ANISOTROPIC MICROCRACK-BASED FRICTIONAL DAMAGE MODEL FOR BRITTLE MATERIALS

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ABSTRACT

An anisotropic constitutive model based on the assumption of linear elastic matrix weakened by microcraks is derived by assuming a tensorial description of the damage and of the unilateral and frictional effects on the displacement jump across the crack faces. Damage and sliding evolutions are obtained by defining proper limit conditions and associated flow rules. This treatment implies a different tensile-compressive response because under triaxial tensile stress states only the damage limit condition is effective, otherwise this condition must be coupled with the frictional sliding one. By considering monotonic loading paths, the constitutive equations provide limit strength domains; limit domains for the case of biaxial stress states are obtained and their dependence on the friction coefficient is shown.

KEYWORDS

Anisotropic damage, microcracked solids, friction, brittle materials, limit strength domain.

INTRODUCTION

Micromechanical models based on a representation of the material meso-structure as a population of growing microcracks embedded in an elastic matrix have been proposed by several Authors (see Krajcinovic [1]) in order to get suitable descriptions of the different response to tensile, compressive and mixed stress states exhibited by brittle materials. A direct approach, that ignores the interactions among microcracks, assumes a vector representation of the anisotropic damage and considers the unilateral frictional sliding between the crack faces; this allows a physical interpretation of the different response to tensile and compressive stress states, load and damage induced anisotropy and energy dissipation at constant damage. Unfortunately, these models involve a high number of internal variables making them useless in computational applications. As a consequence, micromechanically inspired constitutive models characterized by a reduced number of internal variables have been proposed [2,3]. On the other hand, several phenomenological models, involving a limited number of internal variables, have been proposed and applied. These models are based on both scalar-isotropic and tensorial description of damage and take into account the unilateral effect of the crack opening mechanisms by means of the concept of positive and negative projections of stress and strain tensors [4-7].

An attempt to derive a simplified damage model from a complete microcrack one [8] has been carried out by Brencich and Gambarotta [9] by assuming isotropic damage variables as a measure of the average crack size. In this model the opening/sliding effects in the microcracks were considered by two second

order tensors related to the overall normal and tangential traction on the crack faces. Moreover, the evolution equations of the internal variables were deduced by two limit conditions related to damage propagation and frictional sliding. Even if this model provides good results in terms of stress-strain response and limit strength domains, the assumption of isotropic damage implies several validity limits for the model, with particular reference to non proportional loading paths. An extension of the model to include an anisotropic damage description based on a tensorial representation was proposed in [10], but in this approach frictional effects were disregarded.
The model here presented is developed in order to describe both the anisotropic damage and the effects of frictional unilateral conditions on the displacement jumps across the crack faces. The constitutive equations are formulated in terms of a damage tensor evolving from the natural isotropic state and of tensors standing for the overall effects of normal and frictional contact traction on the microcrack faces. A further simplifying hypothesis on the representation of these tensors allows compact constitutive equations and the definition of the tensor of damage energy release rate in terms of the stress, normal and frictional tensors. Damage and sliding evolutions are obtained by coupling a damage and a frictional criterion that provide different evolution modes. Finally, the constitutive equations are applied in case of monotonic loading paths and limit strength domains for biaxial stress states, depending on the frictional parameter, are deduced; they seem to fit both the corresponding ones by the isotropic model [9] and the experimental results.

CONSTITUTIVE MODEL BASED ON THE CONCEPT OF DAMAGE PLANES

The hypothesis of representing brittle materials as an elastic solid containing a population of non-interacting microcracks, isotropically distributed at the natural state, allows the mean strain **E** to be expressed as the sum $\mathbf{E}=\mathbb{K}\mathbf{T}+\mathbf{E}_n+\mathbf{E}_t$ of the mean strain in the elastic matrix and the contributions $\mathbf{E}_n$ and $\mathbf{E}_t$ due, respectively, to normal and tangential displacement discontinuities across the crack faces, being $\mathbb{K}$ the fourth-order elastic isotropic compliance tensor and **T** the mean stress tensor. According to [8], these contributions to the mean strain may be expressed as follows:

$$\mathbf{E}_n=\frac{c_n}{2\pi}\int_\Omega \alpha_n^3(\sigma_n-p_n)\,\mathbf{n}\otimes\mathbf{n}d\Omega \quad , \qquad \mathbf{E}_t=\frac{c_t}{2\pi}\int_\Omega \alpha_n^3 sym[(\boldsymbol{\tau}_n-\mathbf{f}_n)\otimes\mathbf{n}]\,d\Omega \quad , \tag{1}$$

