

# **ANALYSIS OF MICROMECHANICAL TENSILE DAMAGE PROCESS FOR BRITTLE CERAMIC MATERIALS USING FEM**

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## **ABSTRACT**

The computer simulation of physical processes is one of the current directions of research development of body structures behaviour. The method, which was worked out (experimentally confirmed), gives an opportunity to analyse the same problems numerically, without expensive experiment. Recently the finite element method (FEM) has been used to investigate the above problems [1÷4]. The authors of this paper consider crack-face bridging, the stress field and the stress intensity factor ahead of the crack tip in the single-edge notched bend (SENB) ceramics specimen using FEM. In the FEM calculations the cohesive forces caused by the bridging effect were modelled using non-linear elastic springs. To solve this non-linear problem the incremental procedure was used. The results of this analysis are the stresses and displacements on the bridging crack and external load, which allow to compute the stress intensity factor and fracture energy.

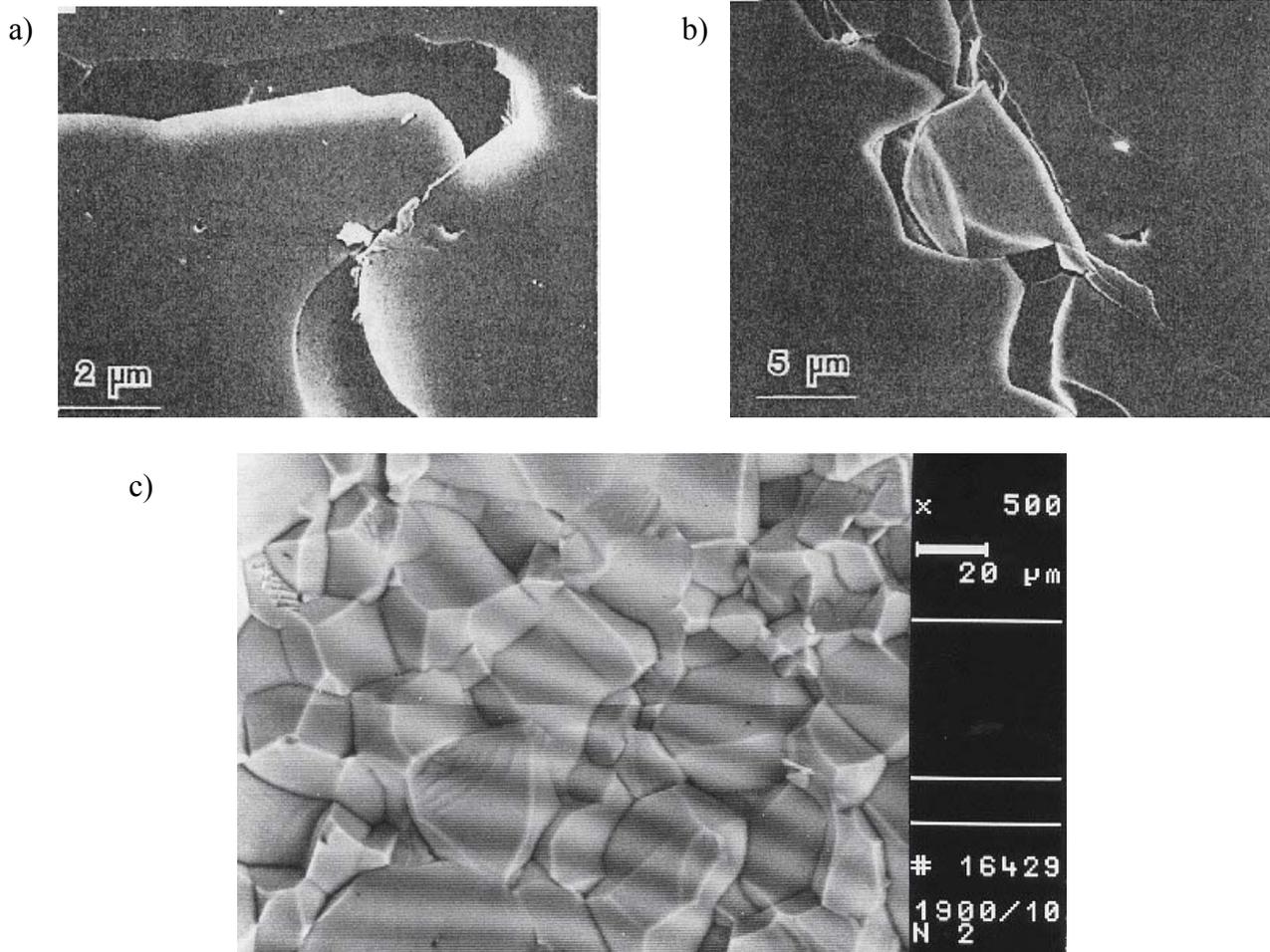
## **KEYWORDS**

Alumina ceramic, crack growth process, bridging effect, FEM simulation

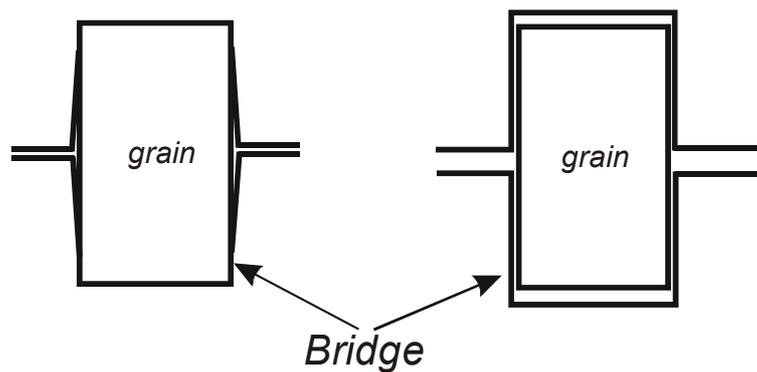
## **INTRODUCTION**

Ceramic materials exhibit several favourable properties (chemical inertness, high temperature capability, hardness stiffness and compressive strength, which make them products to be potentially employed in many different engineering applications. During the crack propagation in alumina ceramics a crack-border interaction zone develops directly behind the crack tip. In this zone, the two crack surfaces are not completely separated, and therefore, crack-surface interactions occur. The origin of this behaviour is well documented by authors [5,6] from the experimental point of view (see Figure 1): the microscopic observations of the crack path showed the existence of crack surface interactions and crack bridging due to serrated grains and unbroken ligaments. There is a compelling evidence of crack surface interaction by frictional tractions around bridging grains, which produces a strong toughening effect. To describe this kind of behaviour (two steps mechanism, see Figure 2) it is possible to use the bilinear softening curve. The curve responds to different mechanisms of energy

