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# A POSSIBLE EXPLANATION OF SIZE EFFECT IN FATIGUE STRENGTH OF METALS

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## ABSTRACT

As is well-known, the experimental fatigue strength of metallic materials tends to decrease with increasing specimen size. Several theories on size effect, such as the Weibull statistical theory, have been proposed to explain this phenomenon. In the present paper, an attempt to analyse size effect in fatigue is made by considering the fractal nature of the reacting cross section of structures, that is, the renormalized fatigue strength is assumed to be equal to a force amplitude divided by a surface with a fractal dimension lower than 2. Such a dimensional decrement depends on a self-similar weakening of the material ligament, owing to the presence of cracks, defects, voids and so on (microscopic level). However, this decrement tends to gradually disappear by increasing the structure size (macroscopic level), this phenomenon being defined as multifractality. Relevant experimental results are examined in order to assess the reliability of the theoretical analysis presented.

## KEYWORDS

Size effect; fatigue fracture phenomenon; metals; fractal geometry.

## INTRODUCTION

According to well-known experimental findings, the fatigue strength of a given material is not a constant mechanical parameter, but it decreases by increasing the specimen size. Such a decrease can be dramatic for very large structures, by provoking sudden catastrophic failures with possible heavy losses of lives and resources involved. Size effect phenomenon was analysed by Griffith [1] for the glass filaments by assuming the presence of microcracks whose size is proportional to the diameter of the filament cross section, whereas Peterson [2] examined the size effect in the case of brittle fracture produced through fatigue loading. Then Weibull [3] proposed the statistical concept of the weakest link in a chain: by increasing the structure volume, the probability of failure increases owing to the higher probability of finding a critical microcrack provoking macroscopic fracture. More recently, the size of the most dangerous defect has been shown to be proportional to the structure size [4]. From such conclusions, it can be derived that the microscopic scale (material microstructure, grain size, microcracks, voids, inclusions, etc.) is significantly connected with the macroscopic scale (structure size), that is, the "disorder" of the material (heterogeneity and/or micromechanical damage) has to be considered when examining critical macroscopic phenomena (like for example fatigue fracture failure of structures).

In the present paper, the fractal nature of the material microstructure [5,6] and the renormalization group theory [7-9] can be considered to analyse the interactions between the two above levels (micro and macro), as has been proposed in Ref.[10]. In other words, the reacting cross section of a given structure shows a self-similar weakening due to the material heterogeneity, cracks, defects, etc., and therefore the fractal dimension of such a surface can be assumed to be lower than 2 [11,12]. Consequently, the damaged ligament of a heterogeneous solid may be modelled through a "lacunar" fractal set, analogous to the mathematical middle-third Cantor set, which presents Hausdorff dimension lower than that of the domain where it is contained. Then, new mechanical properties can be defined with physical dimensions depending on the fractal dimension of the damaged heterogeneous ligament (renormalization procedure), and such properties are scale-invariant constants. According to this approach, the renormalized fatigue strength could be represented by a force amplitude acting on a surface with a fractal dimension lower than 2, as is discussed in the following.

On the other hand, Mandelbrot [13] pointed out a non-uniform (multifractal) scaling of the *natural* fractals (different from the uniform one of the *mathematical* fractals), i.e. in the physical reality a transition occurs from a fractal (heterogeneous) regime for small structures to a Euclidean (homogeneous) one for structures large enough with respect to a characteristic microstructural size. In other words, the effect of the microstructural heterogeneity and/or damage (disorder) of a given material on the macroscopic mechanical behaviour gradually vanishes by increasing the structure size [14].

A monofractal scaling law for fatigue limit of metals is herein proposed, and some experimental results [15] are analysed to show how to apply the theoretical approach adopted.

## FRACTAL NATURE OF FATIGUE FAILURE OF STRUCTURES

According to the concepts previously discussed, the reacting cross section of a disordered material is herein assumed to present a fractal dimension $\alpha = 2 - d$, with $0 \le d < 1$, where the decrement $d$ depends on a self-similar microstructural weakening (heterogeneity and/or damage) [10-12], the value of $d$ being higher when such a weakening is more significant. Let us consider two geometrically similar cylinders ($A$ and $B$), made up of the same material, subjected to cyclic axial loading (Fig.1). On the basis of the theoretical approach proposed for static loading [10], the renormalized fatigue strength $\sigma_a^*$ (the subscript $a$ standing for amplitude) may be assumed as a material constant with physical dimensions given by $[F] \,/\, [L]^{2-d}$, and the following expression can be written :

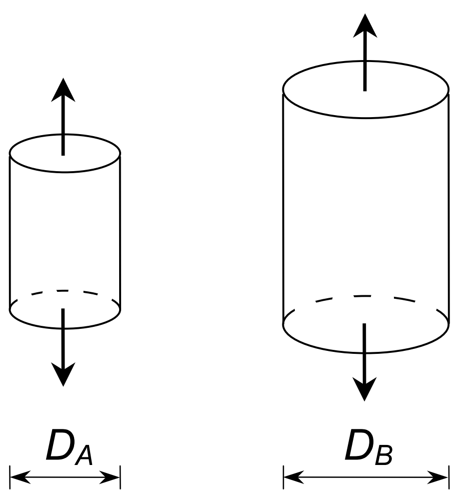


Figure 1: Geometrically similar cylinders under cyclic axial loading

$$\sigma_a^* = \frac{4\ F_{a,A}}{\pi\ D_A^{2-d}} = \frac{4\ F_{a,B}}{\pi\ D_B^{2-d}} \tag{1}$$

where $F_{a,A}$ and $F_{a,B}$ are the axial force amplitudes (acting on the two cylinders, respectively), which provoke fatigue fracture failure.

