

# A $J$ ESTIMATION SCHEME AND ITS APPLICATION TO LOW CYCLE FATIGUE CRACK GROWTH

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## ABSTRACT

A  $J$  estimation scheme is developed that combines the Electric Power Research Institute (EPRI) scheme with the Reference Stress (RS) approach. The hybrid EPRI-RS scheme is validated against elastic-plastic finite element results for small cracks at notches. Rules are given for converting  $J$  to an effective cyclic change in  $J$ ,  $\mathbf{DJ}_{eff}$ , for application to fatigue crack growth (FCG) under low cycle fatigue (LCF) conditions where cyclic plasticity may occur. The  $\mathbf{DJ}_{eff}$  formulation includes the effects of crack closure. It is shown how the scheme can be modified to treat strain-controlled loading situations. The hybrid scheme is validated against laboratory specimen LCF tests and the results of full-scale fatigue tests on mechanically damaged pipes containing notches to simulate gouges.

## KEYWORDS

$J$  estimation scheme, reference stress, cracks at notches, constant cyclic strain, low cycle fatigue crack growth, crack closure

## INTRODUCTION

Over the last decade, part of the fracture mechanics work at Southwest Research Institute (SwRI) has focused on developing practical  $J$  and  $\mathbf{DJ}$  estimation schemes for use in the assessment of elastic-plastic fatigue crack growth (EPFCG) under low cycle fatigue (LCF) conditions involving cyclic plasticity. The driving force for this research has come from a number of different sources, but mainly because it is now widely recognized that FCG approaches based on linear elastic fracture mechanics (LEFM) are often non-conservative when applied to LCF situations. This has led to SwRI's involvement in providing practical solutions to a number of challenging industrial problems. For example, structural integrity issues in advanced space propulsion systems that experience a wide range of severe operating conditions [1], the enhancement of the LEFM based

FCG computer code, NASGRO [2], developed by Forman et al., [3] for NASA, and remaining life assessments of mechanically damaged gas transmission pipelines [4]. In addition to these, the developed methodology is finding direct applications in other industrial areas involving LCF of structures, such as the assessment of the effects of pipe reeling and straightening during the installation of offshore pipelines, and start-up and shutdown of industrial gas turbine engines.

Under small-scale yielding (SSY) conditions the  $\mathbf{DJ}$  methodology reduces to LEFM approaches based on  $\mathbf{DK}$ , the cyclic change in the stress intensity factor,  $K$ . The extension of the elastic-plastic fracture mechanics parameter,  $J$ , to EPFCG based on  $\Delta J$  was pioneered by Dowling [5,6]. The methodology presented herein employs a closure-corrected modification to  $\mathbf{DJ}$  designated as  $\mathbf{DJ}_{eff}$ , developed from the work of Newman [7]. The present paper briefly reviews recent efforts by SwRI in developing and validating  $\mathbf{DJ}_{eff}$  estimation schemes for LCF applications involving cyclic loading of cracks at notches and cracks subjected to constant cyclic strains.

## HYBRID EPRI-RS J ESTIMATION SCHEME

The proposed  $J$  scheme combines the EPRI approach [8] with the RS approach [9] and is herein referred to as the hybrid EPRI-RS method.  $J$  is resolved into elastic and plastic components,  $J_e$  and  $J_p$ , respectively,

$$J = J_e + J_p = \frac{K^2(a_e)}{E'} + \mathbf{m}V J_e(a) \left[ \frac{E \mathbf{e}_{ref}^p}{\mathbf{s}_{ref}} \right], \mathbf{s}_{ref} = \frac{P}{P_o} \mathbf{s}_o, \mathbf{e}_{ref}^p = \mathbf{a} \frac{\mathbf{s}_o}{E} \left( \frac{P}{P_o} \right)^n \quad (1)$$

