dimensional irregular cellular structures as plane models for a metallic foam containing a macroscopic crack under mode-I loading and employ the assumption of small scale yielding. By introducing a local failure criterion, the initiation toughness, crack resistance curves and plastic zone evolution are then calculated numerically.

MODEL SPECIFICATION

We consider two-dimensional hexagonal structures with a macroscopic crack according to Figure 1. The beams (thickness t, average length l; termed 'cell walls' in the sequel) that make up the structure are assumed to be sufficiently slender so that their shear compliance can be neglected compared to their bending compliance ($t/l \ll 1$). The cell wall material is characterised by a uniaxial bilinear stress strain relation of the form

$$\begin{array}{llll} \epsilon = & \sigma/E & \sigma < & \sigma_{\mathrm{y}} \\ \epsilon = & \sigma_{\mathrm{y}}/E + (\sigma - \sigma_{\mathrm{y}})/H & \sigma > & \sigma_{\mathrm{y}} \end{array} \tag{1}$$

in terms of the true stress σ, true strain ϵ, Young's modulus E, yield strength σ_{y} and constant hardening modulus H. The assumption of small-scale yielding allows for the prescription of displacements on a boundary remote from the crack tip as given by the mode-I K-field. The applied K is gradually increased as a loading parameter until the stress in one of the beams attains the fracture strength σ_{f} of the cell wall material. This marks the beginning of a local fracture process which itself is not modelled in detail. Rather, we assume that it affects the macroscopic fracture response of the cellular material only through the amount of work dissipated due to the fracture of the beam. Thus, we take the fracture properties of the cell wall material to be sufficiently described through the fracture energy per unit area of beam cross section Γ_0. Crack propagation is then simulated by disconnecting a beam from the respective vertex with the requirement that the work of fracture equals Γ_0 times the cross section of the beam A.

NORMALISATION AND ESTIMATES

Stress intensity factor

A characteristic value for the fracture toughness which also serves as a normalisation for the stress intensity factor is introduced as

$$K_{\mathrm{Y}} = \sqrt{\pi l}\, \sigma_{\mathrm{y}} (t/l)^2 \quad . \tag{2}$$

This expression can be derived on the basis of elementary considerations by assuming that the local deformation is bending dominated and that the displacement field around the crack tip is that of a linear elastic solid whose stiffness scales with $(t/l)^3$. The dimensionless stress intensity factor is then defined as $\tilde{K} = K/K_{\mathrm{Y}}$.

Dimensionless problem parameters
The parameters of the model allow for the following set of dimensionless quantities upon which the dimensionless fracture toughness depends.

$$\rho = t/l \ , \quad \sigma_{\mathrm{y}}/E \ , \quad \sigma_{\mathrm{f}}/\sigma_{\mathrm{y}} \ , \quad H/E \ , \quad \tilde{\Gamma}_0 = \Gamma_0/(E\sigma_{\mathrm{y}}^2 l) \quad , \tag{3}$$

where $\rho = t/l$, up to a factor of proportionality, can be interpreted as the relative density of the honeycomb; $\tilde{\Gamma}_0$ is, up to a constant factor, the work required to break a beam divided by the elastic strain energy contained in a beam under pure bending when the yield strain is attained at the outer fibres.

Plastic zone
A simple estimate for the plastic zone can be derived by introducing a yield function of the cellular material and inserting the asymptotic elastic stress field into it. In this way, a contour is defined inside of which the stresses exceed the yield value and which thus serves as an estimate for the plastic zone. Assuming a yield function quadratic in mean- and deviatoric stresses, which has been shown to be a good approximation for irregular hexagonal honeycombs CHEN et al. [4], the above procedure leads to the estimate

$$r_{\mathrm{p}}(\varphi)/l = \frac{5.06}{2+\beta^2}\frac{\tilde{K}^2}{\kappa^2}[\sin^2(\varphi/2) + \beta^2(1+\cos\varphi)] \tag{4}$$

for the plastic zone contour in terms of polar coordinates centred on the crack tip. Here, β is related to the ratio of uniaxial to hydrostatic yield strength $\omega \equiv \sigma_u/\sigma_h$ via $\beta^2 = 2\omega^2/(4-\omega^2)$ and κ denotes the ratio of uniaxial yield strength for the irregular and regular honeycombs. In (4), the values $\omega = \beta = 0$ and $\kappa = 1$ correspond to a regular honeycomb.

