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Fracture of Ceramics

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ABSTRACT

The brittle fracture behavior of ceramic materials (Wesgo Al 995, Mykroy 750 and 1100) was studied under non-uniform stress distribution. The results of both notch-tensile and bend tests on these materials indicate that fracture in ceramic materials can also be analyzed by a maximum fracture stress concept provided that the inherent inhomogeneity of these materials is considered. Weibull's statistical theory of fracture provides one very useful basis for such an analysis. A better insight into the physical aspects of the effects of inhomogeneities, the strength under non-uniform stress distributions and the experimental scatter is provided from a consideration of the interaction between the imposed stress distribution and that introduced by individual flaws or inhomogeneities.

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I. Introduction

The analysis of brittle fracture, based on continuum linear elastic theory coupled with a suitable fracture criterion, has been largely conducted on rather brittle metals since they are relatively homogeneous materials with small regions of deviation from linearity (plasticity). These analyses, which have been successfully applied to the fracture of brittle and semi-brittle steels, aluminum and titanium alloys are the well known Griffith criterion (1)* expanded by Irwin (2) and the Ludwik maximum stress concept (3) coupled with Neuber's theory of stress concentrations (4) expanded by Sachs, Weiss and Sessler (5,6). Therefore, it appeared logical and desirable to study the applicability of the analysis of fracture to brittle, non-homogeneous materials, such as ceramics.

All concepts mentioned above predict a loss in strength with increasing crack size. The relationship is generally expressed in the form $\sigma\sqrt{c}$ = constant where σ is the gross fracture stress and 2c the crack length. Thus, Griffith showed that the strength of technical materials is determined by their inherent imperfections and their cohesive energy. This crack length effect becomes a "true" size effect when viewed on a statistical basis in which the probability of finding a large crack increases with the increase of size of the test piece. Recognizing the inhomogeneity of materials, Weibull (7) postulated a statistical theory of failure in an attempt to explain fracture phenomena. Assuming that the size and distribution of the flaws are random and that the weakest-link concept of fracture is applicable, the Weibull theory can be shown to predict a decrease in strength of smooth tensile specimens with an increase in specimen volume according to

$$\frac{\sigma_1}{\sigma_2} = \left(\frac{V_2}{V1}\right)^{\frac{1}{m}} \qquad \text{Equ. 1}$$

where V is the test volume and m is an experimentally determined factor.

Weiss (8) has applied Weibull's statistical theory to the fracture behavior of notched sheet tensile specimens. This was accomplished by utilizing a simplified stress distribution for notched specimens based on Neuber's theory of stress concentrations (4) mentioned above, the value of the stress gradient at the notch root and Westergaard's stress field equation for sharp cracks (9). Equations for the notch strength ratio (net section strength/tensile strength $-\sigma_N/\sigma_T$) in terms of m, the elastic stress concentration factor, K_t, and geometrical variables of the test specimen, are obtained.

In the following, Weibull's statistical theory is applied to the interpretation of the results of notch tensile and flexure data of ceramic materials. For an additional interpretation of the data, a new concept considering the interaction between the stress distributions of a notch and a neighboring flaw is presented.

*Numbers in parentheses refer to references at the end of the text.

II. Experimental Materials and Procedures

The materials in this program included: Wesgo Al 995, Mykroy 750 and Mykroy 1100. The majority of the tests were conducted on the Wesgo material.

Wesgo Al 995 is a high density, high purity multi-crystalline aluminum oxide material produced by the Western Gold and Platinum Corporation (Wesgo) using a proprietary composition. This material is reported to be 99.5 percent Al_2O_3 with a porosity of 3.0 percent or less (10).

Mykroy is a trade name for a glass-bonded mica composites manufactured by Molecular Dielectrics, Inc. These materials are made from a mixture of electrical glass frits and mica flakes under heat and pressure. The grade designation (e.g. 750 and 1100) indicates the approximate thermal limit of the material in degrees Fahrenheit. Mykroy 750 is made with natural mica while Mykroy 1100 is made with synthetic mica (Thermica). Photomicrographs of both Wesgo Al 995 and Mykroy are shown in Fig. 1.

