Analysis of Fracture Mechanics and Fatigue Behavior for EC(T) Specimen

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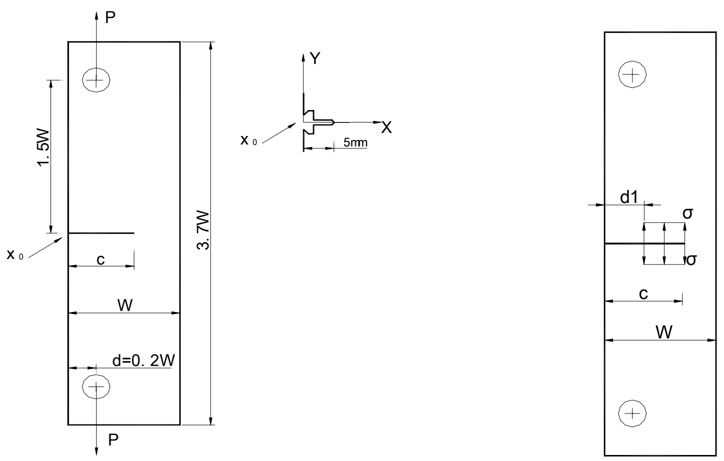
Abstract- In this paper, an approximate weight function (WF) for EC(T) specimen was given and verified. Using the WF, stress intensity factor and crack opening-displacement solutions for the specimen under pin loading and uniform pressure acting on the crack surface were obtained. The plastic-zone sizes from Dugdale model were calculated. Moreover, based on Dugdale model, a plasticity-induced crack-closure model for the specimen under fatigue loads was developed. Using the closure model, fatigue crack-closure behavior of the specimen was studied.

1. Introduction

Recently, an extended compact tension, EC(T), specimen, shown in Fig.1 has been developed for studying fatigue and fracture behavior of materials. The EC(T) specimen is considered an optimum design for laboratory fatigue-crack growth and fracture studies because of its distinct advantages compared to other cracked configurations, i.e. , standard compact tension, single-edge crack, and middle-crack tension specimens. These advantages are giving the experimenter additional working room, requiring low applied loads for an equivalent crack tip stress intensity factor, reducing the T-stress and crack fracture paths being self-similar, etc. [1]. The stress-intensity factor (SIF) solution and crack-surface opening displacements (CODs) at the crack mouth (x/c=0) and near the crack mouth (x/c=0.05) for the specimen under pin loading were derived by using the boundary-force method (BFM)[1,2]. Using an approximate method, the SIF solution for the specimen under pin loading was also obtained by Smith [3]. In this paper, following Smith's idea, an approximate crack surface weight function (WF) for the specimen is given. Using the weight function, SIF solution and CODs for the specimen under pin loading and uniform pressure acting on the crack surface are obtained. The plastic-zone sizes from Dugdale model are calculated. Moreover, based on Dugdale model, a plasticity-induced crack-closure model for the specimen under fatigue loads is developed. Using the closure model, fatigue crack-closure behavior of the specimen is studied.



(a) (b)

Fig.1 Extended compact tension specimen (a). Pin loading and notch details, (b) Uniform segment pressure acting on the crack surface

2. Weight Function For EC(T) Specimen

The site of the load application is far enough away from the site of interest for the EC(T) specimen, the details of the method of load application are unimportant (colloquially, the principle of St. Venant).

Considering this and using the principle of superposition, Smith represented the EC(T) specimen with an edge-cracked long strip loaded by a direct remote end tension load, P, and end bending moment, M (where P is pin load and M=0.3PW) [3]. The SIF solutions by this method agree well with the results from Piasick, et al's BFM. Following this method, crack surface weight function of the EC(T) specimen is assumed to be equal to that of the edge-cracked long strip. That is [4],

$$m(\alpha,x)=\frac{1}{\sqrt{2\pi a}}\sum_{i=1}^{J+1}\beta_i(\alpha)\left(1-\frac{x}{a}\right)^{i-\frac{3}{2}} \tag{1}$$

where the $\beta_i(\alpha)$-function were given in [4,5].

