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A STUDY OF FRACTURE FROM THE VIEWPOINT OF DILATATION

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ABSTRACT

Solid materials possess two modes of elastic deformation to an external load. One is distortional and the other is dilatational. At the limit of elasticity, failure occurs by one of the two modes corresponding to respective elastic ones, i.e., yielding occurs as a limit of distortional deformation and fracture as a limit of dilatational one. Yield condition has been established with deviatric strain and stress in the theory of plasticity, but fracture has not been described with dilatation. In this paper, authors show close connection between dilatation and fracture, and proposes a criterion to predict the failure mode for a given material, non-elastic behavior, with the ratio of the elastic constants G/K, where G and K denote shear and bulk moduli, respectively.

KEYWORDS

Fracture criterion, Dilatation, Distortion, Brittle, Ductile, Elastic failure

INTRODUCTION

Fracture of materials has been discussed in terms of various physical quantities as summarized in Table 1.

TABLE 1
FRACTURE CRITERION FROM VARIOUS VIEWPOINTS

Viewpoint of fracture	Physical quantity	Criterion for fracture
Energy	Surface energy	Griffith's theory [1]
	Elastic energy	J-integral [2]
Stress space	Force	Crack extension force [3]
(Mechanical aspect)	Stress	Stress intensity factor [4]
Strain space	Displacement	Crack opening displacement [5]
(Geometrical aspect)	Strain	Dilatation (and Distortion)*
Property of matter	Elastic constant	Local elastic constant [6]

* The authors are proposing in this paper.

Fracture as a natural phenomenon is supposed to obey a universal principle, but we can describe it in many ways from various viewpoints. Each description is related to a view to visualize respective aspect of fracture through the employed physical quantity.
For example, Griffith's theory visualizes fracture through energics by noting that with an extension of a crack strain energy can be converted to surface free energy. Similarly, description in terms of stress concentration near the crack tip is related to the view that a solid is broken by a force (stress) beyond a critical value. Fracture may be related to degradation of materials around a crack tip, which involves the field of materials science in addition to mechanics. We can also consider fracture a geometrical problem as in the theory with the crack opening displacement. In this paper, we point out that dilatation of a solid body is closely connected with fracture and proposes a criterion to predict the failure mode for a given material as a first step toward establishing a fracture criterion in terms of dilation.

Elastic deformation can be classified into two modes, i.e., dilation (volumetric change) and distortion (shearing deformation). Isotropic solid bodies possess two independent elastic constants, shear modulus G and bulk modulus K, each of which represents resistance to distortion and dilatation, respectively. At the limit of elasticity, failure occurs by one of the two modes corresponding to respective elastic ones, i.e., yielding occurs as a limit of distortional deformation and fracture as a limit of dilatational one. Yield condition has been established with deviatric strain and stress in the theory of plasticity, but fracture has not been described with dilatation. We propose to classify other physical properties also in connection with those two modes as shown in Table 2.

TABLE 2
CLASSFICATION BASED ON THE DEFORMATION MODE

Deformation mode	Elastic constant	Strain energy	Property of material	Failure
Distortion	Shear modulus G	Distortion energy E_s	Ductility	Plastic deformation
Dilatation	Bulk modulus K	Dilatation energy E_v	Brittleness	Fracture

$E_s = G(\varepsilon_{ij}\varepsilon_{ij} - \varepsilon_{ii}\varepsilon_{jj}/3)$, $E_v = K\varepsilon_{ii}\varepsilon_{jj}/2$, where ε_{ij} is the strain component.

RELATED PHENOMENA

We can correlate the present idea with various behaviors of materials as follows:
1) Rubber is nearly incompressible materials [7]. It means that dilatation associated with deformation is negligibly small, hence we expect that deformation will not be easily terminated by fracture.
2) The volume of a body is nearly unchanged during plastic deformation, hence plastic deformation will not be easily terminated by fracture.
3) Materials become more ductile under hydrostatic pressure [8]. This can be interpreted as that fracture is prevented by constraining dilatation.
4) The thick specimen (plane stress) of a ductile material becomes more brittle than the thin one (plane strain). Under uni-axial tensile stress σ, volumetric strain ε_V with the plane stress condition becomes

$$\varepsilon_V = \frac{1-2\nu}{E}\sigma, \quad (1)$$

and volumetric strain ε_V^* with the plane strain condition is

$$\varepsilon_V^* = \frac{1-\nu}{E}\sigma, \quad (2)$$

where E and ν denote Young's Modulus and Poisson's ratio, respectively. When $0 \le \nu \le 0.5$, we get the identical relation, $\frac{\varepsilon_V}{\varepsilon_V^*} = \frac{1-2\nu}{1-\nu} \leq 1$. Therefore, dilatation with the plain stress condition is smaller than that with the plain strain condition.

