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A STIFFNESS RELEASE (FINITE ELEMENT) MODEL FOR HIGH SPEED CRACK PROPAGATION ANALYSIS

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ABSTRACT

In this paper, a finite element model for simulating dynamic crack propagation phenomenon is presented. The mesh is kept stationary and a 1-Dimensional elastic spring element is added at the crack tip node. The gradual propagation of crack is achieved by decreasing its stiffness value from a large (infinite) value to zero as the crack propagates to the next node within the element. This paper presents a model for the stiffness of the spring element and the crack tip element mass as a function of crack length. This method is applied to determine the dynamic energy release rate of DCB specimens made of glass-epoxy subjected to impact. Results show that the energy release rate gives stable and acceptable solution.

KEYWORDS

Computational Fracture mechanics, Dynamic Fracture, Composites, Impact

INTRODUCTION

An understanding of the mechanics of dynamic fracture is necessary for developing sound design methodologies. Nishioka and Atluri [1] gave elaborate information on the analysis of dynamic fracture using FEM. The most common ways to deal with the crack tip region was to simulate crack growth through gradual release of elemental nodal forces or embedding a moving element in which interpolation functions are determined by the continuum near tip fields at the crack tip in the mesh, and J-Integral consideration. Chiang [2] presented a numerical procedure based on eigen function to determine the dynamic stress intensity factor for a crack moving at steady state under anti-plane strain condition. Thesken and Gudmundson [3] worked with elasto-dynamic moving element formulation incorporating a variable order singular element to enhance the local crack tip description. Kennedy and Kim [4] incorporated micropolar elasticity theory into a plane strain finite element formulation to analyse the dynamic response of the crack. Beissel, Johnson and Popelar [5] presented an algorithm, which allows crack propagation in any direction but doesn't require remeshing or the definition of new contact surfaces. The edges of these failed elements simulate crack faces that can sustain only compressive normal traction In the present work an efficient 'Stiffness Release Model', method is proposed to simulate the crack propagation.

FORMULATION

In the present development of the stiffness release model, for Mode-I crack propagation a one-dimensional spring element is attached to the crack tip node, whose stiffness is reduced gradually as crack advances to the next node (Fig.1). The required spring stiffness is very large when crack tip is at the beginning of element and zero when crack tip reaches the other end. Amount of additional stiffness required is a function of crack length and is obtained from quasi-static crack propagation analysis. Fig. 1(a) shows a symmetric part of deformed 2-Dimensional plate in which crack is upto node B, and Fig. 1(b) shows the crack at at intermediate position between node B and node C. Fig.1(a) shows the additional spring element normal to the crack plane. Fig. 1(c) shows the crack reaching node C. Let, K_0 be the stiffness at node B of the original plate in the normal direction without the additional spring; and u_0 be the displacement of node when crack tip reaches the next node as the element opens up completely. The spring stiffness, K_s, corresponding to an intermediate displacement, u, can be found out from equilibrium conditions as

$$K_O(u_O - u) = K_S \tag{1}$$

which gives the equation,

$$K_S = K_O\left[\frac{u_O}{u} - 1\right] \tag{2}$$

Energy in the spring is given as

$$E_s = \frac{1}{2}K_s u^2 = \frac{1}{2}K_0\left[\frac{u_0}{u} - 1\right]u^2 \tag{3}$$

The energy in the system is released due to the decrease in the spring stiffness and thus, the energy release rate can be determined from

$$G = \frac{\Delta E}{\Delta A} \tag{4}$$

where ΔA is the increment in crack area. Defining a non-dimensional parameter, a, as, a = crack length (within element) / element length, d and B is the thickness of plate, the energy release rate for a double cantilever beam can be written as

$$G = \frac{\Delta E}{Bd\Delta a}$$

$$= \frac{1}{2}K_0(-u_0)\frac{1}{Bd}\frac{du}{da} \tag{5}$$

As the variation of G is linear with respect to a, it can be represented as,

$$G = C_1 + C_2 a \tag{6}$$

where C_1 and C_2 are constants. Therefore, from Eqs. (5) and (6) we get,

$$C_1 + C_2 a = -\frac{1}{2}\frac{K_o u_o}{Bd}\left(\frac{du}{da}\right) \tag{7}$$

