A NEW NONLOCAL FRACTURE CRITERION

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ABSTRACT

The paper considers the problem of strength of a body containing an artificial flaw of a definite size and shape. The following questions are formulated: What is the range of allowable dimensions of a flaw of a given shape, which will not lead to the reduction in the strength of a body? How much will be the strength reduction in case when the flaw dimensions exceed the allowable ones? The known nonlocal fracture criteria such as the average stress criterion, the point stress criterion and the fictitious crack criterion can not be used for solving the linked problem of critical loading and critical size of a flaw stated above. To solve this problem the approach is suggested according to which the strength of a material in the stress concentration zone (local strength) is assumed to be dependent on its size. The corresponding fracture criterion is proposed. It is applied to estimating the tensile strength of composite laminates weakened by a single circular hole; the tensile strength of high strength steel bars with a circumferential notch and the tensile strength of polymethylmethacrylate plates with an angled elliptic hole. The expressions for the local-strength function and the failure stress are obtained and good agreement is found between the results of calculations and known experimental data.

KEYWORDS

Strength, brittle fracture, nonlocal criteria, stress concentration, size effect.

INTRODUCTION

The traditional approach to strength calculations is to compare the internal stresses, which occur in a loaded body with their limiting values. The strength condition has the form $\sigma_e < \sigma_0$, and failure occurs when

$$\sigma_e = \sigma_0, \tag{1}$$

where $\sigma_e = f(\sigma_{ij})$ and $\sigma_0 = \text{const}$. The equivalent stress σ_e characterizes the internal stress state of the body and is a function of the stress-tensor components σ_{ij} in the general case. The ultimate stress σ_0 characterizes the average mechanical properties of the body's field and it is assumed to be a material constant. So σ_0 is determined under conditions of the uniform stress state (for example, in uniaxial tension of unnotched specimen). In the traditional approach, strength of a solid in a given point is characterized by the value of equivalent stress in the same point without consideration of the stress state in neighboring points. This is the essence of so-called local strength conditions and corresponding local fracture criteria. They give a good description of experimental data when macro-stress variations are small enough on dimensions of the order of the material structure scale. In other words, the range of application of the traditional approach is restricted to the cases where the dimension of the stress-uniformity zone is quite large to consider that $\sigma_0 = \text{const}$.

The nonlocal strength conditions and fracture criteria have recently been developed intensively [1–5]. The general approaches have been elaborated and the particular problems of strength of a body containing a stress concentrator have been considered. The general feature of nonlocal fracture criteria consists in the introduction of the characteristic length into the function of equivalent stress. That allows to describe the size effect on the strength of a body with a stress concentrator. The ultimate stress is assumed to be a material constant in nonlocal criteria as well as in traditional ones.

As a whole, the nonlocal criteria describe well the fracture initiation in the stress concentration zones. However, in some cases, their use gives rise to contradictory results. In particular, any small flaw located in a body gives rise to strength reduction according to the nonlocal fracture criteria. It is contrary to the modern knowledge about the real solid containing the pre-existing flaws inherent to it. Because of the inherent flaws existence, the small artificial flaws of the size comparable with the size of the inherent ones don't affect on the strength of a body until they rich a definite (critical) size [6–8].

PROBLEM STATEMENT

Consider a linearly elastic body of a brittle material containing an artificial flaw of a definite size and shape subjected to uniform loading. The following questions are formulated: What is the range of allowable dimensions of a flaw of a given shape, which will not lead to the reduction in the strength of a body? How much will be the strength reduction in case when the flaw dimensions exceed the allowable ones?

FRACTURE CRITERION

To solve this problem the approach is suggested [9], the essence of which is to assign the mechanical properties to a certain stressed region of finite dimensions rather than to the material as such, in contrast to the traditional and known nonlocal approaches. This means, in particular, that the strength of a material in the stress concentration zone (local strength) depends on its size.