5) In the fracture of brittle materials, voids are observed in tensile region at high temperature. It seems that dilatation is the driving force to form voids at high temperature, and that elastic dilatation influences fracture at room temperature.

The consideration given above suggests a possibility that the ratio *G/K*(=3(1-2ν)/2(1+ν), which decreases monotonously with Poisson's ratio ν) governs the failure mode, i.e., ductile vs. brittle behavior, of a given material. Table 3 summarizes the values for typical examples [9,10], and we see that this expectation in fact works. Materials which easily change volumes are brittle and materials which can be easily distorted are ductile. This tendency is independent of various classifications of materials such as metal vs. nonmetal, crystal vs. amorphous, and so on. Consequently, we can regard the ratio *G/K* (or Poisson's ratio) as a measure to predict the failure mode for a given material. It is called Pugh's rule [11]. Kelly et al. [12] pointed out the similar result that the ratio *G/E* is the index of ductile-brittle from comparison between shear and tensile stress. The ratio *G/K* and *G/E* are also the monotonous function of ν, hence both indices bring the same result. But the original viewpoint of each index is completely different.

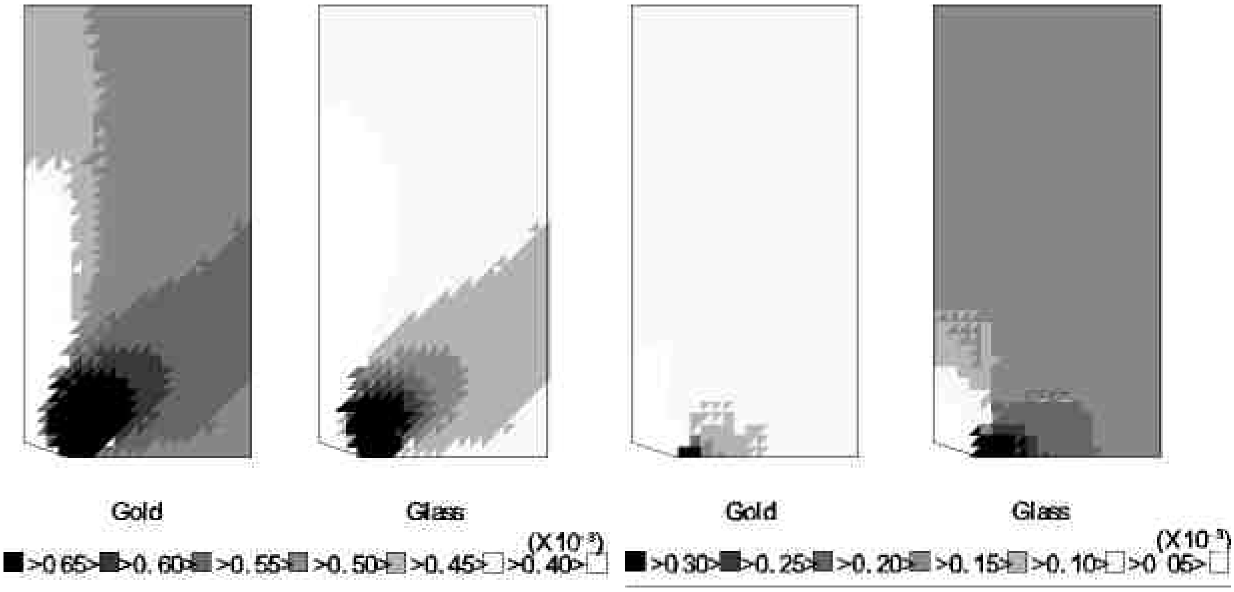
TABLE 3
CORRELATION BETWEEN *G/K* AND FAILURE MODES

Material	*E*(GPa)	*G*(GPa)	ν	*K*(GPa)	*G/K*	
Quartz (fused)	73.1	31.2	0.170	36.9	0.846	
Glass (Crown)	71.3	29.2	0.22	41.2	0.709	↑
Cast iron	152.3	60.0	0.27	109.5	0.548	Brittle
Mild steel	211.9	82.2	0.291	169.2	0.486	
$Fe_{80}B_{20}$ (amorphous)	168.7	64.9	0.30	141	0.460	
Copper	129.8	48.3	0.343	137.8	0.351	
Aluminum	70.3	26.1	0.345	75.5	0.346	Ductile
Brass (70 Zn, 30 Cu)	100.6	37.3	0.350	111.8	0.334	↓
Gold	78.0	27.0	0.44	217.0	0.124	