Notch tension tests and pure bend tests were employed in the investigation. Flat edge-notch specimens were employed for the notch tension tests on Wesgo Al 995, Mykroy 750 and Mykroy 1100. The relative notch depth was kept nearly constant at 30 percent. The root radii* were varied from 0.002 to 0.150 inch corresponding to K_t values between 9.3 and 1.6. Two different specimen thicknesses were included, 0.060 inch and 0.125 inch. The notch tension tests were performed in an Instron testing machine using a Baldwin 1000 lb. SR-4 strain gage load cell for measuring the applied tensile force. Suitable grips and alignment fixtures were employed to minimize eccentricity.

Bend tests were performed only on the Wesgo Al 995. The width (w) of all bend specimens was 0.5 inch; the thickness (t) was varied from 0.040 to 0.250 inch. Both specimen geometries are illustrated in Fig. 2. All bend tests were performed in a 4 point loading bend fixture, designed especially for this purpose. The fixture was used in conjunction with an Instron testing machine for the application of load.

*The notched specimens with small radii were procured with ready-made notches and in some cases were improved by machining with a diamond tool, while specimens with larger radii were produced with the diamond tool.

III. Experimental Results

The results of notch tensile tests on Wesgo Al 995, 0.060 and 0.125 inch specimens are shown in Figs. 3 and 4 where the notch strength or net section stress, σ_N, is plotted against the elastic stress concentration factor, K_t, in log-log coordinates. In this representation the data points appear to fall within a linear scatterband in accordance with the relationship

$$\sigma_N K_t^{0.54} = 21 \text{ ksi} \qquad \text{Equ. 2}$$

The experimental scatter of the thinner specimens, however, considerably exceeds that of the thicker specimens. It was observed prior to testing the thinner specimens that they were not as uniformly flat as the thicker specimens. This apparently resulted in some degree of eccentricity in the loading of the specimens.

In an attempt to assess the amount of experimental scatter to be expected from notch tensile tests on Wesgo Al 995, a series of tests was performed on 18 identical specimens with $K_t = 4.5$ and root radii (r) = 0.010 inch. The results of these tests are given in Table I. The arithmetic mean value of notch strength for the 18 tests was 8,606 psi and individual values ranged from a minimum of 5,580 psi to a maximum of 1,400 psi. The relative standard deviation was calculated to be 0.168 or 16.8% scatter.

The results of notch tensile tests on Mykroy 750 and Mykroy 1100 are illustrated in Fig. 5. Since considerable scatter of the test points was encountered, the results are best represented by scatter bands drawn through the maximum and minimum points. As K_t is increased from 1-10, a loss in notch strength of about 25 percent is observed for the Mykroy 750 material and approximately 40 percent for Mykroy 1100. The results of the thinner specimens fall near the bottom of the scatterbands. The explanation for this behavior lies probably again in the different uniformity of as-manufactured specimens.

Bend tests on smooth specimens were conducted on Wesgo Al 995 in an attempt to ascertain the effect of specimen size on the bend strength. The results are illustrated in Fig. 6, where the nominal bend strength is plotted against the test section volume in log-log coordinates. The observed decrease in strength with increasing volume is in agreement with the existence of a Weibull-type size effect (Equ. 1).

While it is noted that the volume effect was slightly inconsistent at larger volumes, it does serve to indicate the general behavior of the Weibull size effect, i.e., a drop in fracture strength with an increase in volume. The deviation from expected behavior for the largest test volume has been noted elsewhere in a study of the effect of volume on fracture strength (11). From the slope of the curve one can estimate an m value, obtaining an $\underline{m} \approx 5$.