where **n** is the unit vector normal to the crack plane on which the resolved stresses $\sigma_n=\mathbf{n}\bullet\mathbf{T}\mathbf{n}$ and $\boldsymbol{\tau}_n=(\mathbf{I}-\mathbf{n}\otimes\mathbf{n})\mathbf{T}\mathbf{n}$ act; c_n and c_t are the normal and tangential compliance parameters of the set of the plane crack systems not depending on **n** because the hypothesis of isotropy at the natural state; dΩ is the infinitesimal solid angle representing the neighborhood of the unit vector **n**; Ω the unit hemisphere of all the orientations; p_n and $\mathbf{f}_n$ are the normal and tangential tractions acting on the crack faces; $\alpha_n(\geq 1)$ is the damage variable representing the ratio between the actual average size of **n**-oriented cracks and the corresponding one at the reference state. In this approach, α_n and $\mathbf{f}_n$ play the role of internal variables, while the hypothesis of ignoring the sliding induced dilation allows the normal traction to depend on the normal stress $p_n=-\langle-\sigma_n\rangle$ by the MacAuley operator.
To get a simplified model based on a reduced number of internal variables some simplifying hypotheses are put forward. Firstly, the damage variable $a_n\ (=\alpha_n^3)$ related to each microplane is introduced and assumed to depend on **n** in the form $a_n=\mathbf{n}\bullet\mathbf{A}\mathbf{n}$, being **A** ($a_{ij}$) a symmetric positive defined tensor, having $\mathbf{A}_0=\mathbf{I}$ at the natural state. Integration of equations (1) on Ω implies:

$$\mathbf{E}_n=c_n\left(\mathbb{H}_n[\mathbf{A}]\mathbf{T}-\mathbb{P}^*\mathbf{A}\right) \quad , \qquad \mathbf{E}_t=c_t\left(\mathbb{H}_t[\mathbf{A}]\mathbf{T}-\mathbb{F}^*\mathbf{A}\right) \quad , \tag{2}$$

having introduced the positive defined fourth-order symmetric tensors:

$$\mathbb{H}_n[\mathbf{A}]=\mathfrak{H}_n\mathbf{A}=\frac{1}{105}\left\{\frac{7}{3}\,tr\mathbf{A}\ \left(\mathbf{I}\otimes\mathbf{I}+2\mathbf{I}\boxdot\mathbf{I}\right)+2\left(\mathbf{I}\otimes\mathbf{A}'+\mathbf{A}'\otimes\mathbf{I}\right)+4\left(\mathbf{A}'\boxdot\mathbf{I}+\mathbf{I}\boxdot\mathbf{A}'\right)\right\} \quad , \tag{3}$$

$$\mathbb{H}_t[\mathbf{A}] = \mathfrak{H}_t\mathbf{A} = \frac{1}{105}\left\{7\, tr\mathbf{A}\,\left(\mathbf{I}\boxdot\mathbf{I} - \frac{1}{3}\mathbf{I}\otimes\mathbf{I}\right) - 2\left(\mathbf{I}\otimes\mathbf{A}' + \mathbf{A}'\otimes\mathbf{I}\right) + 3\left(\mathbf{A}'\boxdot\mathbf{I} + \mathbf{I}\boxdot\mathbf{A}'\right)\right\} \quad , \tag{4}$$

being $\mathbf{A}' = \mathbf{A} - \frac{1}{3}(tr\mathbf{A})\mathbf{I}$, $(\mathbf{B}\otimes\mathbf{C})\mathbf{X} = \mathbf{B}(\mathbf{C}\bullet\mathbf{X})$ and $(\mathbf{B}\boxdot\mathbf{C})\mathbf{X} = \mathbf{B}(\mathbf{X}+\mathbf{X}^{\mathrm{T}})\mathbf{C}^{\mathrm{T}}/2$. The linear dependence of the previously defined tensors on the damage tensor **A** is expressed through the six order tensors $\mathfrak{H}_n$ and $\mathfrak{H}_t$ [1]:

$$\mathfrak{H}_{n,ijrshk} = \frac{1}{7}I_{ijrshk} \quad , \quad \mathfrak{H}_{t,ijrshk} = \frac{1}{20}\left(\delta_{ih}I_{kjrs} + \delta_{ik}I_{hjrs} + \delta_{jh}I_{kirs} + \delta_{jk}I_{hirs}\right) - \frac{1}{7}I_{ijrshk} \quad . \tag{5}$$

