COMPOSITE LAMINATES LEAST PRONE TO CRACKING UNDER COMBINED MECHANICAL AND THERMAL LOADS

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ABSTRACT

This paper demonstrates the use of fracture mechanics based strength criteria and optimization techniques in the design of fibre-reinforced laminate configurations against cracking. The optimum configurations are sought for multidirectional fibre-reinforced composite laminates under combined in-plane mechanical and thermal loads. The design objective is to enhance the value of the loads corresponding to the first-ply-failure as judged by a transverse failure criterion which contains the *in situ* strength parameters proposed by the authors. The highly nonlinear optimization problems are solved using nonlinear programming incorporating a local-global algorithm of Elwakeil and Arora. It is found that the optimum designs under combined mechanical and thermal loads are not the same as those under pure mechanical loads for three of the four loading cases studied. For all cases the cracking loads are increased several fold in comparison with randomly chosen initial designs. The local-global algorithm can generally improve the computational efficiency of the pure multistart method for the considered optimum strength design of composite laminates.

KEYWORDS

Composite material, laminate, in situ strength, optimum design, thermal load, failure criterion, first-ply-failure, transverse cracking

INTRODUCTION

Most matrices of the advanced composite materials are brittle. They are prone to cracking under very low applied stresses. Cracking not only reduces the overall stiffness, but it can lead to disastrous failure of containers due to leakage. Another characteristic of these composite materials is their design tailorability. For this reason, a composite material or structure can be optimized, for given load conditions, in terms of one or a combination of the following properties: weight, stiffness, strength, toughness, maximization for given material properties and volume, inevitably involve complicated fracture mechanics and/or failure analysis of heterogeneous materials. For this reason, the strength maximization of composite materials has not been as widely studied rigorously as the weight minimization, although optimum strength designs of continuous fibre-reinforced composite laminates have been pursued since the early days of these materials [1,2,3,4,5]. Daniel and Ishai [6] give an optimum design example of a composite material structure – the design of a pressure vessel based upon Tsai-Wu failure criterion and first-ply-failure.

The current failure criteria for fibre-reinforced composite laminates use the basic strength parameters that are measured using a unidirectional lamina (e.g. Daniel and Ishai [6]). Thus the configuration of a multidirectional laminate only influences the stress distribution in the multidirectional laminate. However, it is found that the transverse tensile and shear failure stresses of a unidirectional lamina depend upon the laminate configuration and the lamina thickness. This means that the conventional failure criteria need to be modified. They need to include the *in situ* strength parameters [7,8]. Moreover, in measuring the *in situ* transverse strength of unidirectional laminae in laminates, it was found by Flaggs and Kural [7] that the thermal residual stress resulting from the manufacturing process might consist of a large portion of the *in situ* strength (more than half for $[0_2/90_n]_s$ and $[\pm 30/90_n]_s$ for $n = 1, 2, ..., 8$). A composite structure will also experience temperature variations in service. Because of the remarkable difference in the thermal expansion coefficients as well as the stiffnesses of a unidirectional lamina in its longitudinal and transverse directions, the stresses caused by temperature variations may be quite significant in practice. It is obvious that the thermal stresses in a multidirectional laminate are functions of the laminate configuration, that is, functions of the ply angles in the laminate.

Given that most advanced fibre-reinforced composite laminates are prone to cracking and delamination but that the properties of laminates can be tailored, the present authors have attempted to apply fracture mechanics and optimization techniques to the optimum strength design of fibre-reinforced multidirectional composite laminates (Wang and Karihaloo [9,10,11,12,13]. It is well-known that optimization problems of composite laminates are highly nonlinear. The consideration of the *in situ* strength parameters complicates the problem. In the present paper, we shall demonstrate the optimum *in situ* strength design of multidirectional composite laminates subjected to combined mechanical and thermal loads. We shall first introduce the *in situ* strength parameters, and then incorporate them into the formalism of optimization problems. The optimization problems will be solved by a nonlinear mathematical programming technique incoporating the local-global algorithm proposed by Elwakeil and Arora [14].