IV. Discussion

From considerations of stress-strain curves of poly-crystalline ceramic specimens which are linear to the point of fracture, it might be prematurely concluded that the fracture of such specimens should be predictable on the basis of a maximum fracture stress concept and linear elasticity theory as given in

$$\sigma_N \cdot K_t = \sigma_{max} \qquad \text{Equ. 3}$$

Such a relationship is indeed applicable to brittle metals as shown by Sessler (12). However, as already indicated, ceramics follow more nearly the relationship given in Equ. 2. Since gross plastic flow was not observed, this effect may be suspected to be due to the interaction between the stress fields due to stress concentrations and material inhomogeneities. An analysis of such interaction has been proposed by Weiss (8) who utilizes the statistical approach suggested by Weibull. In this approach, the material inhomogeneity is described in terms of the material constant m which, together with the individual notch geometry, defines the notch strength ratio in a unique fashion, namely

$$\frac{\sigma_N}{\sigma_T} = \left[1 - \frac{r}{4a}(K_t^2 - 1) + \frac{r}{2a}\,\frac{K_t^m - K_t^2}{m-2}\right]^{-1/m} \qquad \text{Equ. 4}$$

where 2a is the net section width of the notch tensile and the tensile specimen and r the root radius of the notch. This relationship is plotted in Fig. 7 for various $\underline{m}$ values. Neglecting the load carrying capacity of the center section of the notch tensile specimen, where the stress drops below the net section stress, one obtains the relationship

$$\frac{\sigma_N}{\sigma_T} = \left[\frac{2}{m-2}\,\frac{K_t^m - K_t^2}{K_t^2 - 1}\right]^{-1/m} \qquad \text{Equ. 5}$$

which yields an m value of approximately $m \approx 8$ on the basis of the data in Table I and an estimated tensile strength of 22 ksi. Since the m-value is, according to Weibull, also uniquely defined by the relative standard deviation it can be determined from the data listed in Table I. The observed relative standard deviation $d \approx 0.168$ also yields $\underline{m} \approx 8$.

While the above analysis appears to be relatively consistent and in agreement with the experimental results it lacks a physical explanation for m. To obtain a better understanding of the fracture process of inhomogeneous solids, Weiss and Schaeffer (13) have proposed an analysis based on a simple model consisting of periodically spaced stress raisers (inhomogeneities) in the stress field of a notch, Fig. 8. These stress raisers are assumed to have a spacing b and a stress concentration factor

K_b. The strength of such a specimen made from a material having a fracture strength σ_F and a stress raiser between x = 0 and x = b from the notch root, can be calculated from the stress distribution in the vicinity of the notch. The value of the stress parallel to the loading axis, σ_y, is given to a fair approximation (8) by

$$\sigma_y = K_t \, \sigma_N \sqrt{\frac{r}{r + 4x}} \qquad \text{Equ. 6}$$

where x is the distance from the notch root. Fracture is assumed to occur when the stress at the flaw equals the fracture strength and is given by

$$\sigma_{NF} = \frac{\sigma_F}{K_t K_b} \sqrt{\frac{r + 4x}{r}} \qquad \text{Equ. 7}$$

Assuming that the inhomogeneity has an equal chance of being anywhere between 0 < x < b, one obtains an average strength σ_N of

$$\sigma_N = \frac{\left(1 + 4\frac{b}{r}\right)^{3/2} - 1}{6\, b/r} \; \frac{\sigma_F}{K_t K_b} \qquad \text{Equ. 8}$$

From the above, the average notch strength ratio is given by

$$\frac{\sigma_N}{\sigma_T} = \frac{1}{K_t} \; \frac{\left(1 + 4\frac{b}{r}\right)^{3/2} - 1}{6\, b/r} \qquad \text{Equ. 9}$$

which is independent of K_b*, and which depends only on the flaw spacing, b. Values of 0.001", 0.01", and 1.0" were assumed for b in an attempt to approximate a value for the observed notch strength ratios. The results indicate that a b value of approximately 0.01 in. provides a suitably decreasing notch strength ratio which is in fair agreement with results, Figs. 3 and 4.