NUMERICAL EVALUATION OF DILATATION AND DISTORTION NEAR CRACK

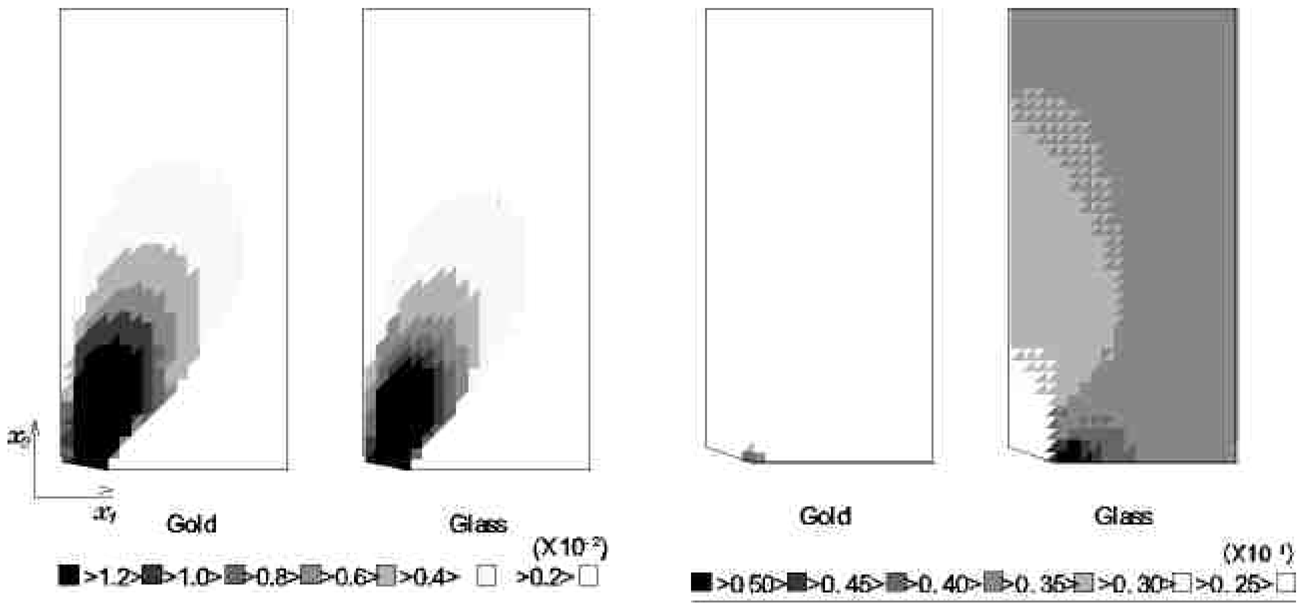
It is interesting to see how the difference in *G/K* affects the distribution of the strain and the strain energy density relevant to each of the two modes of deformation. Taking gold and crown glass as typical examples of ductile and brittle materials, we calculated the distributions by the finite element method for plates with cracks at their centers under 1% uniaxial tension with the plane strain condition. The results are given in Figs.1 and 2, in each of which a quarter of the plate is shown and the strain energy density is normalized with the total strain energy E_t stored in the entire plate. For gold, distortion energy density E_s is high along the 45 degrees direction from the crack tip over a wide region, while high values for dilatation energy density are limited to a small domain around the crack tip. On the other hand, in the case of glass, relatively large amount of strain energy is stored as the dilatation energy over a relatively wide region around the crack tip.

The distributions of distortion, represented by γ_{12} are almost the same for gold and glass because the plates are stretched by the same amount of strain (Fig.2(a)). On the other hand, distributions of dilatation, represented by ε_{ii} are very different between gold and glass (Fig.2(b)). Those numerical results suggest that materials with small *G/K* tend to fail plastically while materials with large *G/K* tend to have cleavage fracture.



(a) Distortion energy density (b) Dilatation energy density

Figure 1: Distribution of strain energy density.



(a) Distortion γ_{12} (b) Dilatation

Figure 2: Strain distribution.

CONCLUSION AND DISCUSSION

We notice the close connection between the deformation mode (distortion and dilatation) and the failure mode (plastic deformation and fracture), and that brittleness and ductility are not only material property. In fracture mechanics, we ignore close connection between dilation and fracture. In the combined mode of fracture, we might discuss the singularity of dilation instead of the stress singularity near crack tip.

Microscopically, topology of atomic array changes in distortion, and many 'meta-stable states' appear in the process. Therefore, we should evaluate the yield condition by (distortion) energy. On the other hand, the inter-atomic distance increases in dilatation, and the 'critical state' appears. Therefore, we could evaluate fracture by (dilatation) strain. Furthermore, lattice defects are also classified into two categories; one is the dislocation, which causes distortion, and the other is the vacancy, which causes dilatation. Therefore, it could be possible to establish new micromechanics from the viewpoint of the distortion and dilatation.

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