The definitions (3) and (4) of the tensors $\mathbb{H}_n[\mathbf{A}]$ and $\mathbb{H}_t[\mathbf{A}]$ are particular cases of the general representation given in [11] because of the assumption of independence of the parameters c_n and c_t on **n** and of the damage description based on a second order tensor. Moreover, the fourth order tensors representative of the overall normal and tangential tractions on the crack faces are defined as follows:

$$\mathbb{P}^* = \frac{1}{2\pi}\int_\Omega p_n\mathbf{n}\otimes\mathbf{n}\otimes\mathbf{n}\otimes\mathbf{n}\; d\Omega \quad , \quad \mathbb{F}^* = \frac{1}{4\pi}\int_\Omega\left(\mathbf{f}_n\otimes\mathbf{n}\otimes\mathbf{n}\otimes\mathbf{n} + \mathbf{n}\otimes\mathbf{f}_n\otimes\mathbf{n}\otimes\mathbf{n}\right)\, d\Omega \quad ; \tag{6}$$

while the tensor $\mathbb{P}^*$ directly depends on **T**, the tensor $\mathbb{F}^*$ assumes the role of further internal variable of the constitutive model.
To limit the complexity of the formulation, it is assumed that the frictional contact traction may be expressed in terms of a symmetric traceless second order tensor $\mathbf{F}^*$ as follows $\mathbf{f}_n = (\mathbf{I}-\mathbf{n}\otimes\mathbf{n})\mathbf{F}^*\mathbf{n}$. A further simplifying assumption is based on the observation that when $\sigma_n \geq 0\ \forall\mathbf{n}$, then $\mathbb{P}^* = \mathbb{O}$ and when $\sigma_n \leq 0\ \forall\mathbf{n}$, then $\mathbb{P}^* = \mathfrak{H}_n\mathbf{T}$. Therefore it is assumed that tensor $\mathbb{P}^*$ may be expressed in the form $\mathbb{P}^* = \mathfrak{H}_n\mathbf{P}^*$, being $\mathbf{P}^*$ a symmetric tensor satisfying the conditions: $\mathbf{P}^* = \mathbf{0}$ if $\sigma_n \geq 0\ \forall\mathbf{n}$ and $\mathbf{P}^* = \mathbf{T}$ if $\sigma_n \leq 0\ \forall\mathbf{n}$. A possible choice for $\mathbf{P}^*$ suggested in [9] is:

$$\mathbf{P}^* = \frac{5}{2}\mathbf{P} - \frac{1}{2}(tr\mathbf{P})\mathbf{I} \quad , \quad \mathbf{P} = \frac{3}{2\pi}\int_\Omega p\mathbf{n}\otimes\mathbf{n}\; d\Omega \quad . \tag{7}$$

Moreover, it may be easily observed by equation (4) that $\mathbb{H}_t[\mathbf{A}]\mathbf{T} = \mathbb{H}_t[\mathbf{A}]\mathbf{T}'$, being $\mathbf{T}' = \mathbf{T} - \frac{1}{3}(tr\mathbf{T})\mathbf{I}$ the stress deviator. On these hypotheses the constitutive equations are derived:

$$\mathbf{E} = \mathbb{K}\mathbf{T} + \mathbf{E}_n + \mathbf{E}_t \quad , \quad \mathbf{E}_n = c_n\mathbb{H}_n[\mathbf{A}]\left(\mathbf{T}-\mathbf{P}^*\right) \quad , \quad \mathbf{E}_t = c_t\mathbb{H}_t[\mathbf{A}]\left(\mathbf{T}'-\mathbf{F}^*\right) \quad , \tag{8}$$

which depend on the damage **A** and friction $\mathbf{F}^*$ internal variable to be evaluated by means of proper evolution equations. Within this model the thermodynamic force associated to the damage variable is derived by the damage energy release rate and is described by the second order symmetric and positive defined tensor **Y** having components:

$$Y_{rs} = \frac{1}{2}c_n\ \mathfrak{H}_{n,hkrsij}\left(T_{ij}-P^*_{ij}\right)\left(T_{hk}-P^*_{hk}\right) + \frac{1}{2}c_t\ \mathfrak{H}_{t,hkrsij}\left(T'_{ij}-F^*_{ij}\right)\left(T'_{hk}-F^*_{hk}\right) \quad , \tag{9}$$

besides the variable associated to the friction tensor is the sliding strain tensor $\mathbf{E}_t$. Once given the applied stress **T,** the damage tensor **A** and the sliding strain tensor $\mathbf{E}_t$, from equation (8.3) the tensor $\mathbf{F}^*$ is obtained.