The relative standard deviation, standard deviation/notch strength, $S/_N$, can also be computed for the model and one obtains

$$\left(\frac{S}{\sigma_N}\right)^2 = 1 - \left[\frac{6\, b/r}{(1 + 4\, b/r)^{3/2} - 1}\right]^2 (1 + 2\, b/r) \qquad \text{Equ. 10}$$

*According to the model the tensile strength is defined by σ_F/K_b

which depends only on the ratio b/r. The results of the proposed simple model can now be compared with the predictions of Weibull's statistical theory of fracture by matching b/r values with m - values yielding the same relative standard deviation, Fig. 9. The results of this comparison indicate that homogeneous materials, i.e. materials characterized by high m-values, must have discontinuity separations which are small compared to the linear dimension of the critically stressed volume. The limiting case is, of course, represented by the interatomic distance of the crystal lattice. Equation 10 and Fig. 9 also explain higher experimental scatter of notch tensile specimens as compared to that obtained on smooth tensile specimens of the same material. It should be pointed out, however, that the simple model used for the present calculations does not take into account statistical variations of the interflaw spacing b or the inhomogeneity stress concentration factor K_b. Such variations will have to be considered for a more accurate and complete analysis of fracture in inhomogeneous materials. For the data given in Table I, r = 0.01 in., i.e. $b/r \approx 1$ Fig. 9 predicts an m value of $m \approx 6$ and a relative standard deviation of approximately 0.2 which is fair agreement with the observed values of $S/\sigma_N = 0.168$ corresponding to $m \approx 8$ calculated from Weibull's theory.

The Weibull approach (Equ. 5) and the model approach suggested by Weiss and Schaeffer (Equ. 8 and 9) can be readily correlated. Both show that the K_t effect will be lessened as the material inhomogeneity increases, decreasing m or increasing b/r. However, while a constant m value is in fair agreement with the results of constant size - increasing K_t test series reported here, an interpretation in terms of the model (Fig. 8) suggests a decreasing m value with increasing K_t since b/r decreases with increasing K_t. This dilemma cannot readily be resolved and is probably due to the oversimplification of the model which utilizes equally severe (K_b = constant) and uniformly spaced inhomogeneities while both values are themselves subject to statistical variations. It is, however, significant that m should vary with the test volume itself since the same material may be quite homogeneous when viewed on a large scale but rather inhomogeneous on a much smaller, microscopic scale.

When considering the factors affecting the results, surface finish must be of prime importance. It has been shown that when the surface is altered the m value and consequently the ultimate strength changes (14). The data indicate that a decrease in strength was observed with an increase in surface roughness. Tensile strengths appeared to be more sensitive to surface finish than did the bend strengths. The type of surface finish was also shown to be an important variable (14). Circumferential grooves of the kind produced in grinding had the most detrimental effect on tensile strength. Evidence has been presented (15,16) which indicates that surface irregularities act as stress concentrations leading to lower fracture strengths for brittle metals. Therefore, it is believed that the large scatter of the Mykroy data could be traced to detrimental effects of the machining on the notch surface. This material, being made of glass frits and mica, was prone to chipping during machining, resulting in a surface finish that yielded a large amount of scatter.

An analysis of the notch data in terms of a "critically stressed volume" may be presented to examine Weibull's predictions of a volumetric size effect of the type shown in Equ. 1. In notched specimens one can calculate the linear extent σ from the notch for which the stress exceeds a critical value σ_c and obtains

$$\sigma = \frac{r}{4}\left[\left(\frac{K_t \quad \sigma_N}{\sigma_c}\right)^2 - 1\right] \qquad \text{Equ. 11}$$

which, assuming a maximum fracture strength concept ($K_t \; \sigma_N$ = const.) is proportional to r the notch root radius. A similar formula is obtained for the stress distribution in the direction of loading and one can therefore obtain an expression for the critically stressed volume as

$$V_c \propto r^2 \; t \qquad \text{Equ. 12}$$

where t is the specimen thickness.