3. SIFs and CODs under pin loading

3.1 SIF solutions

According to two-dimensional weight function theory, the stress intensity factor due to an arbitrary set of applied loads can be obtained by integrating over crack length a product of these loads with the weight function *m*(α*,x)* of the cracked body [4]:

$$K=\sqrt{W}\int_0^{\alpha}\sigma(x)m(\alpha,x)dx \tag{2}$$

where the term σ(x) represents the stress distribution at the prospective crack site in the crack-free body. Under pin loading,

$$\sigma(x)=\frac{P}{BW}(2.8-3.6x) \tag{3}$$

Where P is pin loading, B and W are thickness and width of specimen, respectively. x=X/W. By substituting eqs(1) and (3) into eq(2), SIF solution can be gotten as follow:

$$K=\frac{P}{BW}\sqrt{\pi\alpha W}\,f(\alpha) \tag{4}$$

Where

$$f(\alpha)=\frac{2.8\sqrt{2}}{\pi}\sum_{i=1}^{5}\left[\frac{1}{2i-1}\beta_i(\alpha)\right]+\frac{7.2\sqrt{2}\alpha}{\pi}\sum_{i=1}^{5}\frac{\beta_i(\alpha)}{(2i-1)(2i+1)}$$

$$\alpha=c/W$$

Results from eq(4) and Piascik, et al's BFM[1,2] are given in Table 1, respectively, for comparison. Differences between the results are within 0.8%, very small. By fitting the their numerical solutions, Piasick ,et al got the following SIF expression:

$$K=\left[P\big/\left(B\sqrt{W}\right)\right]F_{EC(T)} \tag{5}$$

Where

$$F_{EC(T)}=(2+\lambda)G\Big/\left[(1-\lambda)^{3/2}(1-d/W)^{1/2}\right]$$

$$G=1.15+0.94-2.48\lambda^2+2.95\lambda^3-1.24\lambda^4$$

$$\lambda=(c-d)/(W-d)$$

Where d is the distance from specimen edge to load line, d=0.2W here. For 0.1≤c/W≤0.9, eq(5) is within ±1.0 % of WF solutions.

3.2 COD solutions

The crack opening displacement between two crack surfaces can be computed by the following equation: Where f(s) is the same as that of eq(4). $E'=E$ for plane stress and $E'=E/(1-\nu^2)$ for plane strain (E is

$$V(c,X) = 2U(c,X) = \frac{2P}{E'B}\int_{\alpha_0}^{\alpha} f(s)\sqrt{\pi s}\, m(s,x)ds \qquad (6)$$

Young's modulus and ν is Possion's ratio). $\alpha_0 = X/W$.

Table 1 A comparison of normalized SIFs and crack-opening displacements under pin loading

C/W	$KBW^{1/2}/P$ (BFM)	$KBW^{1/2}/P$ (WF)	$E'BV_0/P$ (BFM)	$E'BV_0/P$ (WF)	$E'BV_1/P$ (BFM)	$E'BV_1/P$ (WF)
0.1	1.721	1.723	1.664	1.668	1.180	1.586
0.2	2.586	2.590	3.750	3.756	3.194	3.593
0.3	3.571	3.578	6.853	6.865	6.126	6.622
0.4	4.904	4.913	11.99	12.01	10 .96	11.68
0.5	6.907	6.919	21.33	21.34	19.74	20.93
0.6	10.25	10.28	40.30	40.20	37.59	39.71
0.7	16.67	16.73	85.51	84.76	80.21	84.20
0.8	32.21	32.39	227.6	222.3	214.5	221.7
0.84	45.90	46.25	379.4	367.3	358.2	366.6

Normalized displacements (E'BV/P) at crack mouth V_0(X/W=0.) and near the crack mouth V_1(X/c=0.05) from WF and BFM[1,2] , respectively, are also summarized in Table 1 as a function of c/W for comparison. For V_0, WF solutions are within 3.2% of the BFM's results. For V_1, WF solutions agree with those of BFM at 0.2<c/W≤0.84 (Error is within 8%). However, at c/W=0.1, WF solution is obviously different from that of BFM.