IN SITU STRENGTH PARAMETERS

It has been observed in tests that the transverse tensile and shear strengths of a continuous fibre-reinforced unidirectional lamina, when situated in a multidirectional laminate, are functions of the thickness of the lamina itself and the ply angles of its neighbouring laminae (e.g. Flaggs and Kural [9]; Chang and Chen [15]). These strengths of a lamina in a laminate are generally larger than those measured using a thick unidirectional laminate. As a consequence, it is recognised that the transverse and in-plane shear strengths of a lamina cannot be regarded as its intrinsic property. Because of this observation, these strengths of a lamina are referred as *in situ* strengths, when the lamina is situated in a multidirectional laminate.

Chang and Lessard [8] proposed two formulas to calculate the *in situ* transverse and shear strengths by fitting experimental data

$$\frac{Y_t}{Y_t^0} = 1 + \frac{A}{N^B} \sin(\Delta\theta) \tag{1}$$

$$\frac{S_c}{S_c^0} = 1 + \frac{C}{N^D} \sin(\Delta\theta) \tag{2}$$

where Y_t^0 and S_c^0 are the transverse tensile strength and in-plane shear strength measured with a thick unidirectional lamina. A, B, C and D are to be determined by experiments. N is the number of unidirectional laminae in a multidirectional laminate. $\Delta\theta$ represent the minimum difference between the ply angle of a lamina and those of its neighbouring plies.

Wang and Karihaloo [10] studied the physics of the phenomenon of *in situ* strengths using fracture mechanics. Based upon the fracture mechanics analysis, they proposed two formulas to calculate the *in situ* strengths

$$\frac{Y_t}{Y_t^0} = 1 + \frac{A}{N^B} f_t(\Delta\theta) \tag{3}$$

$$\frac{S_c}{S_c^0} = 1 + \frac{C}{N^D} f_s(\Delta\theta) \tag{4}$$

Here, the two functions $f_t(\Delta\theta)$ and $f_s(\Delta\theta)$ represent the influence of the neighbouring laminae on the strengths of a lamina. They are given by

$$f_t(\Delta\theta) = \min\left[\frac{\sin^2(\Delta\theta_a)}{1+\sin^2(\Delta\theta_a)}, \frac{\sin^2(\Delta\theta_b)}{1+\sin^2(\Delta\theta_b)}\right] \tag{5}$$

$$f_s(\Delta\theta) = \min\left[\frac{\sin^2(2\Delta\theta_a)}{1+\sin^2(2\Delta\theta_a)}, \frac{\sin^2(2\Delta\theta_b)}{1+\sin^2(2\Delta\theta_b)}\right] \tag{6}$$

The parameters A, B, C and D in eqns. 3-4 are to be determined from experimental results. As these formulae also contain the ply angle influence functions, i.e. $f_t(\Delta\theta)$ and $f_s(\Delta\theta)$, the investigation of the dependence of A, B, C and D on the laminate configuration is very important. Their dependence upon the laminate configuration is discussed by Wang and Karihaloo [13].

In Figure 1, the *in situ* transverse strength predicted by eqns (1) and (3) are compared with the experimental results of Flaggs and Kural [7] for the material T300/934. In fitting the experimental data, different values of A are used in eqns (1) and (3) (1.7 and 3.4, respectively). Chang & Lessard [8] used $A = 1.3$ and $B = 0.8$ previously to fit the experimental data. It is seen that both of the theoretical formulas fit the experimental data reasonably well. The most important conclusion drawn from Figure 1 is that for the material and laminate configurations studied by Flaggs & Kural [7], the parameters A and B appear to be independent of the laminate configuration. They can therefore be treated as material constants. On the other hand, due to lack of experimental data, the dependence of the parameters C and D on the laminate configuration can not be judged. Chang & Lessard [8] found that formula (2) fits the experimental data well for T300/976 cross-ply laminates with $D = 2.0$ and $C = 1.0$ In the sequel, we shall use formula (3), which has a fracture mechanics basis, to calculate the *in situ* shear strength of laminae in multidirectional laminates with $C = 4.0$ and $D = 1.0$.

