ORAL REFERENCE: ICF 0824OR

An element-free Galerkin method for dynamic fracture in functional graded material

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ABSTRACT

A improved element-free Galerkin method(EFGM) is used as the numerical tool for analyzing dynamic crack propagation problem in functional graded material(FGM). The Element-Free Galerkin Method(EFGM) suggested by T.Belytchko et al[1] is a meshless method, which uses the Moving Least-Squares(MLS) approximation based only on nodes. Since no element connectivity data is needed, the extension of the crack is then treated by the growth of the surfaces of the crack naturally and the remeshing is avoided. This makes the method particularly attractive for moving dynamic crack problems. In this paper, The shear modulus are assumed to vary continuously and Poisson's ratio to be constant. The variation of the material properties is simulated by adopting the material properties of the integration point when forming the stiffness matrix. The dynamic J integral is evaluated. Some numerical results are provided to demonstrate the utility and robustness of the proposed technique.

KEYWORDS

functional graded material, element-free Galerkin method(EFGM), dynamic J integral

INTRODUCTION

The functional graded materials(FGM) have been widely used in technological application . So, it is very important and necessary to study its mechanical behaviors, especially in the fracture mechanics. However the material properties of FGM vary with the coodinates, its mechanical behaviors is very complex. The analytical approach can only deal with some simple and particular problems. Therefore, numerical methods for FGM have to be developed.

The Element-Free Galerkin Method(EFGM)[1] suggested by T.Belytchko et al is a meshless method, which use the Moving Least-Squares(MLS) approximation based only on nodes. Since no element method connectivity data is needed and the extension of the crack is then treated by the growth of the surfaces of the crack naturally, it is very convenient for modeling the crack propagation. This method provides a higher resolution localized derives of strains and stresses. Also it can adopt the material properties of integration points to simulate the variation of the material properties. So, it is very suitable to analyze FGM.

However, in EFGM, the interpolants constructed by the MLS method does not pass through the nodal parameter values, the imposition of boundary conditions on the dependent variables is quite awkward and the computational cost is quite burdensome, which makes EFGM not as fast as FEM. In this paper, the EFGM is coupled to FEM. EFG models are only used near the crack tip where their great versatility and high resolution is needed, FE models are applied in the other domains. Therefore, the boundary conditions can be treated easily and directly by FE models. Meantime, the computation efficiency can be great improved.

Jin and Noda[2] have shown that the singularity and the angular distribution of the stress and displacement near-tip fields for FGM are same as the ones of homogeneous materials. Erdogan and Wu[3] have presented the analytical result. Jian.C et al have given a modified static J integral for FGM. In this paper, based on Moran et al[4], a modified dynamic J integral for FGM is calculated. Numerical results are provided to demonstrate the utility and robustness of the proposed technique.

Element-Free Galerkin Method(EFGM) and Its Coupling to finite elements

The most difference between the EFGM and the FEM is the construction of the shape functions and test functions. In the EFGM, the field variable is approximated by moving least square(MLS) approximations, no element connectivity data is needed, which is necessary in the FE method. The shape functions of EFGM can be written as[1].

$$\mathbf{\Phi}(\mathbf{x}) = \mathbf{p}^T(\mathbf{x})\mathbf{A}^{-1}(\mathbf{x})\mathbf{B}(\mathbf{x}) = \left[\varphi_1 \;\; \varphi_2 \;\; \cdots \;\; \varphi_{\mathrm{n}}\right] \tag{1}$$

The derives of $\mathbf{\Phi}(\mathbf{x})$ are expressed as:

$$\begin{aligned}\Phi(x)_{,i} &= [\mathbf{p}^T(\mathbf{x})\mathbf{A}^{-1}(\mathbf{x})\mathbf{B}(\mathbf{x})]_{,i} \\ &= \mathbf{p}^T(\mathbf{x})_{,i}\mathbf{A}^{-1}(\mathbf{x})\mathbf{B}(\mathbf{x}) + \mathbf{p}^T(\mathbf{x})\mathbf{A}^{-1}(\mathbf{x})_{,i}\mathbf{B}(\mathbf{x}) + \mathbf{p}^T(\mathbf{x})\mathbf{A}^{-1}(\mathbf{x})\mathbf{B}(\mathbf{x})_{,i}\end{aligned} \tag{2}$$

where

$$\mathbf{A}^{-1}(\mathbf{x})_{,i} = -\mathbf{A}^{-1}(\mathbf{x})\mathbf{A}(\mathbf{x})_{,i}\mathbf{A}^{-1}(\mathbf{x}) \tag{3}$$

To impose boundry conditions with as high a degree of accuracy and improve the computation efficiency, the coupled EFG/FE approach is used.