By fitting WF solutions, crack-surface-opening displacement expression under pin loading is obtained as follows:

$$V(c,X) = \frac{4P}{BE'}\sqrt{\frac{c-X}{2\pi W}}\left(A_0 + A_1\left(1-\frac{X}{c}\right) + A_2\left(1-\frac{X}{c}\right)^2\right)F_{EC(T)} \qquad (7)$$

Where $F_{EC(T)}$ is the same as that in eq(5)

$$A_0 = 0.150963 + 17.9227\alpha - 85.4395\alpha^2 + 290.424\alpha^3 - 446.149\alpha^4 + 294.546\alpha^5$$
$$A_1 = -10.0527 + 146.658\alpha - 841.639\alpha^2 + 2358.52\alpha^3 - 3209.77\alpha^4 + 1767.36\alpha^5$$
$$A_2 = 0.874432 - 15.5152\alpha + 114.107\alpha^2 - 382.732\alpha^3 + 599.264\alpha^4 - 369.882\alpha^5$$

When 0.2≤c/W≤0.7, eq(7) is within ±0.4% of WF solutions. At c/W=0.8, the errors are within 13.2%.

The compliance method, that is, by means of measuring crack-mouth-opening displacement to monitor crack length, can be used during EC(T) fatigue crack growth testing. By fitting BFM's solutions, Compliance, in terms of crack length, is given by Piascik, et al [3] as follows:

$$E'BV_0/P = [15.52\alpha - 26.38\alpha^2 + 49.7\alpha^3 - 40.74\alpha^4 + 14.44\alpha^5]/(1-\alpha)^2 \qquad (8)$$

Where α=c/W. Equation (8) is within 0.3% of the same BFM numerical results, within 3% of the corresponding WF solutions aty 0<c/W≤0.9. By fitting WF numerical results, the following expression with high accuracy is given:

$$E'BV_0/P = [-0.0864736 + 17.1971\alpha - 35.1958\alpha^2 + 71.6459\alpha^3 - 64.2242\alpha^4 + 22.5789\alpha^5]/(1-\alpha)^2 \qquad (9)$$

Equation (9) is within 0.6% of the same WF solutions for 0.1≤c/W≤0.9.

4. SIFs and CODs under a segment of uniform pressure in the wake of crack tip

4.1 SIF Solutions

According to eq.(2), let σ(x)=σ, SIF expression for the specimen under a segment of uniform pressure in the wake of crack tip, as shown in Fig.1 (b), is derived as follows:

$$K = \sigma\sqrt{\pi \alpha W}\, f\,(\alpha) \tag{10}$$

Where

$$f\,(\alpha) = \frac{\sqrt{2}}{\pi}\sum_{i=1}^{5}\frac{1}{2i-1}\beta_i(\alpha)\left(1-\frac{d_1}{c}\right)^{i-1/2}, \qquad \alpha = c/w$$

and

$$\beta_1 = 2.0$$

$$\beta_2 = (1.06326 - 3.76571\alpha + 74.003\alpha^2 - 270.01\alpha^3 + 574.281\alpha^4 - 743.859\alpha^5 + 523.98\alpha^6 - 155.07\alpha^7)/(1-\alpha)^{3/2}$$

$$\beta_3 = (0.784116 + 8.8676\alpha - 112.136\alpha^2 + 567.975\alpha^3 - 1471.26\alpha^4 + 2164.99\alpha^5 - 1672.5\alpha^6 + 532.822\alpha^7)/(1-\alpha)^{3/2}$$