dissipation and takes into account the bridging effect of the fibres just behind the crack tip and refers to decohesion and pull out between fibres and matrix.



**Figure 1:** a) crack surface interactions process [5], b) crack bridging due to serrated grains [5] and c) view of the surface after fracture in  $\text{Al}_2\text{O}_3$  ceramic



**Figure 2:** Two steps bridging process in ceramics:  
a) break of a link between grain and matrix, b) pull out between grain and matrix

## THEORETICAL FUNDAMENTALS

The theoretical analysis of crack growth process in alumina ceramic is described by authors [7÷11]. The algorithm of the analysis looks as follows. The existing interaction zone behind crack tip has capability to transmit stresses, which are called bridging stresses. The stresses transmitted due to the crack surface interactions are denoted by  $\sigma_{br}(x)$ , and they are superimposed by the external applied stresses resulting in:

$$\sigma(x) = \sigma_{\text{appl}}(x) - \sigma_{\text{br}}(x) \quad (1)$$

Authors of papers [7,10] indicated, that the stress intensity factor  $K_I$  for SENB loaded by stress distribution  $\sigma(x)$  can be calculated using the fracture mechanical weight function method:

$$K_I = \int_0^a h\left(\frac{x}{a}, \frac{a}{W}\right) \sigma(x) dx \quad (2)$$

where  $h$  is the weight function depending on the geometry of the crack-component configuration. Integration of the equation proposed by Rice [12], which is presented as follows

$$h = \frac{H}{K_I} \frac{\partial \delta}{\partial a} \quad (3)$$

yields the crack opening displacements (COD)  $\delta$  caused by stress  $\sigma$ . A detailed description was gathered from the handbook [13]. Taking into consideration the total stress (Eqn. 1) acting at the crack tip the obtained equation is as follows:

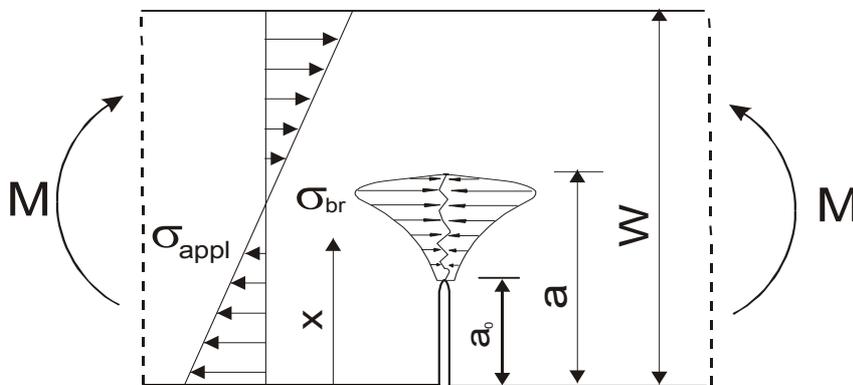
$$\delta = \frac{1}{H} \int_0^a \int_{\max(x, x')}^a h\left(\frac{a'}{W}, \frac{x}{a'}\right) h\left(\frac{a'}{W}, \frac{x'}{a'}\right) (\sigma_{\text{appl}} - \sigma_{\text{br}}) da' dx' = \delta_{00} \exp^{-1}(\sigma_{\text{br}} / \sigma_0) \quad (4)$$

where  $H = E$  (Young's modulus) for plane stress and  $H = E/(1-\nu^2)$  ( $\nu$  -Poisson's ratio) for plane strain,  $x$  is the coordinate of the displacement computed,  $x'$  is the location where the stress  $\sigma$  acts,  $\delta_{00}$  and  $\sigma_0$  are the fracture parameters [7,10]. The solution of the integral Eqn. 4 provides the distribution of the bridging stresses as the function of the stresses applied, and allows obtaining the stress intensity factor basing on the next equation:

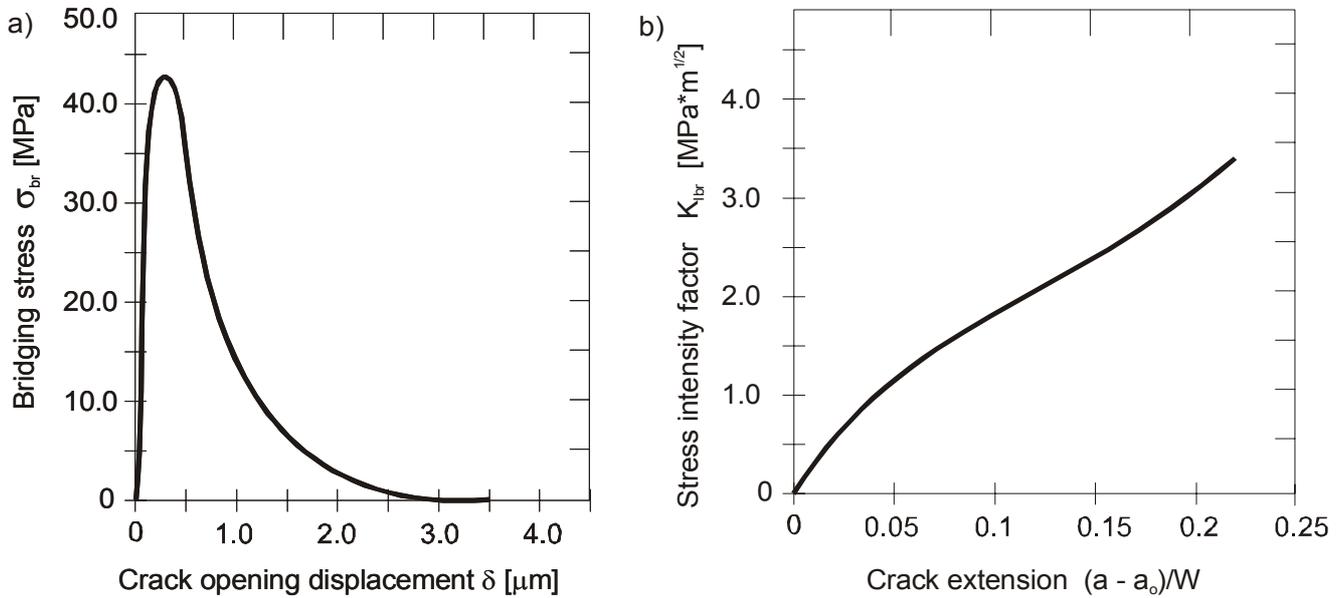
$$K_{I \text{ tip}} = K_{I \text{ appl}} - K_{I \text{ br}} \quad (5)$$

The evaluation of Eqn. 4 using successive approximation requires plenty of computer time. Authors of report [10] have prepared special strategies to limit the number of the computation.