In the meshless domain Ω_E, the MLS method is still adapted to construct the test function of $\mathbf{x}_{eI}$,

$$\mathbf{u}^h(\mathbf{x}_{eI}) = \mathbf{\Phi}(\mathbf{x})\mathbf{u}^* \qquad (4)$$

In the non-interface FEM domain Ω_{FE}, the traditional FEM is used to construct the test function of $\mathbf{x}_{fI}$,

$$\mathbf{u}^h(\mathbf{x}_{fI}) = \sum_{i=1}^{ND} N_i \mathbf{u}_i^* \qquad (5)$$

here ND is the number of element nodes.

For the FEM point $\mathbf{x}_{bI}$ in the interface zone Ω_B, we also use the MLS method,

$$\mathbf{u}^h(\mathbf{x}_{fb}) = \mathbf{\Phi}(\mathbf{x})\mathbf{u}^* \qquad (6)$$

In the FEM domain, the test function can be expressed as,

$$\mathbf{u}^h(\mathbf{x}_{fI}) = \sum_{i=1}^{ND} N_i \mathbf{u}_i(\mathbf{x}_i), \qquad \mathbf{u}_i = \left\langle \begin{array}{l} \mathbf{u}^*, \mathbf{x}_i \in \Omega_{FE} \\ \mathbf{\Phi}(\mathbf{x})\mathbf{u}^*, \mathbf{x}_i \in \Omega_B \end{array} \right. \qquad (7)$$

With these test functions, mass matrix and stiffness matrix can be formed in general way, and boundary conditions can be enforced strictly. Since background finite element is used for quadrature in meshless domain, it is very convenient that in the procedure of numerical implementation, the material properties of the integration point are adopted not only in finite element domain but also in meshless domain.

NUMERICAL EXAMPLE

A single cracked panel (SECP, Fig.1) of unit thickness in elastic plane stress conditions is considered.

l and w are the length and the width of the plate. a is the edge crack. Poisson's ratio υ and mass density ρ are constant. Young's modulus is given by the following expressions:

$$E(x) = E_1 \exp\left(\frac{x}{h} \ln \frac{E_2}{E_1}\right) \qquad (8)$$

In this paper, $l = 4, h = 1, a = 0.5$, $E_1 = 1.11 \times 10^{11}$, $E_2 = 2.22 \times 10^{11}$, $\upsilon = 0.3$, $\rho = 7800$ and $\sigma = 1.0$