The size of the highly stressed region is denoted by L_e; if it is quite large compared to the dimensions of the microstructural components of the material, including the inherent flaws, i.e., the conditions of averaging of the mechanical properties are satisfied, the value of the local strength differs little from σ_0. On the contrary, if L_e is comparable with the dimensions of the microstructural components, their influence on the local strength becomes noticeable. This influence is the stronger, the smaller the size L_e relative to the characteristic length of the material L_0. Thus, the local strength of the material should depend not only on the size of the highly stressed region L_e but also on the ratio L_0 / L_e. The fracture criterion can be stated as follows: The failure of a macroelement at the notch root is governed by the size of a highly stressed region to characteristic length of a material ratio. With allowance for this, we write the fracture criterion

$$\sigma_e = f(\sigma_0, L_0 / L_e). \tag{2}$$

Consider a tensile loaded body containing a smooth symmetrical stress concentrator as a basic problem for determining the local-strength function $f(\sigma_0, L_0 / L_e)$. Stress concentrator becomes a crack when $K_t \to \infty$ (K_t is the stress concentration factor). Asymptotic analysis of the critical (failure) stress $\sigma_c = f(\sigma_0, L_0 / L_e) / K_t$ behavior results in follows requirements:

$$\sigma_c = \sigma_0, \quad \text{for } K_t = 1; \tag{3}$$

$$\sigma_c \to \text{const} > 0, \quad \text{for } K_t \to \infty. \tag{4}$$

The requirement (3) ensures the transition of the nonlocal to the traditional criterion in the case of the uniform stress state. The requirement (4) ensures the relation between the nonlocal criterion and linear elastic fracture mechanics (LEFM). A constant in expression (4) depends on the cracking resistance of a material and the crack size and shape. We present the critical size of the flaw l_c in the form

$$l_c = l_0\left(1+\frac{\beta}{K_t}\right), \quad \beta \geq 0, \tag{5}$$

where l_0 is the critical size of the crack and β is a numerical parameter. The physically consistent values of β lie in the domain $\beta \geq 0$.

Since the local stress distribution in considered problem depends on the curvature radius of the concentrator to a large extent than on other geometrical parameters; therefore, in the first approximation, one can use the curvature radius of the concentrator ρ at a dangerous point to estimate L_e. For estimation of L_0, the critical size of the flaw l_c is used. We present the function $f(\sigma_0, L_0 / L_e)$ in the form

$$f(\sigma_0, L_0/L_e) = \sigma_0 f(l_c/\rho). \tag{6}$$

Bearing in mind that the stress concentration factor is an increasing function of l/ρ (l is the size of the concentrator)

$$K_t = f_t(l/\rho), \tag{7}$$

it is easy to see that it suffices to use the function f_t as $f\ (l_c/\rho)$ to satisfy the requirements (3) and (4):

$$f(l_c/\rho) = f_t(l_c/\rho). \tag{8}$$

The function given by Eqn. (8) is unique because $\sigma_c = \sigma_0$ for $l = l_c$, for any ρ. Thus, with allowance for Eqns. (6) and (8), the nonlocal fracture criterion takes the form

$$\sigma_e = \sigma_0 f_t(l_c/\rho). \tag{9}$$

Therefore, the critical stress is determined by the expression $\sigma_c = \sigma_0 \dfrac{f_t(l_c/\rho)}{f_t(l/\rho)}$, and the ratio $\dfrac{f_t(l/\rho)}{f_t(l_c/\rho)}$ can be regarded as an effective stress concentration factor.

EXAMPLES OF FRACTURE CRITERION APPLICATION

A plate with an elliptic hole under tension

The stress concentration factor can be presented in the form [10]

$$K_t = 1 + \sqrt{\alpha l/\rho}, \tag{10}$$

where α is a numerical coefficient which depends on the elastic constants of a material and the dimensions of a plate. For an infinite isotropic plate $\alpha = 2$ [10] and for an infinite orthotropic plate $\alpha = \sqrt{E_1/E_2} - \nu_1 + E_1/(2G)$ [11], where E_1, E_2, ν_1 and G are the elastic constants. The local-strength function $f(\sigma_0, L_0/L_e) = \sigma_0\left(1+\sqrt{\alpha l_c/\rho}\right)$. The critical stress has the form

$$\sigma_c = \sigma_0 \frac{1+\sqrt{\alpha l_c/\rho}}{1+\sqrt{\alpha l/\rho}}, \quad \text{for } l > l_c; \tag{11}$$

$$\sigma_c = \sigma_0, \quad \text{for } l \le l_c. \tag{12}$$

The critical size l_c

$$l_c = \frac{2K_c^2}{\pi\sigma_0^2}\left(1+\frac{\beta}{1+\sqrt{\alpha l/\rho}}\right), \quad \beta \ge 0, \tag{13}$$

where K_c is the critical stress intensity factor. To obtain the lower limit for σ_c or l_c that would define the margin of safety, the parameter β should be taken equal to zero. If K_c is unknown then l_c is found experimentally. With allowance for Eqn. (10) we can write Eqn. (11) in the form

$$\sigma_c = \sigma_0\left(\frac{1}{K_t}+\sqrt{\frac{l_c}{l}}\left(1-\frac{1}{K_t}\right)\right). \tag{14}$$

Eqns. (10)−(14) are also applicable to concentrators of non-elliptic shape, for which one can introduce the notion of equivalent elliptic hole or equivalent elliptic notch [10]. The latter concerns both flat and cylindrical specimens with a circumferential notch, including a V-shaped notch with a small opening angle.