On integrating Eq. (7) and using the conditions that at u=0 when a=0 and condition $u=u_0$ when a=1, we get

$$\frac{u}{u_o} = \frac{2C_1 a + C_2 a^2}{2C_1 + C_2} \tag{9}$$

which can be expressed in the form,

$$\frac{u}{u_0} = a + C\,(a - a^2) \tag{10}$$

Thus, the equation for the spring stiffness, (Eq. (2)), finally can be written as

$$K_s = K_o\left(\frac{1-f}{f}\right) \tag{11}$$

where $f=u/u_0$ as given by Eq. (10). Given the static stiffness, K_0, the energy release rate is determined using Eq. (5). Or, we can directly determine the dynamic energy release rate, G using the basic equation

$$G\Delta A = \Delta W_{ext} - \Delta U - \Delta T \tag{12}$$

where ΔT is the increment in kinetic energy in the body. If the crack moves with velocity v, then,

$$G = \frac{1}{Bv}\frac{d}{dt}\left(W_{ext} - U - T\right) \tag{13}$$

Constant 'C' in Eq. (10) can be determined from the quasi-static case so as to get the energy release rate curve is continuous at element boundaries. For quasi-static case this value is very small. However it was observed that C has considerable effect in dynamic crack propagation. It can be seen that this constant, C, is affected by element size and time step Δt. For a given dynamic problem, its value need to be chosen so that final energy release rate curve is smooth without any fluctuation.

It is also important to take into account the changes in the effective mass of the crack tip element. In order to take into account the mass variation, shape function N_1 of the element at the crack tip is modified such that it is a function of crack length 'a' as,

$$N_1 = \left(\frac{1-\xi}{2}\right)^{\alpha}\left(\frac{1-\eta}{2}\right) \tag{13}$$

where , ξ and η are natural co-ordinates and α is chosen as,

$$\alpha = \left(\frac{10(1.1-a)}{a} \right)^{0.9} \tag{14}$$

N_I=1 at (ξ,η) is (-1,-1) and N_I= 0 when $\xi = \eta = 1$ and α is chosen such that, for a given crack length, a, within an element N_I=1 when ξ= -1, and N_I=0 when ξ≥(2a-1). Thus, as the crack advances mass of the element increases gradually. Finally when a=1 i.e., when crack is at the end of the element, shape function N_I take its usual bilinear form. Thus, crack node mass is zero when the crack is at the beginning of the element and increases gradually as crack reaches the end of the element.

RESULTS AND DISCUSSION

The important aspects of the results are: i) Determination of the constant C (Eq. (10)) for quasi-static crack propagation; ii) crack propagation analysis at constant speed under dynamic input force pulse; and iii) Crack propagation analysis of a DCB impact experiment. Material and geometric properties of the specimen are as follows: Length of specimen, L = 0.06m; width of specimen, W = 0.005m; Thickness of specimen, B = 0.024m; Crack length, a' = 0.03m; Modulus of Elasticity, E=210x10^9 Pa; Poisson's ratio, ν = 0.3. The DCB specimen is analysed using 4-noded isoparametric element. The mesh has 600 elements of 1.0 mm X 0.5 mm. Static stiffness, K_0, was determined for the given mesh by applying force at the node preceding crack tip node and determining corresponding displacement at that node. Energy release rate variation with crack length for different values of constant C are determined and it was observed that for C= -0.028, that the energy release rate curve was smooth within the element and continuous at element boundaries.