Here  $E' = E$ , Young's modulus, for plane stress and  $E/(1-\mathbf{n}_e^2)$  for plane strain, where  $\mathbf{n}_e$  is Poisson's ratio.  $P$  is the applied load,  $P_o$  is the plastic limit load and  $V$  is a dimensionless engineering parameter and  $\mathbf{m} = 1$  for plane stress,  $(1-\mathbf{n}_p^2)/(1-\mathbf{n}_e^2)$  for plane strain, where  $\mathbf{n}_p$  is the plastic Poisson's ratio. The strain  $\mathbf{e}_{ref}^p$  is the plastic component of the reference strain corresponding to the reference stress,  $\mathbf{s}_{ref}$ , on the stress-strain curve. The effective crack depth,  $a_e = a + \mathbf{f}r_y$ , depends on a plastic zone size  $r_y$  and a load-dependent parameter  $\mathbf{f}$  that are defined

below for a Ramberg-Osgood material where  $\frac{\mathbf{e}}{\mathbf{e}_o} = \frac{\mathbf{s}}{\mathbf{s}_o} + \mathbf{a} \left( \frac{\mathbf{s}}{\mathbf{s}_o} \right)^n$  and  $\mathbf{a}$ ,  $\mathbf{s}_o$  and  $n$  are material constants and  $\mathbf{e}_o = \mathbf{s}_o/E$ .

$$\mathbf{f} = \frac{1}{1 + \left( \frac{P}{P_o} \right)^2}, r_y = \frac{1}{\mathbf{b}\mathbf{p}} \left[ \frac{n-1}{n+1} \right] \left( \frac{K}{\mathbf{s}_o} \right)^2 \quad (2)$$

In this equation,  $\mathbf{b}$  equals 2 for plane stress and 6 for plane strain.

The limit load  $P_o$  can be obtained from the EPRI handbooks of  $J$  solutions [10] or the review of solutions performed by Miller [11]. An optimization procedure proposed in [12] used 189 finite element analysis (FEA) solutions for  $J_p$  covering an extensive assortment of structures, crack shapes and sizes, and applied load types to determine values for  $P_o$  and  $V$  that gave the best fit between Eqn. 1 and the FEA results. This scheme enables Eqn. 1 to be generalized to arbitrary stress-strain behaviors. The mean value of  $V$  was 1.169, which is close to the value of 1 generally assumed in

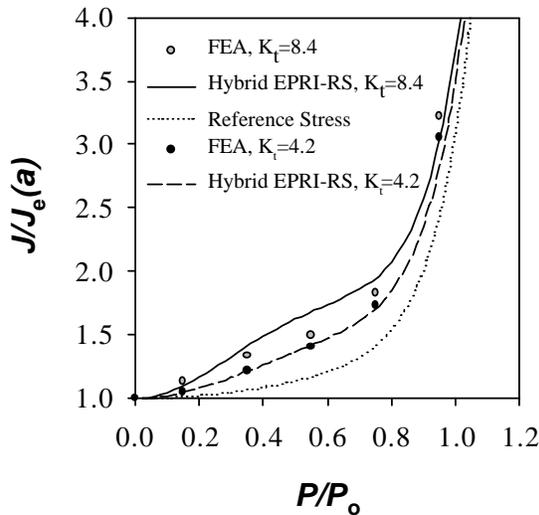
the RS approach. The effectiveness of the optimization scheme in reproducing FEA generated  $J_p$  values for a range of crack shapes and sizes is demonstrated by the results shown in [13].