The following procedure was applied to analyse the stress state and displacement of crack surface (COD) behind the crack tip in the single-edge notched bend (SENB) alumina ceramic specimen. The average grain size in analysed ceramic was 13  $\mu\text{m}$ . The performed calculations provided the distribution of bridging stress as a function of crack opening displacement, see Figure 4a. The bridging crack intensity factor  $K_{I \text{ br}}$  was obtained, see Figure 4b.



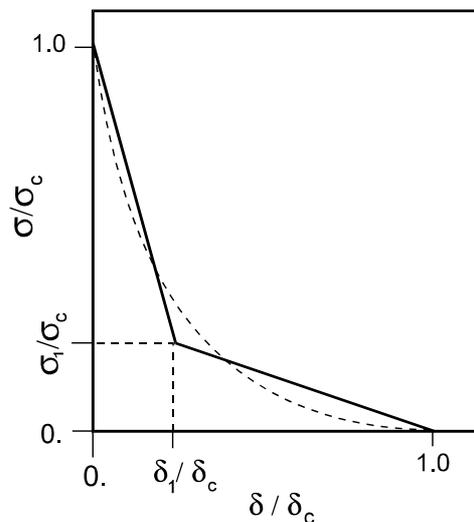
**Figure 3:** Crack starting from a notch in a bending test [7,10]



**Figure 4:** a) Distribution of bridging stress and b) bridging stress intensity factor as a function of crack extension

## NUMERICAL METHOD

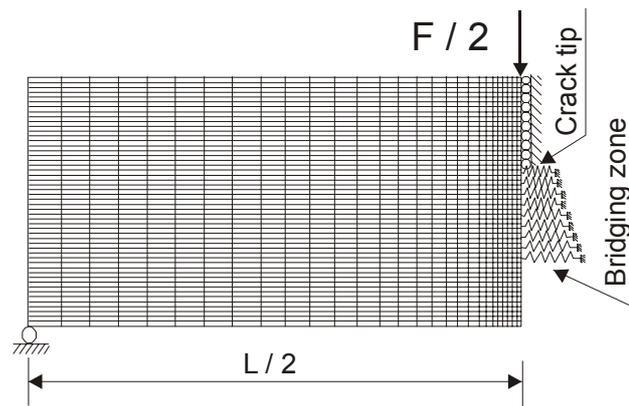
The Finite Element Method was applied to crack growth process simulation in alumina ceramic. The structure's deformation under consideration is localised within the crack. The relationship between the crack opening displacement (COD)  $\delta$  and the transferred stresses  $\sigma$  by bridging effect was described using the softening curve (see Figure 5). The authors of papers [9,11] indicated, that the curve must always fit some conditions. Firstly, when  $\delta = 0$ , the stresses transferred through the crack have to be equal to  $\sigma_c$ . Secondly, there is a critical value of  $\delta$ , named  $\delta_c$ , defined as follows if  $\delta \geq \delta_c$  then  $\sigma = 0$ . Finally, the area under the softening curve is the fracture energy  $G_F$ , that is the energy needed by the unit area to create a new separated surface. The parameter  $\sigma_c$  was obtained from the three point bending test for the bars without the notch, and performed from the alumina ceramics being analysed. The value of the parameter  $\sigma_c$  is given in the Table 1. The critical value of the crack opening displacement  $\delta_c$  was measured by the authors [9] for different grain size polycrystalline alumina. The authors have found out that it is equal to one quarter of the mean a grain size.



**Figure 5:** Strain softening curve (— bilinear, ---- exponential) [11]

The behaviour of the material behind the crack tip (influence between the newly separated surfaces due to the existing cohesive forces) was modelled using non-linear elastic springs. The profile of the

curve (relation force-displacement) was obtained basing on formula presented in papers [4,11]. Numerical model of the three-point bending bar was performed from 8-nodes plain stress elements. Due to the body symmetry (see Figure 6) one-half of the bar was analysed only.