Equation 11 predicts that a plot of log V_c versus log ($K_t \; \sigma_N$) should result in a straight line with a slope of $-1/\underline{m}$. Such a plot is shown in Fig. 10 for Al 995 (t = 0.125). The constant m from this analysis is approximately 10.

Finally, there is a graphical method for determining the constants of Weibull's distribution function, i.e., σ_o and m. It requires a plot of log log $\frac{1}{1-S}$ versus log (σ - σ_u)* with the result that the slope and the intercept of the curve yield the constants m and σ_o, respectively. Armour Research Foundation using Wesgo Al 995 samples of 0.469 in^3 gage section found that for 54 samples tested in bending the values of the material constant m was 3.23 (10). Data from tensile tests on Al 995 indicate m values of 6.5 and 5.4 for crosshead rates 0.012 and 0.030 $\frac{\text{in.}}{\text{min}}$ respectively using 0.113 inch diameter gage section tensile specimens (14). Unfortunately, in this work, there was not a sufficient number of bend tests performed at one test volume to yield good data for such an analysis.

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*S is the probability of rupture and is given by (7) S = n/ (N+1) where N is the total number of specimens and n the n-th specimen considered. σ_u is the lower stress limit.

VI. Summary and Conclusions

A study of the fracture behavior of brittle non-homogeneous materials in the presence of non-uniform stress systems is presented. It is shown that fracture of ceramic materials can be analyzed by a maximum fracture stress concept provided the material inhomogeneity is taken into account.

Weibull's statistical theory of fracture applied to the stress distribution of notched sheet specimens provides a satisfactory basis for an analysis of the experimental data and correlates with the statistical analysis. The determination of m from considerations of the critically stressed volume of notch tension specimens leads to m values which are also in fair agreement with the statistical determination.

More information about the interaction of defects with externally introduced stress concentrations is obtained from a simple model which considers the stress distribution not only of the notch but also of the internal stress concentration due to flaws. The predictions obtained from the model are very similar to those obtained from the Weibull analysis applied to notch specimens and are also in fair agreement with the results of the statistical analysis of the data. They are:

1.) For a model material containing equally spaced like inhomogeneities, m is a function of the flaw spacing to root radius ratio (b/r) only; i.e., m is a function of the test volume.

2.) Increasing inhomogeneity (decreasing m or increasing b/r leads to a lessening of the notch effect.

3.) Increasing size leads to a loss in strength of geometrically similar specimens (K_t = const.).

ACKNOWLEDGEMENT

The authors gratefully acknowledge the support of the U.S. Air Force Research and Technology Division, Materials Laboratory, especially the interest and guidance offered by Mr. Walter Trapp and Mr. Gordon Atkins. The work reported here resulted from contract No. AF 33(657)-11154.

The authors are also indebted to Dr. Gene T. Schaeffer, previous graduate student in the Syracuse Solid State - Metallurgy Program, for his many valuable suggestions in the preparation of the report.