EVOLUTION EQUATIONS AND LIMIT STRENGTH DOMAINS

The evolution equations are obtained by assuming both a damage and a frictional sliding criterion. The first one is based on the following assumption:

$$\phi_d\left(\mathbf{Y},\mathbf{A}\right) = \left|\mathbf{Y}\right| - R(\mathbf{A}) \leq 0 \quad , \tag{10}$$

being $R(\mathbf{A})$ the overall damage toughness function to be properly chosen. When the limit condition $\phi_d = 0$ is attained then damage evolution equation is assumed:

$$\dot{\mathbf{A}} = \mathbf{V}_d \ \dot{d} \quad , \quad \mathbf{V}_d = \frac{\partial \phi_d}{\partial \mathbf{Y}} = \mathbf{Y}/|\mathbf{Y}| \quad , \quad \dot{d} \geq 0 \quad , \tag{11}$$

that guarantees a positive energy dissipation $w_d = |\mathbf{Y}|\dot{d} \geq 0$. Moreover, the damage toughness function $R(\bullet)$ is assumed depending on the scalar variable D=$1/3|\mathbf{A}|^2$ (D$\in[1,\infty)$), providing a measure of the overall damage (at the natural state D=1 and R(1)=0). The progressive strength deterioration exhibited by brittle materials up to the limit strength and the following strain softening phase may be modeled by the toughness function, that must be chosen to attain a maximum value R_c followed by a decreasing phase for increasing damage as long as it attains vanishing values (see for instance [12]).

When compressive stress acts on some crack plane, the limit damage condition must be coupled with the frictional sliding condition. In this case tensor $\mathbf{P}^*$ does not vanish and 1/3tr$\mathbf{P}^*$<0 represents the average compressive stress on the set of compressed planes. As a result, the crack sliding is partially restrained by the frictional traction **f**. In the frame of the present model, the limit condition should be formulated in terms of the global friction tensor $\mathbf{F}^{*\prime}$ and the average compressive hydrostatic pressure 1/3 tr$\mathbf{P}^*$. A simplifying assumption could concern an overall frictional limit condition in a form analogous to the Drucker-Prager criterion [10]:

$$\phi_s = |\mathbf{F}^*| + \mu \ tr(\mathbf{P}^*) \leq 0 \quad , \tag{12}$$

where μ plays the role of friction coefficient. In this case, when the limit condition is attained $\phi_s = 0$ the sliding rule is assumed:

$$\dot{\mathbf{E}}_t = \mathbf{V}_s \ \dot{\lambda} \quad , \quad \mathbf{V}_s = \frac{\partial \phi_s}{\partial \mathbf{F}^*} = \frac{\mathbf{F}^*}{|\mathbf{F}^*|} \quad , \quad \dot{\lambda} \geq 0 \ . \tag{13}$$

On these hypotheses it is possible to derive the evolution equations for the damage tensor and the friction tensor following two different possibilities depending on whether overall compression or tension is active.

In the case of tensile traction on any crack planes ($\sigma_n \geq 0$), it follows $tr\mathbf{P}^* = tr\mathbf{P} = 0$ and so also the friction tensor is vanishing $\mathbf{F}^* = \mathbf{0}$. Only the damage limit condition (10) must be considered; when the limit condition is attained $\phi_d = 0$, then the damage rate is obtained by solving the LCP:

$$\dot{\phi}_d = c_n \mathbf{T} \bullet \mathbb{H}_n[\mathbf{V}_d]\dot{\mathbf{T}} + c_t \mathbf{T}' \mathbb{H}_t[\mathbf{V}_d]\dot{\mathbf{T}}' - \tfrac{2}{3} R' \mathbf{V}_d \bullet \mathbf{A} \dot{d} \leq 0 \quad , \quad \dot{d} \geq 0 \quad , \quad \dot{\phi}_d \dot{d} = 0 , \tag{14}$$

that has a single stable solution if $R' = dR/dD > 0$. The condition $R' = 0$ define a limit state for the damage process corresponding to an upper limit for the stress rate; since it may be observed that this condition does not depend on the load history, it is here assumed as a limit strength condition and is expressed in the simple form $|\mathbf{Y}| = R_c$.