For dynamic crack propagation, a short pulse is applied on the specimen (Fig. 2). The pulse was modelled by F(1-cos(ωt)), where frequency is taken as 125KHz with a pulse duration of 8μs, amplitude of 50N. The crack was initiated after 8μs and moves at speed of 1500m/s and. The time steps used was Δt=1.0x10^{-6}sec. The value of C= -0.028 obtained for quasi static crack propagation case, if used for dynamic case of a crack velocity of 1500m/s gives energy release rate as shown in Fig 3. It can be seen that there are large oscillations at the element boundaries which shows that C value for the quasi-static case is not suitable for the dynamic case and has to be modified. C value is modified so that the energy release rate curve becomes smoother. Effect of change in C value on energy release rate curve is shown in Fig. 4. It can be seen that C=0.4 gives rise to less oscillations in G_I in comparison to other values and it is chosen for further analysis. Fig. 5 shows effect of averaging of 'f ' on energy release rate variation leading to a better solution. The effect of modification of crack tip element mass by changing the shape function N_I on energy release rate can be seen in Fig 6. This figure reveals that energy release rate curve has smoother variation within element and fairly stable solution across element boundaries It can be seen very clearly that solution with mass modification is better than one without mass modification.

The present method is applied to determine the dynamic G_I for the impact experiment on DCB specimen made of Glass Fabric/Epoxy DCB composite [6]. Following data was used as input for finding energy release rate. Density, ρ =1825Kg/m^3; E_L=26x10^9 N/m^2, E_T=6.6x10^9N/m^2; G_{LT}=3.5x10^9N/m^2; ν_{LT}=0.21; ν_{TL}=0.053. Fig 7 give the dynamic G_I from the regular energy release rate determination. Fig. 8 shows the results by the present model. From these it can be seen that the present model is more accurate compared with the force release model.

CONCLUSIONS

A new model is proposed to investigate high speed crack propagation.

1. Model gives fairly smooth and more stable variation of energy release rate.
2. Experimental results reveal that unlike force release model energy release rate doesn't drop to a low value in just a few time steps but falls gradually which is a more practical result.
3. It can be observed that the size of the element and time step should be chosen such that the effective stiffness used in the integration scheme decreases fairly monotonically.

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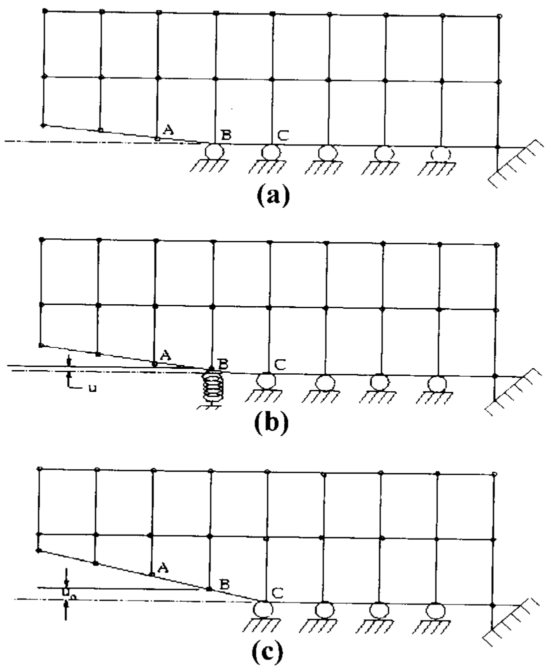

Fig. 1: Crack opening pattern in model

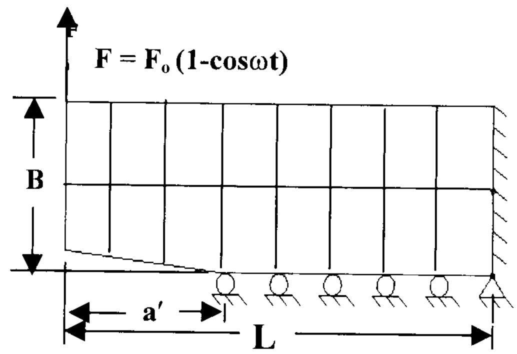

Fig. 2: DCB specimen (Half)