In most cases, transverse cracking is the first noticeable damage in a laminate. Although the transverse cracks generally do not result in the immediate failure of the whole laminate, they have the potential to induce failure by stress concentration and delamination. In the optimum strength design to follow,

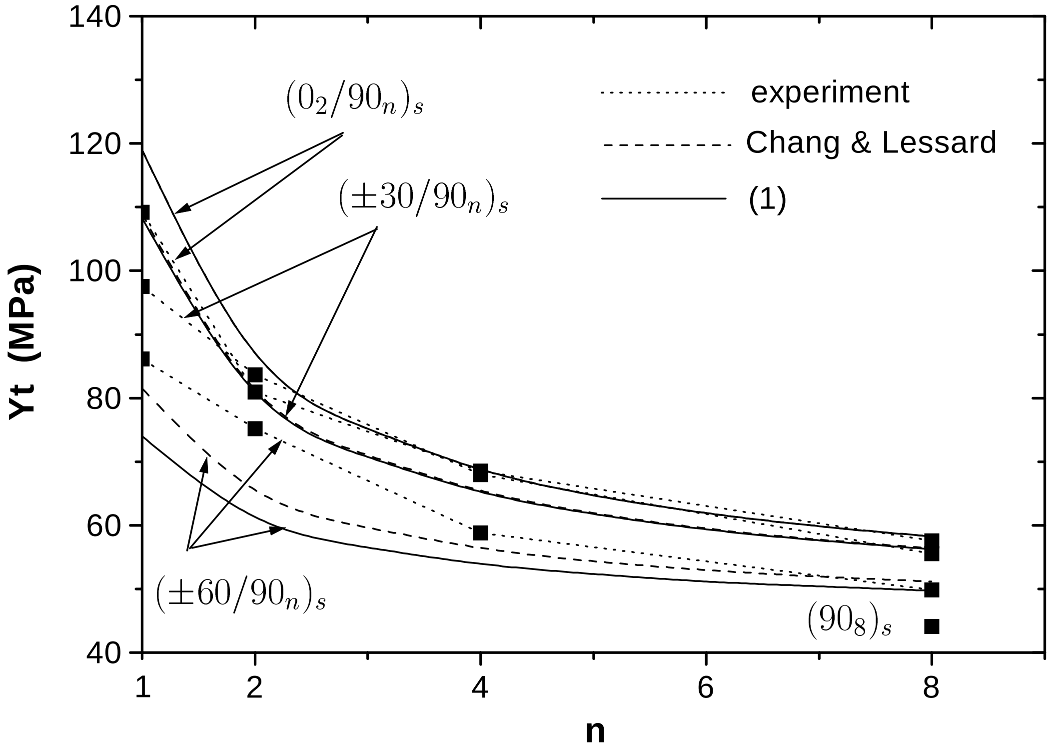

Figure 1: Comparison of theoretical and experimental results of the in situ transverse tensile strength. $A = 3.4$ and $B = 0.8$ are used in formula (1) (after Wang and Karihaloo [13]).

we shall use a transverse tensile failure criterion [8] to judge the transverse failure of a unidirectional lamina in a multidirectional laminate. This criterion, into which the *in situ* strengths are incorporated, is written as

$$q_i^2 \equiv \left(\frac{Y}{Y_t}\right)_i^2 + \left(\frac{S}{S_c}\right)_i^2 \leq 1; \qquad (i = 1, 2, ..., L) \tag{7}$$

where Y and S are the in-plane transverse and shear stresses in the lamina. L is the total number of unidirectional laminae in the laminate.