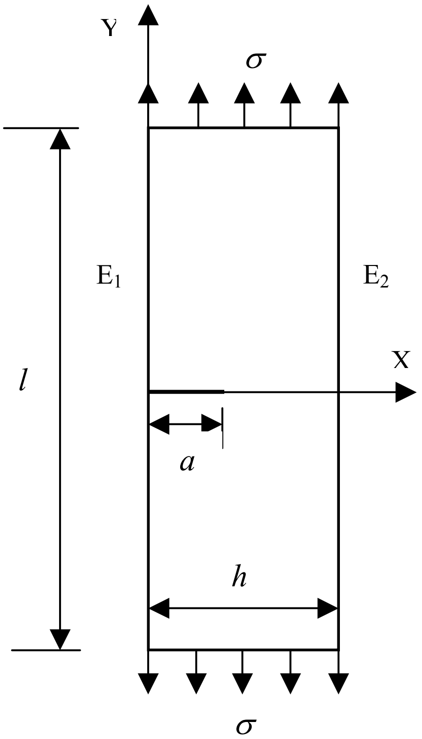

Fig.2 A single edge FGM panel

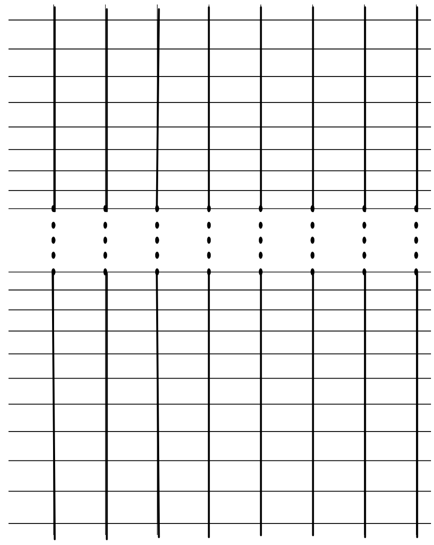

Fig.2 Node distribution

The distribution of nodes is like fig.2 . The plate is divided into 30×40 elements, including the background finite elements only for quadrature. The number of all nodes is 1271. The nodes of the EFGM is only distributed near the crack tip. The meshless domain is divided into 30×4 background finite elements mesh. The rest of the plate is divided into 30×36 finite elements. The nearer to the crack surface, the finer the nodes are distributed along Y. Otherwise, the material properties vary along X, so finer nodes are distributed along X than along Y.

J integral is often used to evaluate the stress intensity factor. Based on Moran[5], we obstaine modified dynamic J integral for FGM

$$J_t = \iint_S [-(W+L)q_{,1} + \sigma_{ij}u_{i,1}q_{,j} + \rho(\ddot{u}u_{i,1} - \dot{u}_i\dot{u}_{i,1})q - \frac{1}{2}(\varepsilon_{ij}D_{ijkl,1}\varepsilon_{kl} + \rho_{,1}\dot{u}_i^2)q]dS \quad (9)$$

It is obvious that the term $\iint_S \frac{1}{2}(\varepsilon_{ij}D_{ijkl,1}\varepsilon_{kl} + \rho_{,1}\dot{u}_i^2)qdS$ only exists for FGM. For homogeneous materials, since $D_{ijkl,1} = 0$， and $\rho_{,1} = 0$， the term vanishes.

In term of (9), we evaluate the dynamic J integral for FGM on two different contour Γ_1 and Γ_2 with increment of the time step. The numerical results are drawn in

Fig. 3.

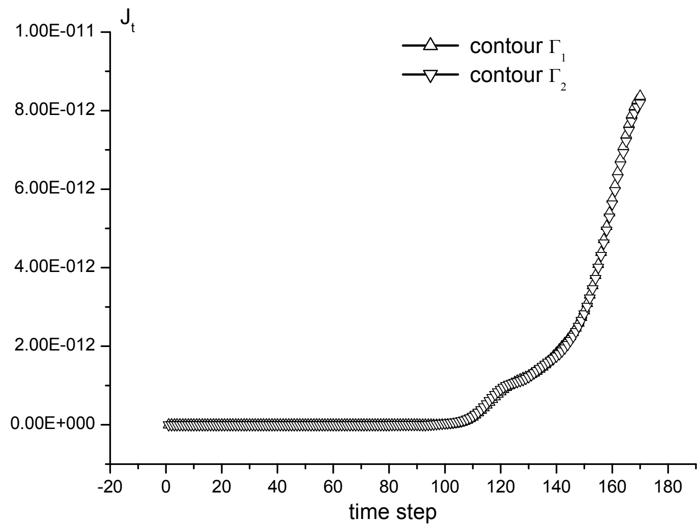

Fig.3 Variation of J integral with time step

These curves in fig.3 show that when the strength waves don't reach crack surface, the value of J integral is zero, when the strength waves reach crack surface, the value of J integral become increasing. Because the velocity of strength waves varies with the Young's modulus, a little difference between the J integral along Γ_1 and the J integral along Γ_2 exists. However, when the strength waves influence the whole area for evaluation, the values of the J integral on contour Γ_1 and Γ_2 are almost equal, namely the J integral is path independent. So, the trend of variation of the J integral is rational. The results show that the method in the paper is efficient for FGM.

CONCLUSION

In this paper, EFGM is used for analyzing dynamic fracture problem in FGM and meanwhile, EFGM is coupled to FE for enforcing boundry conditions strictly and improving the computation efficiency. The numerical results show that the technique is efficient. Otherwise, in the procedure of forming mass matrix and stiffness matrix and evaluating the dynamic J integral, we adopt the material properties of Gauss integration points. So, not as a great number of nodes are needed as in conventional technique for FGM. Not only computation efficiency is improved once again, but also high accuracy is achieved.

Since EFGM facilitates the modeling of growing crack problem, the technique proposed in the paper is promising in dealing with dynamic crack propagation of FGM.

Although only Mode I cracks are reported, it is straightforward to employ this method to more complicated crack configurations.

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