## SMALL CRACKS AT NOTCHES

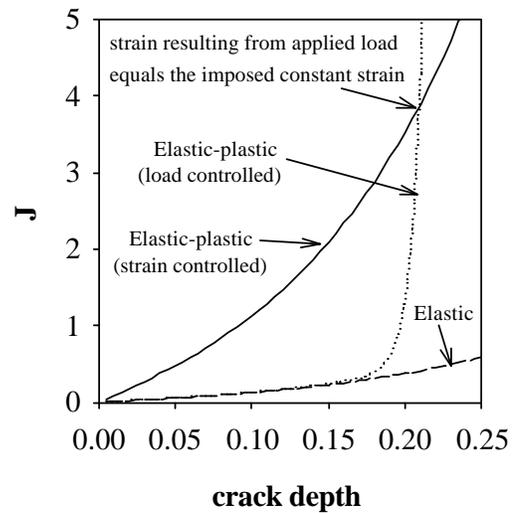
The hybrid EPRI-RS scheme captures in  $J_e(a_e)$  first-order crack-tip plasticity effects that govern the transition from LEFM to fully-plastic behavior while providing a widely applicable scheme through the RS expression for  $J_p$ . This capability is shown in the following example calculations of  $J$  for small cracks at notches. The double edge notched tension (DENT) problems analyzed are described in detail in [14]. Plane stress FEA were performed for notches of constant depth  $d=0.3b$  (where  $b$  is half the width of the plate) with various root radii,  $r$ . A wide range of  $J$  solutions were generated for various  $a/r$  and  $d/r$  values and for strain hardening exponents,  $n$ , of 5, 10, and 15. The FEA computations and the hybrid EPRI-RS solutions for  $J$  are displayed in Figure 1 for  $a/r$  ratios of 0.1195 and 0.115 and  $d/r$  ratios of 2.39 and 11.625 corresponding to elastic stress concentration factors (notch stress/remote stress) of 4.2 and 8.4, respectively. In the figure  $J/J_e(a)$  is plotted against the normalized load,  $P/P_o$ . It is clear that the hybrid approach captures the influence of the high stresses near the notch root, but that the RS approach fails to do this. This point has been made previously by Smith [15].

## STRAIN CONTROLLED LOADING

Under some circumstances, cracks may be subjected to constant strain LCF where the elastic strain is small compared to the plastic strain and the maximum load in the cycle decreases as a crack grows so that the term  $E\epsilon_{ref}^p / \sigma_{ref} = \Psi$  remains constant during growth, and  $J_p = mW J_e(a)\Psi$ . Examples are constant-strain LCF tests and the reeling and straightening of pipes as they are reeled on and off a large diameter spool as part of the pre-installation and at sea installation processes, respectively. The difference between  $J$  evaluated under constant load and strain conditions is illustrated in Figure 2.



**Figure 1:** Comparison of  $J$  solutions for small cracks at notches.



**Figure 2:** Schematic comparison of strain and load controlled  $J$  values.

## RULES FOR DETERMINING $\mathbf{DJ}_{EFF}$ FROM $J$ SOLUTIONS

The closure-corrected EPFCG parameter,  $\mathbf{DJ}_{eff}$ , can be derived from  $J$  by employing a set of relatively simple rules, as shown below.

- (1) Convert the monotonic  $\mathbf{s-e}$  curve to the hysteresis  $\mathbf{Ds-De}$  curve. For example, the Ramberg-Osgood equation becomes  $\frac{\Delta \mathbf{e}}{2\mathbf{e}_0} = \frac{\Delta \mathbf{s}}{2\mathbf{s}_0} + \mathbf{a} \left( \frac{\Delta \mathbf{s}}{2\mathbf{s}_0} \right)^n$ .
- (2) Convert the LEFM based FCG equation to a  $\mathbf{DJ}_{eff}$  based equation which, in the case of the Paris equation, will take the form  $\frac{da}{dN} = C (\Delta J_{eff})^m$ . (The Paris constants  $C$  and  $m$  can be estimated from LEFM FCG data [2]).
- (3) Replace  $J_e(a_e)$  by  $\Delta J_{e,eff}(a_e^\Delta)$ , where

$$\Delta J_{e,eff}(a_e^\Delta) = \frac{[U\Delta K(a_e^\Delta)]^2}{E'}, \quad a_e^\Delta = a + \Delta f^\Delta \Delta r_y^\Delta, \quad \Delta f^\Delta = \frac{1}{1 + \left( \frac{\Delta P}{2P_o(a)} \right)^2}, \quad \Delta r_y^\Delta = \frac{1}{bp} \cdot \frac{n-1}{n+1} \left( \frac{U\Delta K}{2s_o} \right)^2$$