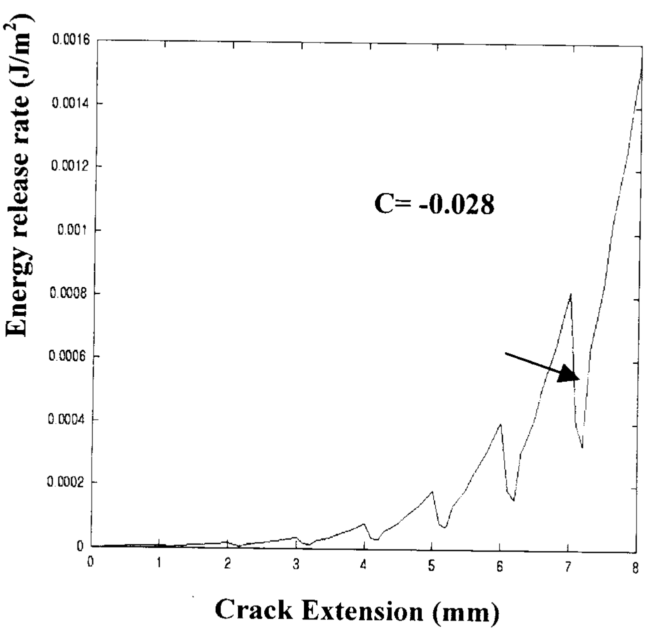

Fig. 3: Energy release rate variation for Crack-velocity 1500m/s

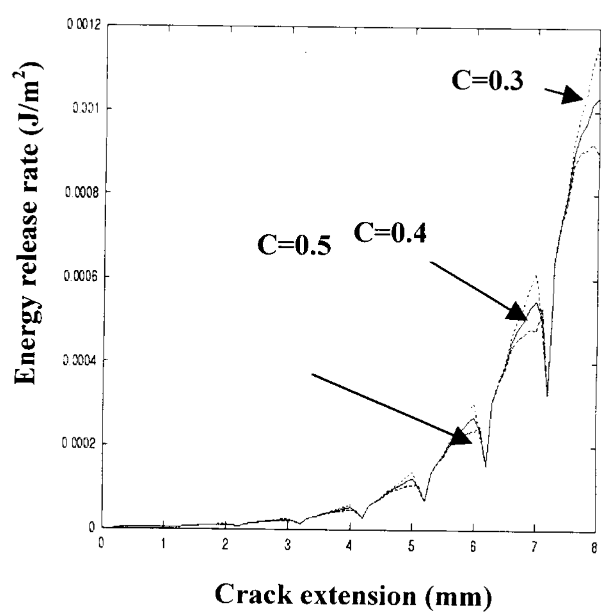

Fig. 4: Effect of change in C on energy release rate

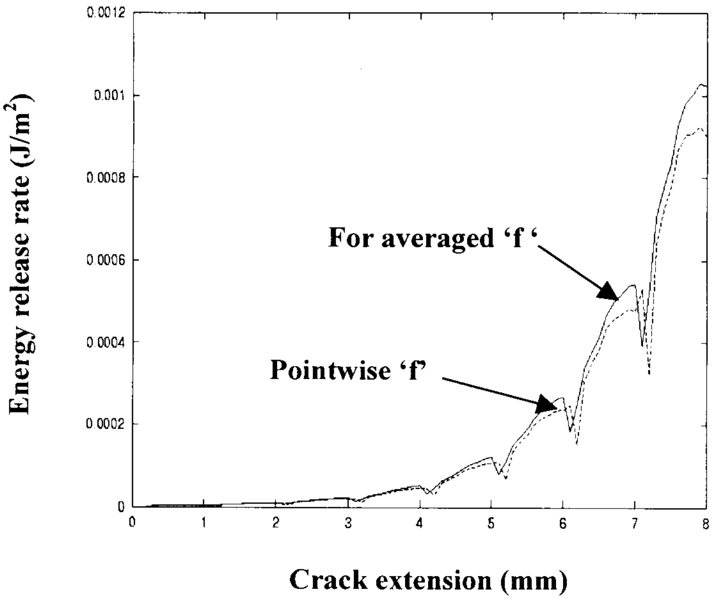

Fig. 5: Effect of averaging 'f' values on energy release rate