An isotropic plate with angled elliptic hole under tension

Consider an isotropic plate with an elliptic hole, which is oriented at an angle ω to the direction of loading. The local-strength function for the basic problem in symmetric tension $\left(\omega = \frac{\pi}{2}\right)$ has the form

$$f(\sigma_0, L_0/L_e) = \sigma_0\left(1+\sqrt{2l_c/\rho}\right). \tag{15}$$

The critical stress is determined by the expression

$$\sigma_c = \min\left\{\frac{f(\sigma_0, L_0/L_e)}{\sigma_e/\sigma}\right\} > 0, \tag{16}$$

where σ is the tensile stress applied to the plate. We assume that failure determined by normal tensile stresses, i.e., $\sigma_e = \sigma_\theta > 0$ (σ_θ is the tangential stress on the hole boundary). The problem of σ_c determination is to find the minimum

$$\sigma_c = \min\left\{\sigma_0 \frac{1+m^2-2m\cos 2\theta+\sqrt{2l_c(1-m)/a}\,(1+m)(1+m^2-2m\cos 2\theta)^{1/4}}{1-m^2+2m\cos 2\omega-2\cos(2\theta-2\omega)}\right\} > 0. \tag{17}$$

Here the well-known expression for the stress σ_θ on the boundary of an elliptic hole [12] and the expression for the curvature radius of the hole boundary $\rho_\theta = a\frac{(1+m^2-2m\cos 2\theta)^{3/2}}{(1+m)^2(1-m)}$, where $m = \frac{a-b}{a+b}$; a and b are the major and minor semiaxes of the ellipse; θ is the varied parameter, were used.

COMPARISON BETWEEN PREDICTED AND EXPERIMENTAL DATA

Eqns. (14) and (17) for the critical stress, which were obtained on the basis of the nonlocal fracture criterion (Eqn. (9)), were used to estimate the strength of a plates with a circular or elliptic hole and bars with a circumferential notch subjected to uniaxial tension. The results of calculations are shown in Figs. 1–3.

A plate with a circular hole

Hyakutake, Hagio and Nisitani [8] tested quasi-isotropic FRP plates containing a circular hole of a different diameter. The critical stress variation with respect to the hole diameter given in Eqn. (14) is plotted in Fig. 1 (the solid curve) and compared with experimental data (points). The critical size (diameter) $l_c = 0.7$ mm was evaluated from best-fitting data. The dashed line was obtained with the use of the traditional criterion.

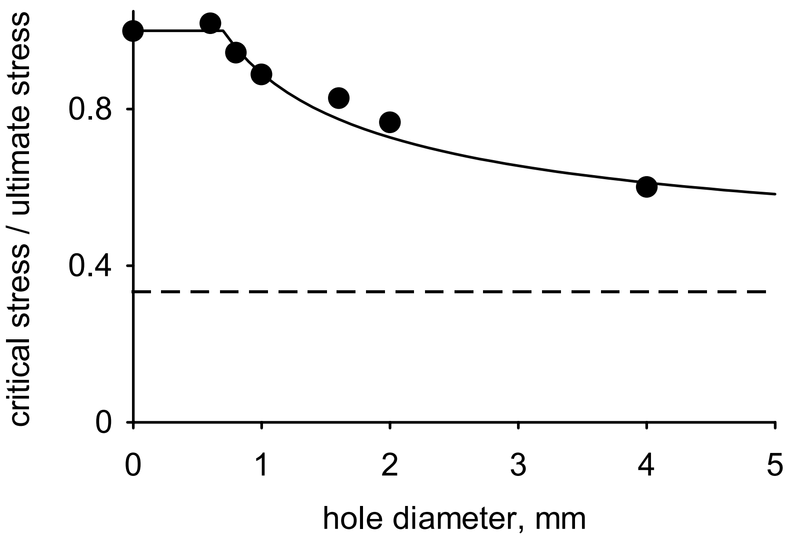

Figure 1: Critical stress variation with hole diameter.