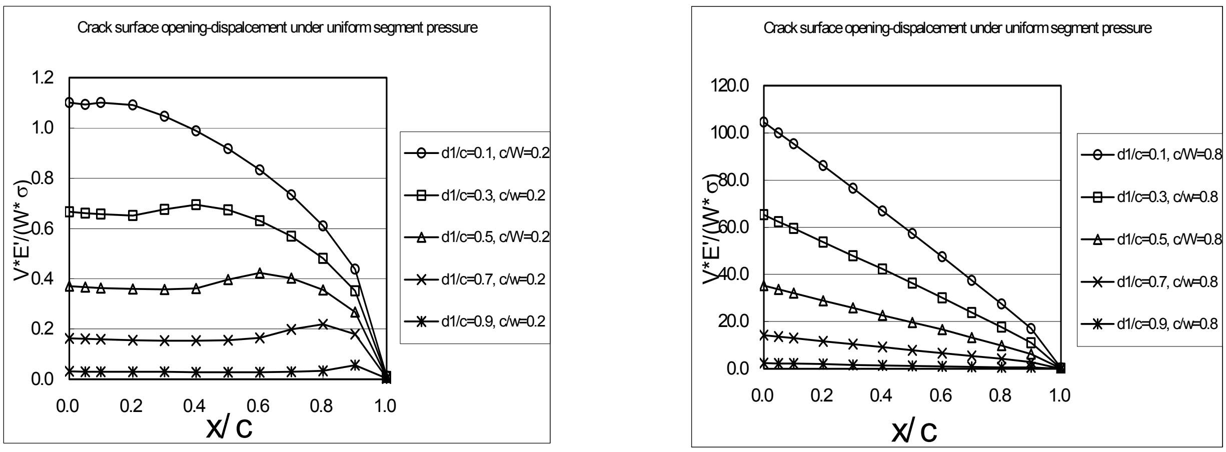
$$\beta_4 = (0.0381597 - 11.516\alpha + 127.158\alpha^2 - 628.338\alpha^3 + 1674.3\alpha^4 - 2507.3\alpha^5 + 1964.73\alpha^6 - 634.102\alpha^7)/(1-\alpha)^{3/2}$$

$$\beta_5 = (-0.224561 + 4.32355\alpha - 42.822\alpha^2 + 22.819\alpha^3 - 574.885\alpha^4 + 867.481\alpha^5 - 683.491\alpha^6 + 221.584\alpha^7)/(1-\alpha)^{3/2}$$

Equation (10) is within 0.06% of the corresponding WF numerical results at $0.1 \le c/W \le 0.8$ and $0 < d_1/c < 1$. d_1 is the distance from crack mouth to initiating load position.

4.2. COD solutions

The crack opening displacement between two crack surfaces for the specimen under a segment of uniform pressure in the wake of crack tip, as shown in Fig.1 (b), can be also computed by substituting f(s) in eq(10) into eq(6) and letting $a_0=d_1/W$ in eq.(6). The WF solutions of normalized COD for the specimen with several d1/c at c/W=0.2 and 0.8, are shown in Fig.2. Unfortunately, it is very difficult to get a COD expression with high accuracy by fitting the corresponding WF numerical solutions under this loading case.



(a) (b)

Fig.2 Normalized crack-surface-opening displacement for EC(T) specimen with uniform pressure applied to crack wake for various d_1/c. (a) c/w=0.2, (b) c/W=0.8

5. Plastic-zone from Dugdale model for EC(T) specimen

The Dugdale model for EC(T) specimen requires that the "finiteness" condition of Dugdale be satisfied. This condition state that K at the tip of the plastic zone (at c+ρ) is zero. Thus,

$$\frac{P}{BW}\int_0^{\alpha}(2.8-3.6x)m(\alpha,x)dx=\int_{\alpha_0}^{\alpha}\sigma_0 m(\alpha,x)dx, \qquad (11)$$

Where $\alpha=(c+\rho)/W$, $\alpha_0=c/W$, σ_0 is flow stress, which is taken to be an average of the yield and ultimate strength. By eq.(11), the plastic-zone size (ρ) is calculated for various c/W and $P/(BW\sigma_0)$ ratios. An equation is then fitted to these results and is

$$\frac{\rho}{a}=\frac{\pi W}{8c}\left(\frac{PF_{EC(T)}}{BW\sigma_0}\right)F_0 \qquad (12)$$

Where

$$F_0=\left(\sum_{i=0}^{6}A_i\alpha^i\right)/(1-\alpha)^{3/2}$$

and $A_0=1.2231$, A_1 =-23.3888, A_2 =226.401, A_3 =-942.615, A_4 =2080.0, A_5 =-2301.44, A_6 =1036.04. Equation (12) is within 3% of the corresponding numerical results at $0.1\le c/W\le 0.8$ and $\rho/(W-c)\le 0.55$.