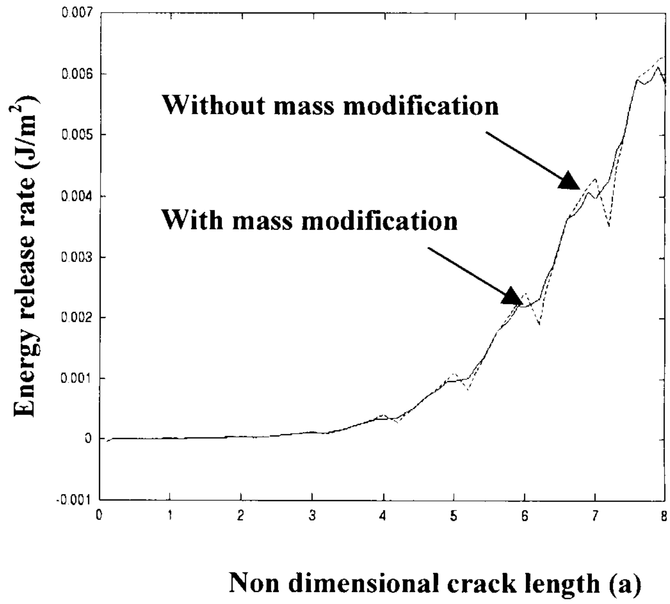

Fig. 6: Energy release rate for crack velocity 1250m/s

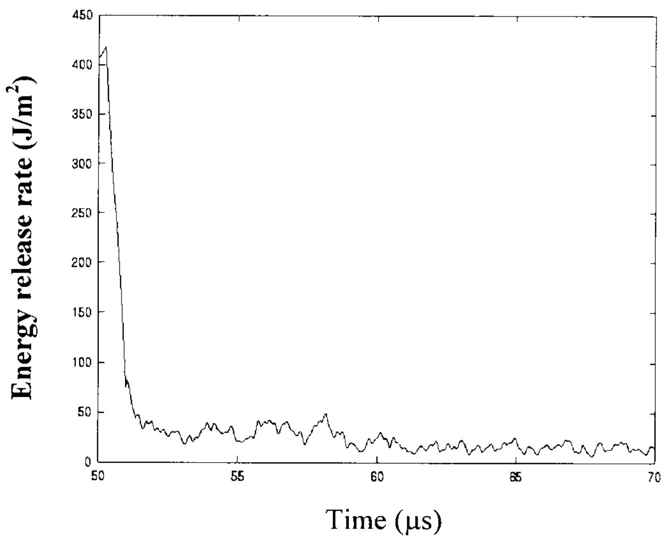

Fig. 7: Energy release rate for Expt. 1 [6] By Force release model

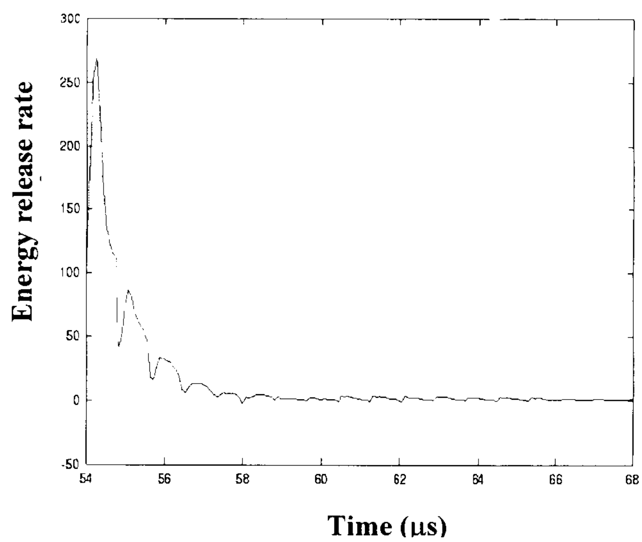

Fig. 8: Energy release rate for Expt. 1 (Present study)