The closure parameter,  $U$ , is defined as

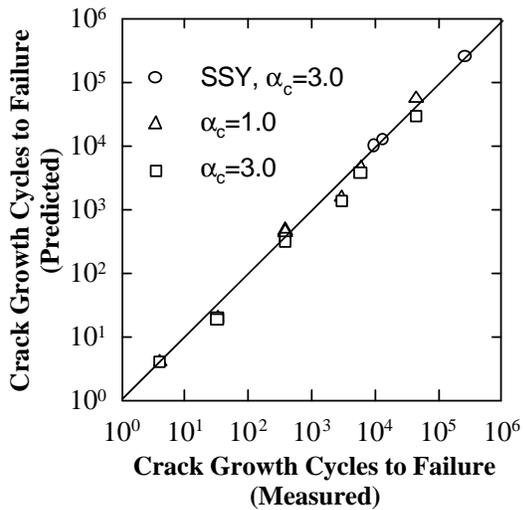
$$U = \frac{1 - K_{open} / K_{max}}{1 - R} \quad R = \frac{K_{min}}{K_{max}}$$

where  $K_{open}$  is evaluated at the point the crack opens, and  $K_{min}$  and  $K_{max}$  are evaluated at the minimum and maximum loads in the cycle, respectively. A detailed expression for  $U$  derived from the work of Newman [7] is given in [14].

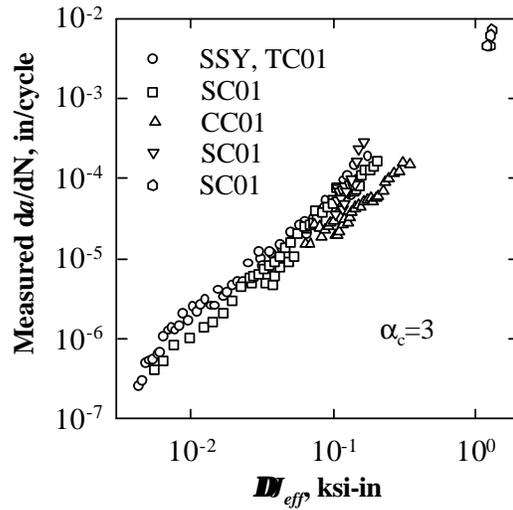
- (4) Replace  $J_p$  by  $\Delta J_{p,eff} = U \mathbf{m} \Delta J_e(a) \cdot \frac{E \Delta \mathbf{e}_{ref}^p}{\Delta \mathbf{s}_{ref}}$  where  $\Delta \mathbf{s}_{ref} = \frac{\Delta P}{2P_o} \mathbf{s}_o$  and  $\Delta \mathbf{e}_{ref}^p$  is the corresponding reference plastic strain range determined from the hysteresis stress-strain curve.

## VALIDATION: LOAD CONTROL INCONEL 718 MATERIAL

The  $\Delta J_{eff}$  solutions were applied to calculating FCG rates and lifetimes in tests conducted by SwRI for NASA on surface cracks (SC), corner cracks (CC), and central through cracks (TC) in IN718 plates under SSY, intermediate- and large-scale yielding conditions [2]. A comparison between experimentally measured fatigue cycles to failure and predicted cycles is given in Figure 3. The term  $\alpha_c$  appears in Newman's expression for  $U$  and takes a value of 1 under plane stress and 3 under plane strain conditions. The results in Figure 3 demonstrate that, in this case, the choice of value for  $\alpha_c$  has little effect on the predicted cycles to failure. In general, all of the predictions are excellent, over a very wide range of cyclic lives. FCG rate data based on  $\mathbf{DJ}_{eff}$  are shown in Figure 4 and demonstrate a very strong correlation of FCG rates over more than four orders of magnitude with  $\mathbf{DJ}_{eff}$ .