**Figure 6:** Finite element model with crack-face bridging during modelling crack growth process

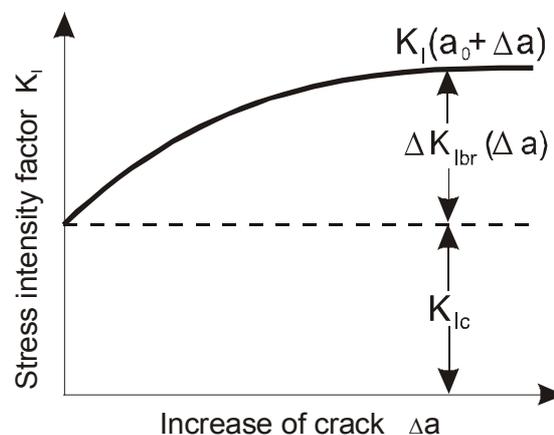
TABLE 1  
SOFTENING CURVE PARAMETERS FOR ALUMINA CERAMICS

Mean grain size ( $\mu\text{m}$ )	$\sigma_c$ (MPa)	$\delta_c$ ( $\mu\text{m}$ )	$\sigma_1$ (MPa)	$\delta_1$ ( $\mu\text{m}$ )
13	200	3.25	18.06	0.4

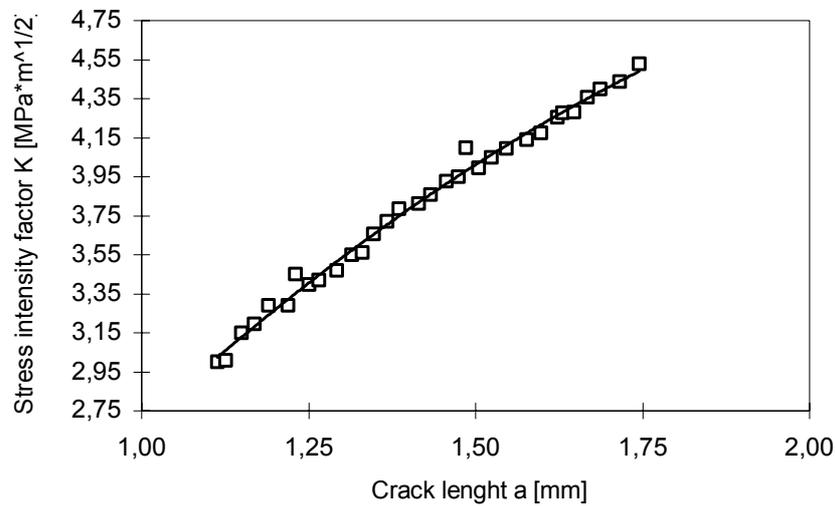
A finite element model for crack bridging systems has been proposed and performed in this work on the basis of the results from single-edge notch bend experiment. Existing in the process of the crack propagation bridging effect was modelled using non-linear springs elements (see Figure 6). It was used the incremental procedure to solving this problem. The results of this analysis are stresses and displacements on the bridging crack and external load. A numerical simulation of crack growth shows that the stress bridging zone moves along the crack, behind the crack tip.

Basing on obtained results the stress intensity factor was calculated and than R-curve was created. Typical profile of R-curve with existing toughening effect is presented on Figure 7. Bridging zone causes the increase of stress intensity factor  $\Delta K_{lbr}$ .

Initial value of stress intensity factor for analysed ceramics equals  $K_I \approx 2,95 \text{ MPa}\times\text{m}^{1/2}$  (see Figure 8). From the performed calculations for alumina ceramics with different grain sizes it was apparent, that the value of  $K_I$  is strongly depended on grain size in analysed material. The values obtained from numerical analysis are in a good agreement with results, which were described by the authors [1,7,10].



**Figure 7:** A profile of R-curve for materials with existing toughening effect (for instance bridging zone behind crack tip) [1]



**Figure 8:** *R-curve* for alumina ceramics

## SUBMISSION

The method of numerical analysis with the use of FEM proposed in this paper allows for reduction of analytical computation, which was presented in this paper. From the macroscopic point of view we have possibility take into consideration the effect, which appeared in the microstructure of alumina ceramics during the crack growth (bridging effect). This numerical method permits to analyse much more complex structures with real loading. There is also a possibility to obtain *R-curve* for alumina ceramic (see Figure 8).

### *Acknowledgements*

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