NUMERICAL TECHNIQUE

The numerical evaluation of the above described model is performed with a finite element method. The beams are discretised with up to 7 cubic elements and the honeycomb slab is bounded by a circle of radius $R \approx 85l$ where the displacements are prescribed according to the mode-I K-field. When the fracture strength σ_{f} is reached in one of the elements, that element is removed from the finite element model – thereby disconnecting the fractured beam from a vertex. This element removal is performed with a built-in routine of the finite element code ABAQUS which first replaces the element with the forces and moments it exerts on its neighbouring nodes and subsequently reduces these section forces to zero over a prescribed load-parameter interval. By choosing this interval, i.e. ΔK, appropriately, $\tilde{\Gamma}_0$ can be adjusted to the desired value. In this way, crack propagation is simulated and K-resistance curves are calculated.

RESULTS

The following cell wall material parameters are chosen as representative of those for aluminium alloy foams:

$$\sigma_{\mathrm{y}}/E \ = 0.1\% \ , \quad \sigma_{\mathrm{f}}/\sigma_{\mathrm{y}} \ = 2.0 \ , \quad H/E = 0.1 \ , \quad \tilde{\Gamma}_0 = 3.0 \quad . \tag{5}$$

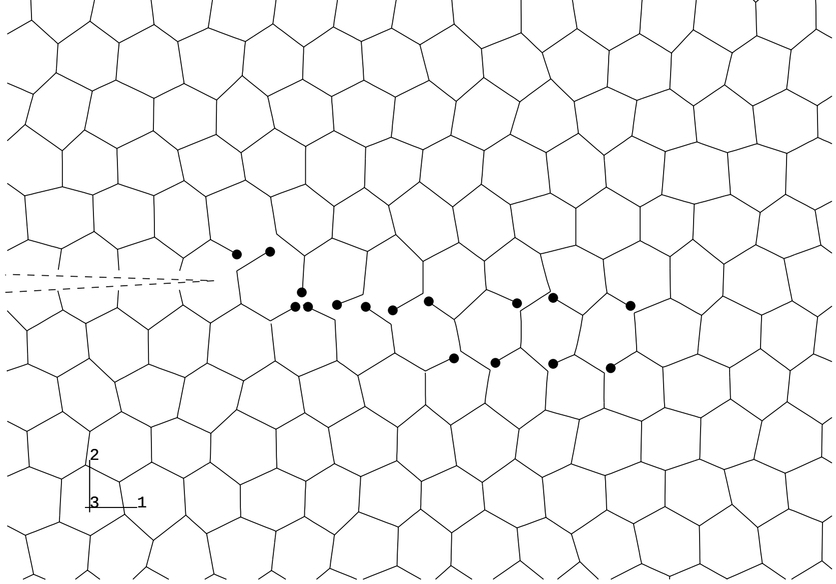

Figure 2: Predicted crack path in an irregular honeycomb

Crack path and plastic zones

A typical prediction of crack path is shown in Figure 2 with loading taken to the point where 15 cell walls have failed. This structure is generated by randomly perturbing the vertex positions of a regular hexagonal honeycomb, the amount of perturbation being uniformly distributed between -15% and $+15\%$ of the average beam length. Note that the broken beams do not form a continuous crack path with leading to a unique position of the macroscopic crack tip; the broken beams divide the structure into a multiply connected domain and this disconnected crack advance can be viewed as a crack bridging phenomenon. The plastic zones at initiation and at $\Delta a = 8l$ are shown in Fig. 3 together with the estimate (4), in which $\omega = 0.7$ and $\kappa = 0.8$ have been used (These values have been extrapolated from the calculations in Chen et al. [4] for the chosen level of imperfection in the foam.) The plots in Fig. 3 have been obtained by marking with a cross the plastic vertices of 5 different realisations of an irregular honeycomb in the same plot. Thus, the intensity of the markers can be viewed as the ensemble average for the magnitude of the plastic strain around the crack tip. The estimate (4) for the plastic zone is in reasonable agreement with the numerical results with regard to both shape and size.