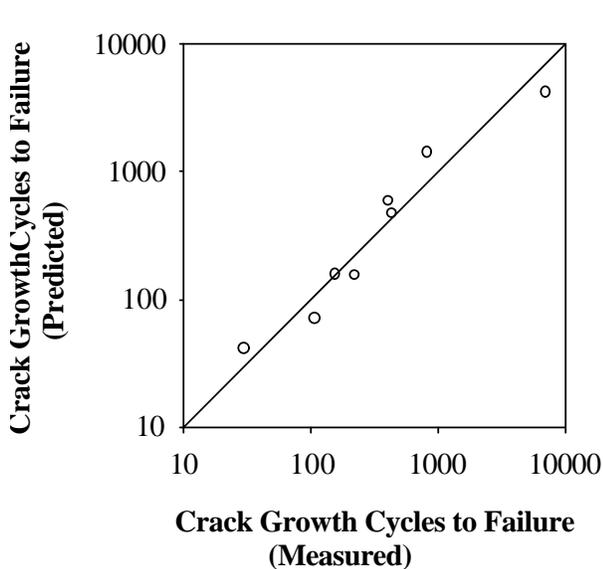
**Figure 3:** Comparison of predicted and measured cycles to failure.



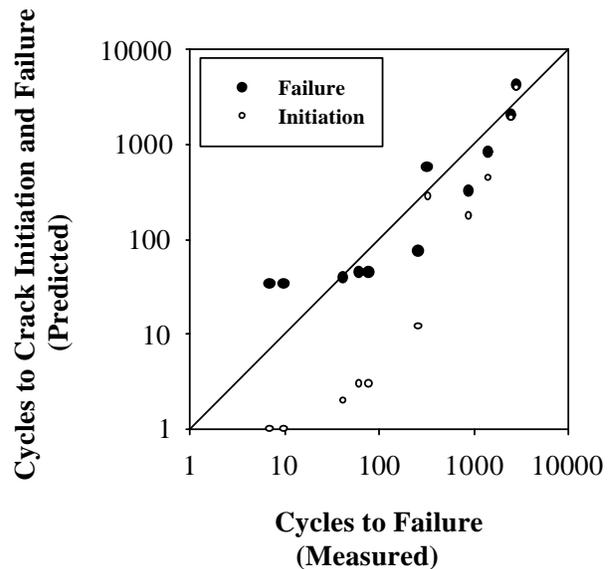
**Figure 4:** Correlation of measured FCG rates with calculated  $DJ_{eff}$  values.

### VALIDATION: CONSTANT STRAIN LCF AND PRESSURIZED NOTCHED PIPES: X52 PIPE MATERIAL

As part of an investigation by SwRI for the Gas Research Institute (GRI) into the effects of mechanical damage on the remaining life of gas transmission pipelines, LCF tests were performed on X52 steel (see [4] for details). The tests were performed on round bars of diameter 2.54 mm (0.1 inches) under constant strain range conditions. Crack initiation was detected from the reduction in applied maximum load. The cycles to propagate initiated thumbnail cracks of depth 108  $\mu\text{m}$  (4.25 mil) to failure were measured and an EPFCG equation was determined using the constant strain formulation for  $DJ_{eff}$ . The results, shown in Figure 5 as a plot of predicted against measured cycles to failure, provide a self-consistency check on the derived EPFCG equation and  $DJ_{eff}$ . The derived growth rate equation was then used to predict the remaining fatigue lives of dented pressurized pipes containing machined notches. The combined effects of the notch and the dent produced LCF conditions at the notch tip during pressure cycling. The predicted crack initiation and propagation cycles to cause a leak (defined as failure) are plotted against the measured cycles in Figure 6. As can be seen, under severe LCF conditions only a few pressure cycles are needed to initiate cracking at the notches, and the majority of the lives of the damaged pipes are spent in propagating the initiated cracks to failure. The results in Figure 6 demonstrate good agreement between the calculated and measured cycles to failure, verifying the proposed EPFCG methodology and  $DJ_{eff}$ -schemes for both the strain-controlled round bar tests and the analysis of the pressurized notched pipes.



**Figure 5:** Comparison of predicted and measured cycles to failure under strain-controlled LCF conditions in round bar test specimens.



**Figure 6:** Comparison of predicted and measured cycles to failure of notches in mechanically damaged pipes.

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