ALTERNATIVE METHODS FOR DERIVING AND FITTING J-R CURVES: HOW THEY AFFECT STRUCTURAL INTEGRITY ASSESSMENT

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ABSTRACT

In this paper, results from Reese & Schwalbe’s linear normalization (LN) methodology for deriving J-R curves are compared, related to J-∆a (J-integral-ductile crack growth) data, to those obtained from traditional unloading elastic compliance (UEC) technique. Research results regarding to a nuclear grade steel exhibiting a wide range of elastic-plastic fracture resistance, agree quite well for both techniques until a certain level of toughness of the material. Below this critical level, linear normalization produces too conservative and inconsistent results for sub-sized compact testpieces. Power-law, linear and logarithmic fits were applied to the J-∆a data points within well-known limits of validity of deformation-J (JD). The results were assessed in terms of two typical J-integral criteria of the nuclear industry, namely, the crack initiation J (Ji) and the so-called Paris & Johnson’s J50 for ductile instability of cracks. It was concluded that the logarithmic fit produces conservative values for both Ji and J50 criteria, when compared to power-law, whereas the linear fitting method provides the most non-conservative failure predictions.

KEYWORDS

Fitting and extrapolation methods, J-R curve techniques, Structural integrity assessment

INTRODUCTION

J-R Curves

The J-integral is the most important parameter for characterizing the elastic-plastic fracture resistance of structural materials and efforts have been continuously conducted to develop simplified methodologies for determining the so-called J-R curves. The most recent and promising trend in this field is the use of single-specimen normalization techniques, which simply demand the determination of the load versus displacement record and both initial and final crack lengths. Reese & Schwalbe [1] developed a method, named linear normalization (referred LN hereafter), which is based on the original Landes’ work (LMN function [2]). LN is grounded upon the principle of load separation [3,4], which has been proved for all specimen geometry [5,6]. This principle allows the load, P, to be written as a function of the crack length, a, and the corresponding applied plastic displacement, vpl, by two separate multiplicative functions:

\[ P = G(a/W).H(v_{pl}/W) \]
W is the specimen width, and $G(a/W)$ is the geometry calibration function, which is dependent on the specimen configuration and can be determined from the J calibration [5,6].

$$G(a/W) = B \cdot W \cdot (b/W)^{\eta_{pl}}$$  \hspace{1cm} (2)

B is the specimen thickness, b the uncracked ligament length, $b = W - a$, and $\eta_{pl}$ the geometry correction (plastic) factor, which is assumed to depend weakly on material properties. For compact specimens it is generally assumed the value of 2.13 [2,5,6].

Reese & Schwalbe focused their attention on the correlation between the change or gradient in normalized load, $\Delta P_N$, and the respective crack extension, $\Delta a$. The gradient in the normalized load owing to a slight crack growth from the initial (pre)crack length ($a_0$) is:

$$\Delta P_{N(i)} = P_N(a_i) - P_N(a_0) = P/G(a_i/W) - P/G(a_0/W)$$  \hspace{1cm} (3)

A well-defined linear dependency of $\Delta P_{N(i)}$ on $\Delta a_i = a_i - a_0 = b_0 - b_i$ has been shown for large amounts of crack growth in elastic-plastic fracture toughness testing (J-R curve) [1]. This linear relationship allows the complete J-R curve of these materials to be obtained by means of a special graphical procedure.

**Fitting Methods**

Power-law is the most widely employed method for both J-R curve fit and extrapolation [7,8], and it is even mandatory in current high demanding components codes [9]. J-R data extrapolation for higher levels of crack growth is used to compensate insufficiently extensive data obtained in laboratory testing, when the failure criteria, e.g., in the ductile instability assessment of a cracked component, is beyond the limits of validity of deformation-J ($J_D$), as defined on $J$-$\Delta a$ space [10]. By virtue of the downwards concavity typically exhibited by J-R curves, which effect is further intensified by the $J_D$-saturation phenomena [11], extrapolation through power-law may be a quite non-conservative approach and the higher the degree of non-conservatism, the shorter is the crack extension level attained by fracture toughness testing [8]. In a previous paper [12], the authors have claimed that the logarithmic fit may be a worthwhile alternative method to the power-law, as long as it produces more conservative results with regard to predictions of ductile instability events for cracked components, specially when data extrapolation is necessary. Other methods used in some extent to fit J-R curves include polynomial and linear fits.