A plate with angled elliptic hole

Wu, Yao and Yip [13] tested PMMA plates 380 mm long, 152 mm wide and 3.2 mm thick. The semiaxes of the elliptic hole were $a = 12.7$ mm and $b = 2.5$ mm. The failure stress for varying ω was experimentally determined. Fig. 2 shows experimental data (points) and the critical stresses calculated by Eqn. (17) for $\beta = 0$ (the solid curve). The dotted curve is calculated according to the traditional criterion.

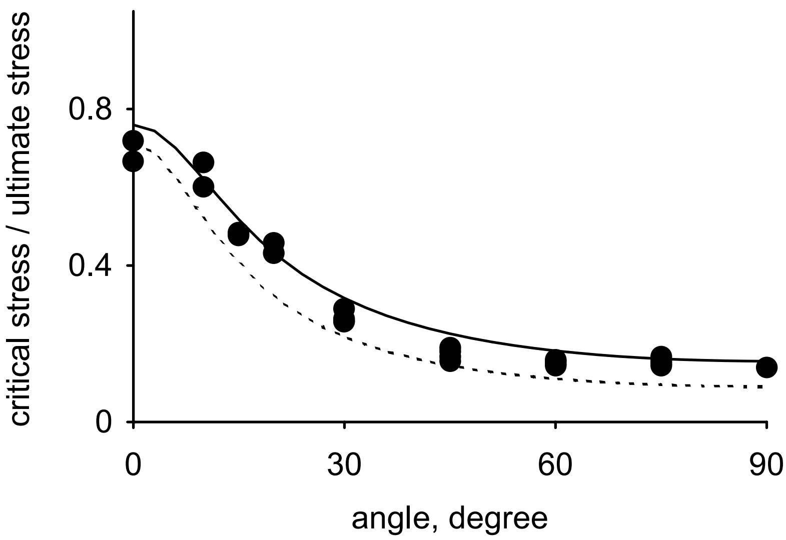

Figure 2: Critical stress variation with angle ω.

A bar with a circumferential notch
Nisitani and Noguchi [14] tested cylindrical bars made of high strength steel. The specimens had a circumferential V-shaped notch with opening angle $\psi = 60°$ and radius of curving ρ at the notch root. Specimens with notch depth $a = 0.2$ mm were tested by varying ρ within 0.056–2.1 mm. Fig. 3 shows the values of σ_c calculated by Eqn. (14), as a function of the stress concentration factor for $\beta = 0$ and $\beta = 1$ (curves 1 and 2). Curve 1 limits from below the domain of σ_c, and curve 2 approximates the experimental data represented by the points. As $K_t \to \infty$, the calculated curves approach asymptotically the value found in accordance with LEFM (dashed straight line). The dotted curve is calculated according to the traditional criterion.

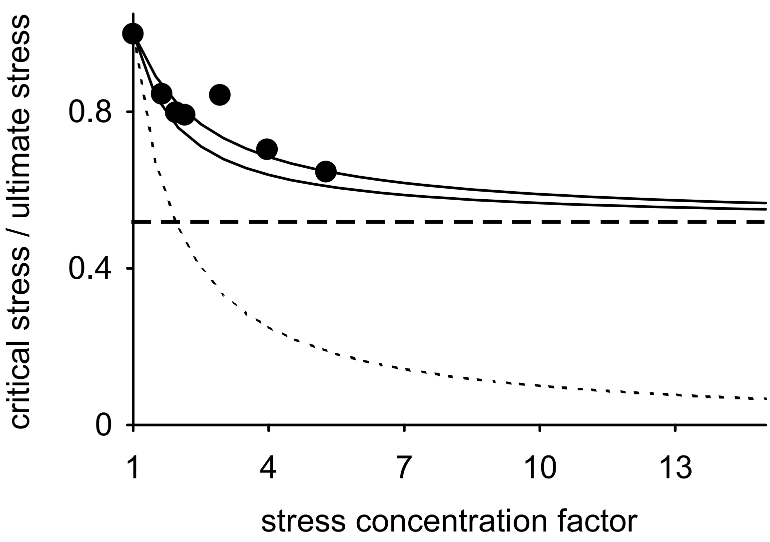

Figure 3: Critical stress variation with stress concentration factor.

The results of calculations are in good agreement with the experimental data on brittle fracture under conditions of stress concentration.

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