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TABLE I

RESULTS OF DISTRIBUTION STUDY OF NOTCH TENSILE TESTS ON WESGO A1995

Spec. No.	r (in)	D (in)	d (in)	t (in)	Load(2)	Notch Strength σ_N(psi)	K_t
C6-1	0.010	0.496	0.346	0.127	245	5,600	4.5
C6-2	0.010	0.496	0.346	0.127	430	9,800	4.5
C6-3	0.010	0.494	0.346	0.127	400	9,100	4.5
C6-4	0.010	0.497	0.350	0.127	460	10,400	4.5
C6-5	0.010	0.497	0.346	0.127	250	5,700	4.5
C6-6	0.010	0.497	0.348	0.126	393	8,950	4.5
C6-7	0.010	0.497	0.347	0.127	445	10,000	4.5
C6-8	0.010	0.497	0.348	0.127	330	7,450	4.5
C6-9	0.010	0.496	0.346	0.126	363	8,350	4.5
C6-10	0.010	0.496	0.345	0.127	426	9,750	4.5
C6-11	0.010	0.497	0.346	0.127	430	9,800	4.5
C6-12	0.010	0.497	0.345	0.126	458	10,000	4.5
C6-13	0.009	0.496	0.346	0.126	303	6,950	4.7
C6-14	0.010	0.496	0.346	0.126	330	7,550	4.5
C6-15	0.009	0.496	0.347	0.126	420	9,600	4.7
C6C	0.010	0.497	0.347	0.128	380	8,600	4.5
C6A	0.010	0.497	0.347	0.127	382	8,700	4.5
C6D	0.009	0.499	0.349	0.128	384	8,600	4.7

Notes

(1) specimen failed in grips

(2) crosshead speed was 0.010 in/min

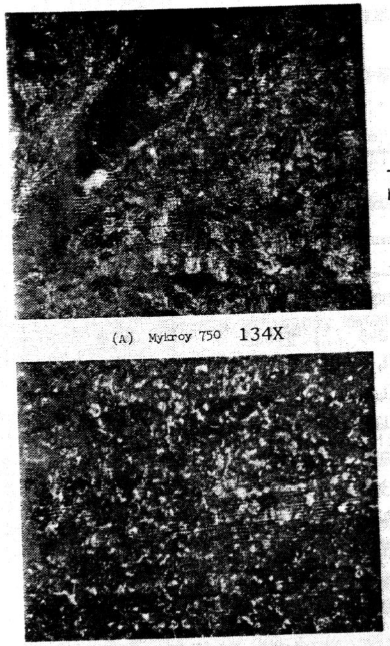

FIG. 1 Photomicrographs of (A) Mykroy 750 and (B) Wesgo Al 995 (As-received surface).

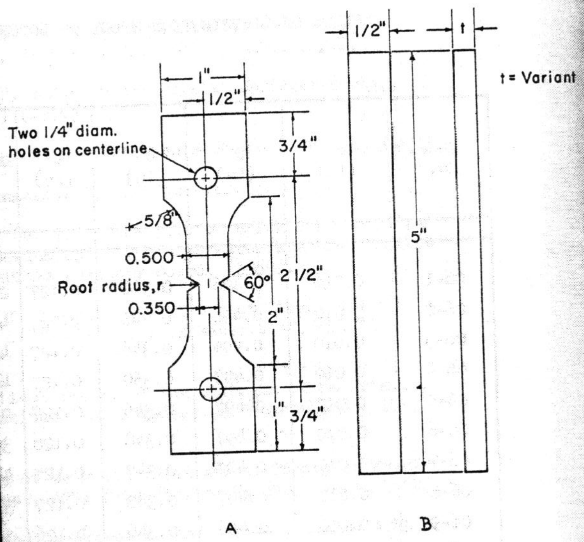

FIG. 2 Flat notch tensile (A) and smooth bend (B) test specimen geometries.

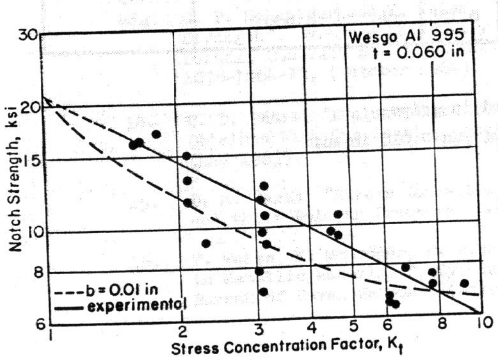

FIG. 3 Effect of stress concentration on notch strength of 0.060 inch Wesgo Al 995 ceramic.