If a compressive traction is acting on some planes, it follows $tr\mathbf{P}^* < 0$ and the strain rate $\dot{\mathbf{E}}_t$ associated to the crack frictional sliding must be obtained by equation (13). In this case from equations (8), (11) and (13) one obtains:

$$\dot{\mathbf{T}}' - \dot{\mathbf{F}}^* = \tfrac{1}{c_t} \mathbb{H}_t[\mathbf{A}]^{-1} \left\{ \mathbf{V}_s \dot{\lambda} - c_t \mathbb{H}_t[\mathbf{A}](\mathbf{T}' - \mathbf{F}^*)\dot{d} \right\} \quad , \tag{15}$$

that allow us to formulate the evolution equations according to four different possible initial states:

(a) *Elastic state*: $\phi_s < 0, \ \phi_d < 0$. In this case, being $\dot{\mathbf{E}}_t = \mathbf{0}$ it follows $\dot{\mathbf{F}}^* = \dot{\mathbf{T}}'$.

(b) *Friction limit state with stable damage*: $\phi_s = 0, \ \phi_d < 0$. The sliding rate $\dot{\lambda} \geq 0$ is obtained by solving the LCP:

$$\dot{\phi}_s = b_{ss}\ \dot{\lambda}\ + \dot{t}_s \le 0 \quad , \quad \dot{\lambda} \ge 0 \quad , \quad \dot{\phi}_s\,\dot{\lambda} = 0 \quad , \tag{16}$$

where

$$b_{ss} = \mathbf{V}_s \bullet \mathbb{H}[\mathbf{A}]^{-1}\mathbf{V}_s \quad , \quad \dot{t}_s = \mathbf{V}_s \bullet \dot{\mathbf{T}}' + \mu\ tr\,\dot{\mathbf{P}}^* \quad , \quad \dot{\mathbf{P}} = \frac{3}{2\pi}\int_\Omega \dot{p}\,\mathbf{n}\otimes\mathbf{n}\ d\Omega \quad , \tag{17}$$

that always provides a single solution. While the sliding strain rate is given by equation (13), the opening strain rate is not vanishing only if $\mathbf{P}^* \ne \mathbf{T}$ and in this case $\dot{\mathbf{E}}_t = c_n \mathbb{H}_n[\mathbf{A}](\dot{\mathbf{T}} - \dot{\mathbf{P}}^*)$.

(c) *Damage limit state with no sliding*: $\phi_d = 0,\ \phi_s < 0$. Since this implies $\dot{\mathbf{E}}_t = \mathbf{0}$, this possibility is meaningful only when some plane are compressed, that is $\mathbf{P}^* \ne \mathbf{T}$. In this case, remembering equation (15), the LCP (10) assumes the form:

$$\dot{\phi}_d = b_{dd}\,\dot{d} + \dot{t}_d \le 0 \quad , \quad \dot{d} \ge 0 \quad , \quad \dot{\phi}_d\,\dot{d} = 0 \quad , \tag{18}$$

where

$$b_{dd} = -\frac{1}{c_n}\,\tilde{\mathbf{E}}_t \bullet \mathbb{H}_t[\mathbf{A}]^{-1}\tilde{\mathbf{E}}_t - \frac{2}{3}R'\mathbf{V}_d \bullet \mathbf{A} \quad , \quad \dot{t}_d = c_n(\dot{\mathbf{T}} - \dot{\mathbf{P}}^*) \bullet \mathbb{H}_n[\mathbf{V}_d](\mathbf{T} - \mathbf{P}^*) \quad , \tag{19}$$

having defined $\tilde{\mathbf{E}}_t = \mathbb{H}_t[\mathbf{V}_d]\mathbb{H}_t[\mathbf{A}]^{-1}\mathbf{E}_t$. In the case $\mathbf{P}^* = \mathbf{T}$ no damage evolution is allowed. Moreover, the solution of problem (18) is unique until $b_{dd} < 0$ and the condition $b_{dd} = 0$, that depends on the load path, characterizes a limit for the stress rate.