OPTIMUM DESIGN

For a composite laminate under given in-plane loads, if the ply angles and thicknesses of the constituent laminae are so chosen that the values of q_i^2 for all laminae are reduced, then the loads corresponding to the transverse cracking or failure will be enhanced. This objective is achieved by minimizing the maximum value of q_i^2. Following the procedure in the work by Wang and Karihaloo [11], the optimization problem is formulated as

$$\underset{\theta_i, t_i, \gamma}{\text{Min}} \quad \gamma \tag{8}$$

subject to

$$q_i - \gamma \leq 0 \tag{9}$$

$$-\frac{\pi}{2} \leq \theta_i \leq \frac{\pi}{2} \tag{10}$$

$$\sum_i t_i = h \qquad (i = 1, ..., L) \tag{11}$$

$$\underline{t} \leq t_i \leq \overline{t} \tag{12}$$

The optimization problem (8) is highly nonlinear with multiple local minima making the search for the global minimum difficult. Therefore, the above optimization problem is solved using nonlinear programming (constrained variable metric method) in conjunction with the so-called domain elimination method for global optimization proposed by Elwakeil and Arora [14].

The above optimization procedure was applied to the optimum design of an 8-ply symmetric multidirectional laminate $(\theta_4/.../\theta_1)_s$. The stiffness and strength constants used in the calculation of the *in situ* strengths are adapted from the work by Chang and Lessard [8] on T300/976. The thermal expansion coefficients are taken as those of T300/934 [7], i.e. $\alpha_L = 0.09\,\mu$strain/^{o}C, $\alpha_T = 28.8\,\mu$strain/^{o}C. The thickness of a single ply is assumed to be 0.14 mm. The temperature variation is taken as $\Delta T = -147^o$C, i.e. the temperature drop in the manufacturing process [7]. It can be arbitrary otherwise. Given a mechanical load $[N_1^0, N_2^0, N_6^0]$, the improvement in the design is represented by

$$k = \frac{1}{\max q_i}, \qquad (i = 1, 2, ..., 4) \tag{13}$$

The results of optimization with respect to the failure criterion (7) are shown in Figure 2 and Table 1. Figure 2 shows the changes of the load factor k during the optimization process for four in-plane loading combinations. Table 1 shows the initial, pseudo-randomly chosen guesses to ply angles, their final optimum values, and the optimum load factor k_{max}.