The apparent fatigue strengths for such bodies are equal to :

$$\sigma_{a,A} = \frac{4\ F_{a,A}}{\pi\ D_A^2} \tag{2}$$

$$\sigma_{a,B} = \frac{4\ F_{a,B}}{\pi\ D_B^2} \tag{3}$$

Therefore, recalling eqns (1) and (2), equation (3) becomes :

$$\sigma_{a,B} = \frac{4}{\pi\ D_B^2} \left\{ F_{a,A} \left( \frac{D_B}{D_A} \right)^{2-d} \right\} = \sigma_{a,A} \left( \frac{D_B}{D_A} \right)^{-d} \tag{4a}$$

and in a logarithmic form :

$$\ln \sigma_{a,B} = \ln \sigma_{a,A} - d\ \ln\left(D_B / D_A\right) \tag{4b}$$

By assuming $D_A = 1$ and $D_B = D$, where $D$ is a generic value of the bar diameter, the last two expressions can be written in a more general form :

$$\sigma_a = \sigma_{a,1}\ (D)^{-d} \tag{5a}$$

$$\ln \sigma_a = \ln \sigma_{a,1} - d\ \ln D \tag{5b}$$

where the latter equation represents a straight line with slope equal to $-d$ in the diagram shown in Fig.2, $\sigma_{a,1}$ being the fatigue strength for a cylinder with $D = D_A = 1$.

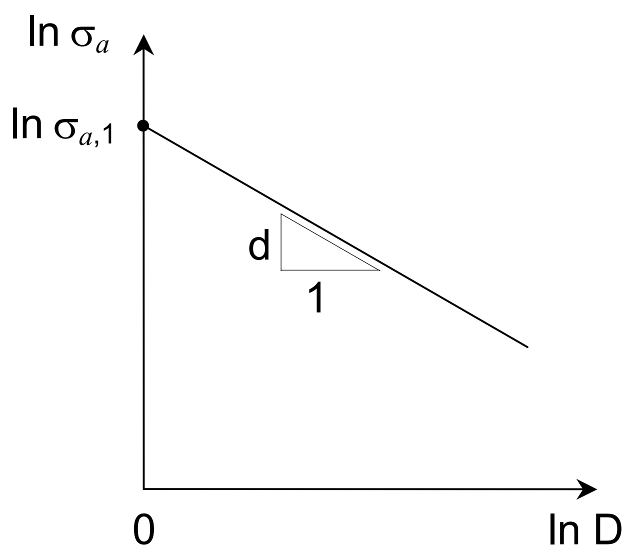


Figure 2: Monofractal scaling law for fatigue strength $\sigma_a$

In the case of $\sigma_a = \sigma_{af}$, where $\sigma_{af}$ is the fatigue limit, equations (5) become :

$$\sigma_{af} = \sigma_{af,1} \; (D)^{-d} \tag{6a}$$

$$\ln \sigma_{af} = \ln \sigma_{af,1} - d \; \ln D \tag{6b}$$

with $\sigma_{af,1}$ equal to the fatigue limit for $D = D_A = 1$. Note that, through a reasoning similar to that described above, equations analogous to those for push-pull loading (eqns (1) to (6)) can be obtained in the case of rotary bending.

## ANALYSIS OF SOME EXPERIMENTAL RESULTS

Now some experimental results are examined to show how to apply the above equations. As is well-known, several aspects (material properties, manufacturing process, specimen shape, testing conditions) play a role in determining the amount of fatigue limit decrease by increasing the structural size, but the analysis of the specific influence of each aspect is beyond the scope of the present paper.

Hatanaka et al. [15] performed fatigue tests on smooth specimens made up of two different materials : a cast steel (JIS SCMn 2A) originally including many defects (comparatively disordered material), and a forged steel (JIS SF 50) with a quite homogeneous microstructure (comparatively ordered material). The mechanical properties of these two types of steel are shown in Table 1.