6. Fatigue crack closure behavior

Based on two dimensional weight function method, a new crack closure analytical model was developed by two of the present authors, Liu and Wu[4,5] in order to extend the Newman model to various cracked geometries. Following the method, using the weight function method as explained above, a crack closure model for EC(T) specimen was established. In the model, the applied stress level $P_o/(BW)$, at which the crack surfaces are fully open, is obtained on crack surface-opening displacement. To find the applied stress level needed to open the crack surface at any point, the displacement at that point due to an applied stress increment (Po-Pmin)/(BW) is set to equal to the displacement at that point due to the contact stresses at Pmin/(BW). Thus,

$$\left(\frac{P_o}{BW}\right)_i=\frac{P_{\min}}{BW}-\sum_{j=16}^{n}s_j g(x_i,x_j)/f(x_i) \qquad for \quad i=16 \quad to \quad n \qquad (13)$$

Where $f(x_i)$ is the crack surface-opening displacement at the point x_i due to unit pin load P per unit thickness (B) and unit width (W). $g(x_i,x_j)$ is the displacement at the point x_j due to unit uniform stress acting on a segment of the crack surface with the center at x_i. n is the total number of elements modeling crack-tip plastic-zone and residual plastic deformation along the crack surface. The maximum value of $(P_o)_i$ gives the crack open load, P_o.

In this paper, EC(T) specimen is assumed to be made of 2024-T351 aluminum alloy. The mechanical properties of the material are UTS σ_u=457 MPa, yield stress σ_y=364Mpa, Young's modulus of elasticity E=69Gpa. Constraint factor, alf, ahead of crack tip is assumed to be equal to 1 and 1.73, respectively.

Normalized crack opening loads under different stress ratios and constraint factors were obtained by the model above, and are given in Fig.3. From the figure, it is found that obvious effect of both stress ratio and maximum stress on crack closure exists. Crack closure is more distinct under lower stress ratio. The effect of maximum stress on crack closure is significant at low stress ratio and small crack-tip constraint factor. These results can be used to explain fatigue crack growth behavior of EC(T) specimen at different stress ratio and specimen thickness.

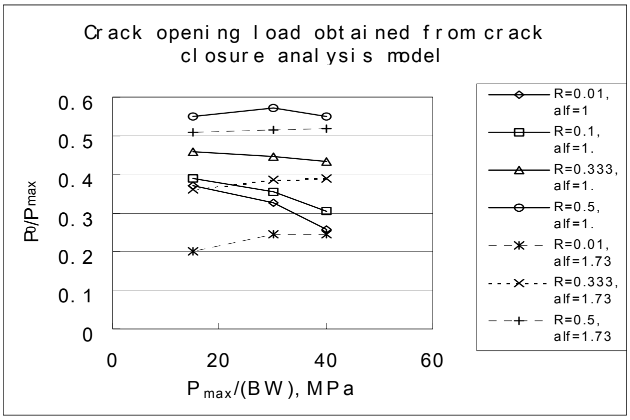


Fig.3. Normalized crack opening loads for EC(T) specimen made of 2024-T351 aluminum alloy under different stress ratios and constraint factors

7. Conclusions

(1) According to the principle of St. Venant, a crack-surface weight function of EC(T) specimen was assumed to be same as that of an edge-cracked long strip. Stress intensity factor and crack mouth opening-displacement solutions, obtained by using the WF, agreed well with the corresponding BFM's solutions. Thus, the WF is verified to be with high accuracy, can be used for EC(T) specimen.

(2) Using the WF, SIF and COD solutions of EC(T) specimen under pin loading and uniform pressure acting on the crack surface were obtained. The plastic-zone sizes based on Dugdale model were calculated. By fitting the numerical results, simple expressions with high accuracy were obtained for COD under pin loading, SIF under a segment of uniform pressure in the wake of crack tip, and the Dugdale plastic-zone size.

(3) A plasticity-induced crack-closure model for the specimen was developed. Using the model, fatigue crack closure behavior of the specimen was analyzed. The results showed that an obvious effect of both stress ratio and maximum stress on crack closure exists. Crack closure is more distinct at lower stress ratio. The effect of maximum stress on crack closure is significant at low stress ratio and small crack-tip constraint factor. These results can be used to explain fatigue crack growth behavior of EC(T) specimen at different stress ratio and specimen thickness.

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