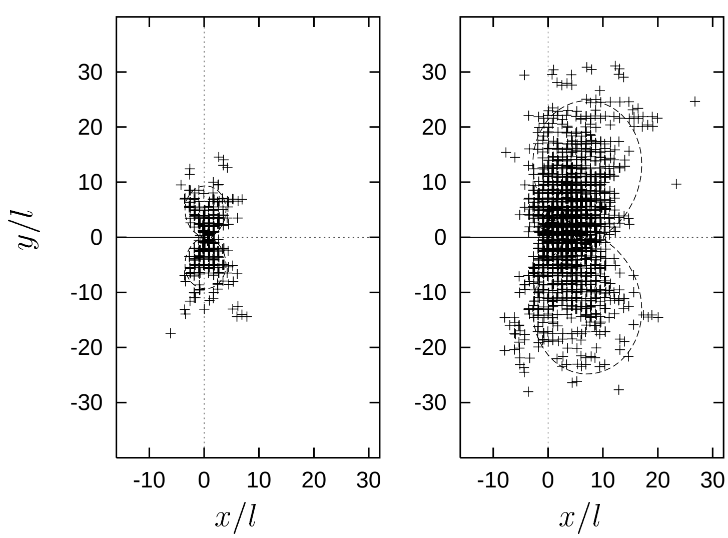

Figure 3: Plastic zones for the irregular honeycomb at initiation (left) and for $\Delta a = 8l$ (right)

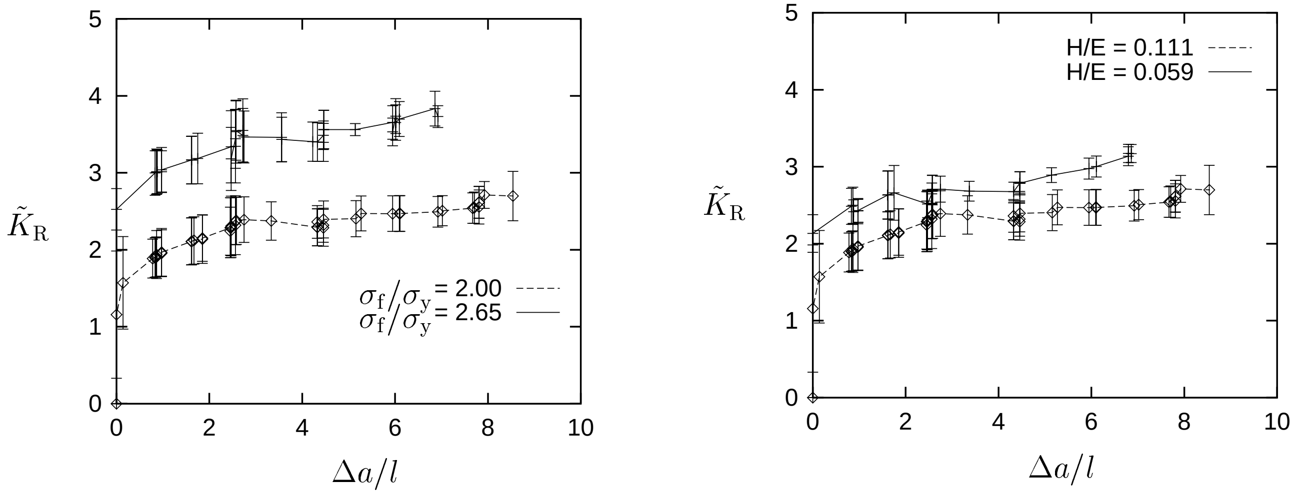

Figure 4: K-resistance curves for two values of fracture strength (left) and hardening modulus (right)

Crack resistance

Crack resistance curves in the form of a Stress intensity factor vs. crack elongation plots have been obtained by averaging, at each stage of the crack propagation, the K_R-values of 5 different realisations. Typical results are shown in Figure 4 where the error-bars indicate the standard deviation from the corresponding mean value. The left plot illustrates the influence of an altered cell wall fracture strength ($\sigma_\mathrm{f}/\sigma_\mathrm{y}$ 2.0 $\rightarrow$ 2.65) and the plot on the right that of an altered hardening modulus (H/E 0.11 $\rightarrow$ 0.059). As could be expected, increasing the former and decreasing the latter both leads to elevated K_R values: In the first case, a larger applied K is needed to break the first beam because these can sustain larger stresses. In the second case, the deformation of the mesh boundary and, hence, K needs to be larger for the stresses to reach the unaltered fracture strength.