In this work, the performance of the methods for both J-$\Delta a$ data determination and fitting are evaluated for a nuclear grade steel exhibiting microstructures with a wide range of elastic-plastic fracture resistance. None of the microstructures tested exhibited cleavage (catastrophic fracture) and in all cases unloading elastic compliance (UEC) provided confident results for dealing of close comparison between both techniques.

**MATERIALS AND TESTPIECES**

Seven miniaturized testpieces (0.4TC[S]) were machined from a thick forged plate of a nuclear grade steel in the as-received (AR) and several thermally embrittled (TE) conditions, the latter achieved by special heat treatments. They were fatigue precracked to an $a_0/W$ ratio of 0.55, side grooved (SG) to a 20 or 33% reduction of their gross-thickness ($B=10$ mm) and thereafter tested at 300°C. The mechanical properties of the materials and the testpieces’ specifications are listed in Table 1. Notice that the reduction in area of the tensile specimens precisely ranks the elastic-plastic crack resistance of the six tested microstructures.

**EXPERIMENTAL AND ANALYTICAL PROCEDURES**

**Unloading Elastic Compliance (UEC)**

J-R curve testing was conducted by clip-gage-controlling elastic unloadings, under a strain rate of 0.3 mm/min. J-$\Delta a$ data points were obtained according to ASTM E1820 standard [10], i.e. corrected for crack growth. Initial and final crack length predictions by elastic compliance measurements loosely satisfied minimum accuracy requirements established by ASTM standard.
Linear Normalization (LN)

Linear-normalized J-R curves were derived by making use of the load versus load-line displacement diagrams resulting from compliance technique, and following the Reese & Schwalbe’s analytical procedure, which is fully described elsewhere [1]. Initial and final crack lengths, $a_0$ and $a_f$, respectively, were obtained from the broken specimens, by means of observation in a stereo-microscopy. Figure 1 displays the linear dependence of $\Delta P_N$ on $\Delta a$, as described in Eqn. 3, for all steel structures and specimens tested in this study.

J-R Fitting Methods

Power-law, logarithmic and linear fits were applied to J-$\Delta a$ data points within limits of validity of deformation-J ($J_D$), as delineated by exclusion off-set lines, at respectively 0.15 and 1.5 mm of crack growth. Given the reduced testpieces’ size, only the specimen correspondent to the lowest fracture toughness level fulfilled both J maximum capacity and minimum thickness requirements established in Ref. 10. Once the J-R curves were fitted, the J value for crack initiation, $J_i$ [10], and the so-called Paris & Johnson’s $J_{50}$ [13] for healthy conservative prediction of ductile instability of cracked components, were determined.

**TABLE 1**

MATERIALS AND TESTPIECES CHARACTERIZATION

<table>
<thead>
<tr>
<th>Microstructural Condition</th>
<th>Testpiece Designation</th>
<th>SG (%)</th>
<th>Yield Strength (MPa)</th>
<th>Ultimate Strength (MPa)</th>
<th>Elongation (%)</th>
<th>Reduction in Area (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A (AR)</td>
<td>JRT7</td>
<td>33</td>
<td>362</td>
<td>548</td>
<td>11</td>
<td>77</td>
</tr>
<tr>
<td>A (AR)</td>
<td>JRT8</td>
<td>20</td>
<td>362</td>
<td>548</td>
<td>11</td>
<td>77</td>
</tr>
<tr>
<td>B (TE)</td>
<td>JRT27</td>
<td>20</td>
<td>361</td>
<td>621</td>
<td>17</td>
<td>71</td>
</tr>
<tr>
<td>C (TE)</td>
<td>JRT32</td>
<td>20</td>
<td>344</td>
<td>611</td>
<td>16</td>
<td>63</td>
</tr>
<tr>
<td>D (TE)</td>
<td>JRT36</td>
<td>33</td>
<td>370</td>
<td>620</td>
<td>12</td>
<td>54</td>
</tr>
<tr>
<td>E (TE)</td>
<td>JRT41</td>
<td>33</td>
<td>376</td>
<td>626</td>
<td>12</td>
<td>49</td>
</tr>
<tr>
<td>L (TE)</td>
<td>JRT86</td>
<td>33</td>
<td>701</td>
<td>810</td>
<td>08</td>
<td>44</td>
</tr>
</tbody>
</table>