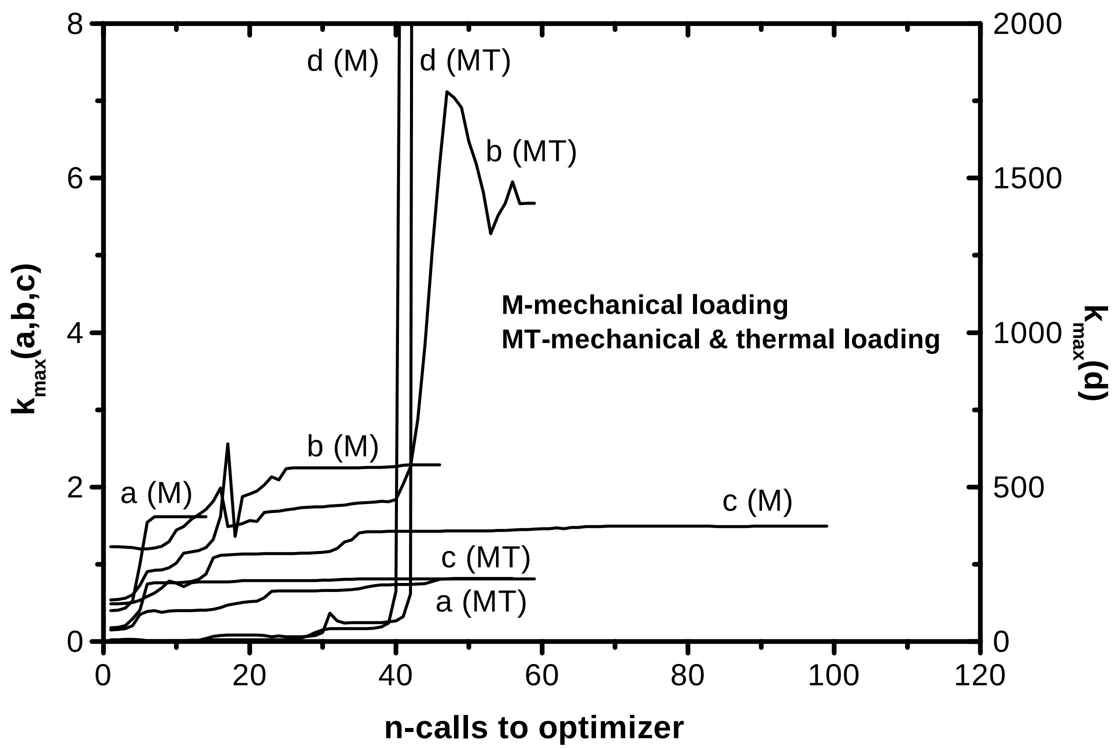

Figure 2: Evolution of load factor k for a symmetric laminate of 4 ply angles for four mechanical loading cases without and with thermal effect: (a) $[N_1^0, N_2^0, N_6^0]^T = [200, 200, 0]^T$ kN/m; (b) $[N_1^0, N_2^0, N_6^0]^T = [200, 0, 200]^T$ kN/m; (c) $[N_1^0, N_2^0, N_6^0]^T = [400, 200, 0]^T$ kN/m; (d) $[N_1^0, N_2^0, N_6^0]^T = [200, 200, 200]^T$ kN/m.

It is seen from Figure 2 that for each of the loading cases (a), (b) and (c), the mechanical load corresponding to the first-ply-failure in the optimally designed laminate is increased several fold compared reached from many initial designs. It is hard to compare the results with and without thermal effect. However, the authors have previously found that for loading cases (a), (b) and (c), the same initial designs lead to different final designs for pure mechanical load and mixed mechanical and thermal load (Wang and Karihaloo [13]). For loading case (d), there is a global optimum design, namely, the configuration where all the ply angles are in the 45^0 direction. When the plies are so arranged, for the transverse criterion (7), the absolute global minimum value of the objective function is identically zero. This minimum value is captured by the optimizer. In all designs, in order to reduce the value of the objective function, the optimizer aims at reducing the transverse and in-plane shear stresses and distributing the stress in the fibre direction of a lamina in the laminate. The optimizer always distributes the stresses according to the strengths in different directions of the anisotropic material. An examination of the elimination procedure described above shows that the efficiency of the local-global algorithm depends upon the number of design variables, the elimination factor and the time taken to find a local minimum.

TABLE 1

SUMMARY OF OPTIMIZED PLY ANGLES IN A SYMMETRIC 8-PLY LAMINATE

	without thermal effect			with thermal effect		
loading case	Initial design $\theta_1, \theta_2, \theta_3, \theta_4$	Final design $\theta_1, \theta_2, \theta_3, \theta_4$	k_{max}	Initial design $\theta_1, \theta_2, \theta_3, \theta_4$	Final design $\theta_1, \theta_2, \theta_3, \theta_4$	k_{max}
a	-43,-52,-47,5	-43,-86,-42,34	1.62	-74,-82,14,67	-36,-61,-42,45	0.8
b	-18,75,76,45	-58,32,86,32	2.3	-12,-34,40,13	32,-61,31,32	5.7
c	-62,60,-82,63	-36,50,-55,12	1.5	-74,-69,59,87	-44,51,-44,19	8.2
d	88,5,16,59	45,45,45,45	∞	47,29,29,49	45,45,45,45	∞

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