Because the shape of the R-curves remains essentially the same in all of the plots in Figure 4, we can characterise the influence of the cell wall material parameters by calculating only the initiation toughness as a function of these parameters. The corresponding results are depicted in Figure 5 in a log-scale representation, showing that $\tilde{K}_\mathrm{c}$ increases linearly with the normalised fracture strength and decreases with the hardening modulus according to a power law with an exponent $\approx -1/3$.

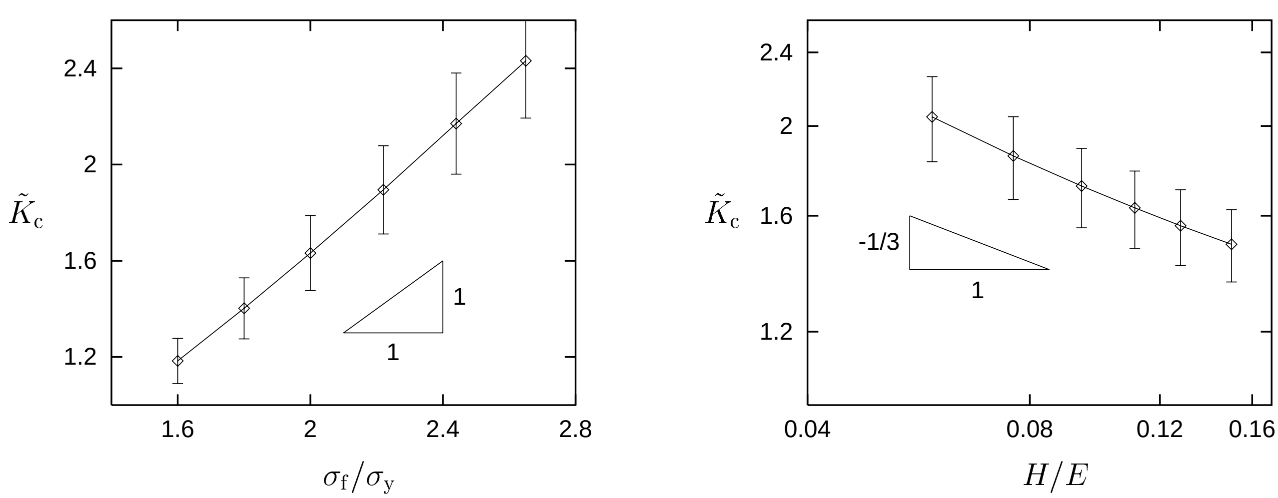

Figure 5: Initiation toughness vs. normalised fracture strength (left) and hardening modulus (right)

A result not shown here is that the dimensionless crack resistance remains practically unchanged upon changing the relative density ρ from 5% to 10%. This means that the influence of the relative density is adequately described through the normalisation introduced earlier, i.e. the dimensional fracture toughness of the honeycomb scales quadratically with ρ. Further results include the finding that, of the parameters listed in (3), the yield strain σ_y/E has negligible influence on the crack resistance. In contrast, numerical experimentation revealed that an increase in $\tilde{\Gamma}_0$ leads to large changes in the applied stress intensity factor during the process of removing an element (i.e. fracture of a beam) – with the effect that, at the end of a particular removal step, the stress may have attained the fracture strength in *several* other elements. In this case, the proposed method fails to describe the process adequately since it does not allow for the sequential fracture of beams at the crack tip. Nevertheless, we conclude that the slope of the R-curve will increase with increasing $\tilde{\Gamma}_0$.

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