TABLE 1 - Mechanical properties of two steels tested by Hatanaka et al. [15]

| *Material* | *Yield stress (MPa)* | *Ultimate tensile strength (MPa)* | *Elongation (%)* |
|---|---|---|---|
| SCMn 2A | 325 | 576 | 18.2 |
| SF 50 | 283 | 484 | 39.1 |

Cylindrical smooth specimens with diameter $D$ equal to 8, 20, 30 and 40 mm, respectively, were employed. The S-N curves for the two steels tested under rotating bending are shown in Ref.[15]. Note that, for both materials, the fatigue strength decreases by increasing the specimen size. In particular, the amount of decrease in the value of fatigue limit $\sigma_{af}$ by increasing $D$ from 8 to 40 mm is equal to about 24% for SCMn 2A steel and about 13% for SF 50 steel.

If experimental results of $\sigma_{af}$ against $D$ reported in Ref.[15] are plotted in a bilogarithmic diagram (Fig.3), two straight lines can be determined through the least squares method : the straight line slope, $-d$ (see eqn(6b)), for the cast steel is equal to – 0.162, whereas that for the forged steel (dashed line) is equal to – 0.085. Consequently, the reacting cross section presents a fractal dimension $\alpha = 2 - d$ equal to 1.838 and 1.915, respectively: in other words, the ligament for an ordered material is more similar to a two-dimensional Euclidean surface than that for a disorder material. Such a conclusion is consistent with the concepts discussed in the previous section. Furthermore $\ln \sigma_{af,1}$, defined in eqn(6b), is equal to 5.76647 for SCMn 2A steel and 5.63681 for SF 50 steel, that is, $\sigma_{af,1}$ is equal to 319.4 MPa and 280.6 MPa, respectively.

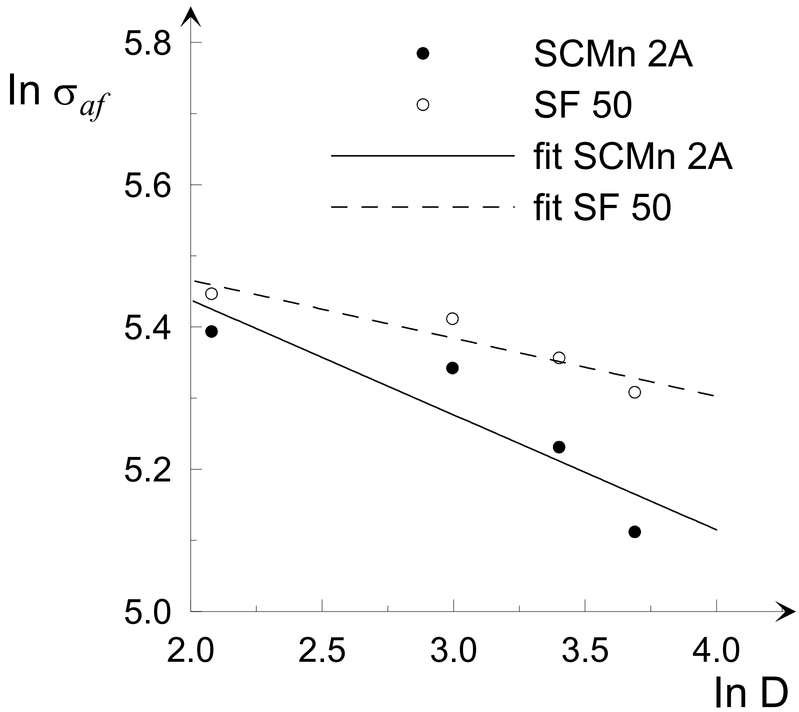


Figure 3: Monofractal scaling law for fatigue limit $\sigma_{af}$ of two steels tested by Hatanaka et al. [15]

Note that the experimental points in Fig.3 are not perfectly aligned (the correlation coefficient is equal to 0.906 for the cast steel and 0.937 for the forged steel), which could mean that the monofractal scaling of $\sigma_{af}$ is valid only in a narrow size range where the fractal dimension $\alpha$ is about constant. In other words, a non-uniform (multifractal) scaling of $\sigma_{af}$ may be assumed, with a gradual decrease of $d$ as the scale $D$ increases. As a matter of fact, the material microstructure is independent of the macroscopic scale of the specimens tested; consequently, the influence of the microstructural disorder (heterogeneity and/or damage) on fatigue behaviour may progressively diminish by increasing the specimen size, and may become practically negligible for cylinder sizes large enough with respect to a characteristic microstructural size.

## CONCLUSIONS

Experimental tensile strength and fatigue strength decrease by increasing the specimen size, and this decrease is more pronounced for comparatively heterogeneous and/or damaged materials, i.e. the so-called “disordered” materials.

The problem of size effect in fatigue has been herein analysed through fractal geometry concepts, by assuming a self-similar weakening of the reacting cross section of structures, due to the material disorder (microscopic level). A monofractal scaling law for fatigue limit $\sigma_{af}$ has been proposed. The fatigue strength decrease may gradually tend to disappear by increasing the structure size $D$ (macroscopic level) with respect to a characteristic microstructural size.

Experimental fatigue data related to two different steels have been examined to discuss the theoretical approach adopted. Such an approach seems to be a possible alternative method to analyse the size effect problem in fatigue fracture failure of structures.

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