$L_0 = 4D_0 = 40$ mm

RESULTS AND DISCUSSION

Incremental Crack Length

In this analysis, the number of load (P)-displacement ($\delta$) data points, taken evenly spaced regarding to $\delta$, was kept fixed and the incremental crack length extension ($da = d\Delta a$) was taken at the values of, respectively, 0.1, 0.01 and 0.001 mm on the fully computerized iterative data processing. Percentage differences among the three used approaches were then calculated. Figure 2 points out that crack increments smaller than 0.01 mm do not promote significant changes on J values. Higher errors invariably occurring post-maximum load capacity of the specimen, are certainly due to the effects of both spread plasticity and relatively large amounts of crack growth on data processing. However, even for the least accurate approach ($da = 0.1$ mm) such errors have never exceeded $\pm0.06\%$, which is a very stringent criterion for purposes of comparing J values.

Number of Load-Load Line Displacement Data

For a fixed crack increment length of 0.01, J-R curves were generated by randomly choosing several different number of P-$\delta$ data points along the loading curve of the specimen. Figure 3 shows that, as a general rule, the larger the number of J (i.e., P-$\delta$) data points, the higher is the J value for a constant $\Delta a$ analysis. This can be explained in terms of the crack growth correction factor in the deformation-J ($J_D$) concept [3,10]. Thus, the larger the number of chosen P-$\delta$ data points, the smaller the average crack growth correction (i.e., reduction) factor and, consequently, smaller is its cumulative effect in lowering the J-R curve.
Figure 1: Linear relationship between $\Delta P_N$ and $\Delta a$, as determined by on-line UEC monitoring.

Figure 2: Incremental crack length affecting LN J-R curve. (a) Testpiece JRT36 with a large number of J data points. (b) Associated errors. Arrows indicate maximum load positions.
The $\eta_{pl}$ factor as an indicative of the LN worthiness
The applicability of the LN technique in the assessment of different fracture resistance behaviors was confirmed for very most of the microstructures tested. However, as shown in Fig. 4, LN failed in deriving the J-R curve for the least elastic-plastic fracture resistant microstructure. An $\eta_{pl}$ factor of 2.13 was assumed for the miniaturized specimens herein tested. As a coincidence, or not, it was found out that good results regarding to the LN technique were obtained from testpieces which the best linear correlation between the normalized load gradient ($\Delta P_N$) and the ductile crack extension ($\Delta a$) is achieved for $\eta_{pl} \leq 2.13$. Conversely, bad LN results were invariably associate to $\eta_{pl} > 2.13$, for a maximum $\Delta P_N - \Delta a$ linear correlation. This empirical rule could serve as an indicative of the applicability of the LN technique for this class of material.

$J_i$ and $J_{50}$ Criteria for Structural Integrity Assessment
The results concerning $J_i$ and $J_{50}$ criteria, obtained from both UEC and LN techniques, are furnished in Fig. 5. They are plotted against the Charpy impact energy of standard bend bar specimens precracked in fatigue with the same side-grooving level of the correspondent sub-sized compact J-testpiece. It can be seen that quasi-static fracture toughness results correlate rather well with the absorbed energy under dynamic
conditions. It is worthy of note that the LN methodology produces slight conservative results if compared to those obtained from the UEC technique. It is also observed that the degree of conservatism of both J criteria is strongly dependent on the fracture resistance of the tested microstructure.

![Graph showing J-integral criteria for all testpieces](image)

Figure 5: J$_i$ and J$_{50}$ criteria for all testpieces, as predicted by the LN technique. Dashed lines correspond to UEC results under the same testing conditions. $R$ is the determination coefficient of the LN straight lines.

CONCLUSIONS

The following conclusions have been drawn during this comparative study:

1. Computer programming renders to LN a trustworthy and very simple methodology for deriving J-R curves within a broad range of elastic-plastic fracture resistance of low-alloy steels.
2. A simple empirical rule has been derived to determine the applicability of the LN methodology.
3. A 0.01 mm crack increment is suitable, in the data processing, to produce precise LN J-R curves.
4. Even a few load-load line displacement data points allow the generation of sufficiently accurate J-R curves through the linear normalization approach.
5. There is a trend of LN technique in producing slightly conservative results of J-integral criteria for structural integrity assessment, as compared to elastic compliance method.

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REFERENCES