(d) *Damage and friction limit state*: $\phi_d = 0,\ \phi_s = 0$. The evolution of the internal variables is obtained by solving the coupled LCP:

$$\begin{aligned} \dot{\phi}_s &= b_{ss}\ \dot{\lambda}\ + b_{sd}\,\dot{d}\ + \dot{t}_s \le 0 \ , \quad \dot{\lambda} \ge 0 \ , \quad \dot{\phi}_s\,\dot{\lambda} = 0 \ , \\ \dot{\phi}_d &= b_{ds}\,\dot{\lambda}\ + b_{dd}\,\dot{d}\ + \dot{t}_d \le 0 \ , \quad \dot{d} \ge 0 \ , \quad \dot{\phi}_d\,\dot{d} = 0 \ , \end{aligned} \tag{20}$$

where

$$b_{sd} = b_{ds} = \frac{1}{c_t}\,\mathbf{V}_s \bullet \mathbb{H}_t[\mathbf{A}]^{-1}\tilde{\mathbf{E}}_t \quad . \tag{21}$$

In this case the solution is unique for load control paths until:

$$\det\mathbf{B} = \frac{1}{c_t^2}\left[(\mathbf{V}_s \bullet \mathbb{H}_t[\mathbf{A}]^{-1}\mathbf{V}_s)(\tilde{\mathbf{E}}_t \bullet \mathbb{H}_t[\mathbf{A}]^{-1}\tilde{\mathbf{E}}_t) - (\mathbf{V}_s \bullet \mathbb{H}_t[\mathbf{A}]^{-1}\tilde{\mathbf{E}}_t)^2\right] + \frac{2}{3c_t}R'\mathbf{V}_d \bullet \mathbf{A} > 0 \,, \tag{22}$$

a condition that is depending on the load history.

By substituting the sliding tensor rate $\dot{\mathbf{E}}_t$ and the damage tensor rate $\dot{\mathbf{A}}$ in the incremental form of equations (8) one obtains the complete constitutive equation to be applied in case of general loading histories. In the particular case of proportional load histories it is possible to obtain the limit strength condition defined as the maximum value of the stress intensity allowable by the model. In fact, when tensile stresses are acting on all crack planes ($tr\mathbf{P}^* = 0$) the limit stress state is path independent and corresponds to the condition $|\mathbf{Y}| = R_c$. If some or every crack planes are compressed ($tr\mathbf{P}^* < 0$), the limit strength domain is obtained by simultaneously imposing equations $\phi_d = 0,\ \dot{\phi}_d = 0$ and $\det\mathbf{B} = 0$, that corresponds to assume $\mathbf{F}^* = -\mu(tr\mathbf{P})\mathbf{T}'/|\mathbf{T}'|$; this implies the limit strength criterion to be expressed in the general form $|\mathbf{Y}| = R_c$, being the internal variable in the definition (9) directly dependent on the stress tensor. It follows that the limit strength domain only depends on three material parameters: the ratio $\rho = c_t/c_n$, the friction coefficient μ and the uniaxial tensile strength σ_T. In order to show some features of the model, limit domains referred to biaxial stress states are shown in the diagrams of figure 1a.
Finally, a further simplifications can be obtained by assuming the scalar description of damage **A**=*a***I**, that provides the isotropic damage constitutive model proposed by Gambarotta and Brencich [9].

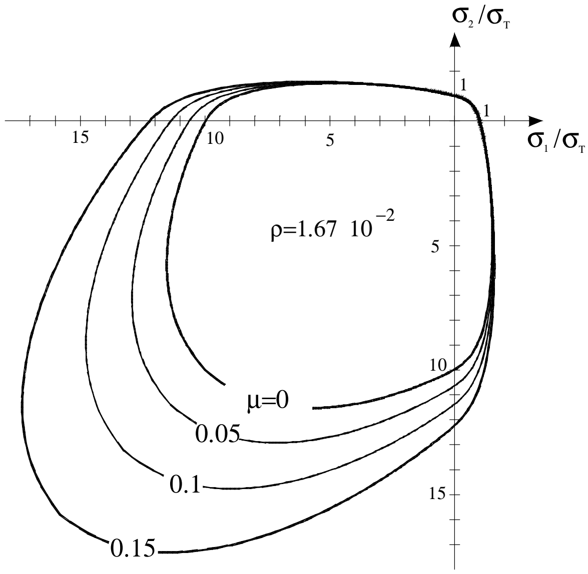

Figure 1: Biaxial limit strength domain for varying friction coefficient.

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