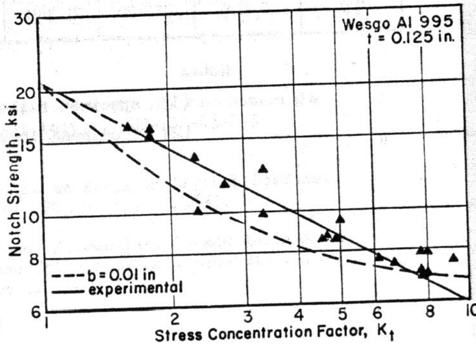

FIG. 4 Effect of stress concentration on notch strength of 0.125 inch Wesgo Al 995 ceramic.

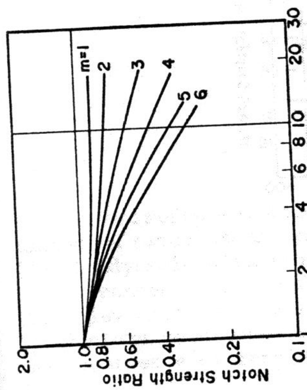

FIG. 7 The effect of m on the predicted notch strength ratio vs. K_t curve (8).

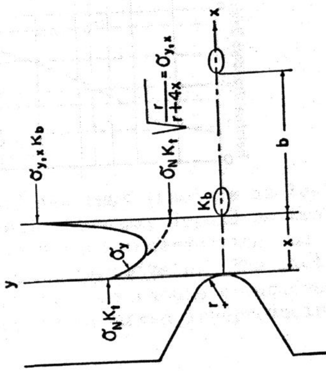

FIG. 8 SCHEMATIC "MODEL" CERAMIC SHOWING DISTRIBUTION DUE TO NOTCH AND FLAW

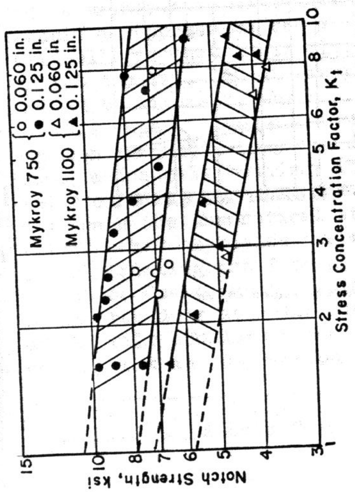

FIG. 5 Effect of stress concentration on notch strength of Mykroy 750 and Mykroy 1100.

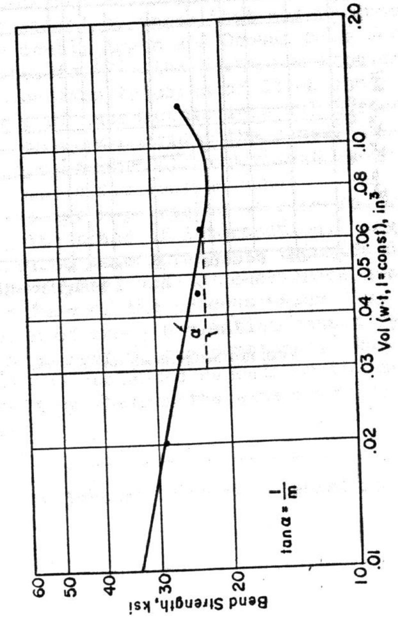

FIG. 6 Effect of volume on bend strength for Wesgo Al 995.

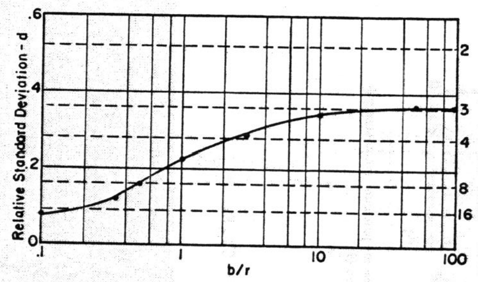

FIG. 9 Effect of b/r on relative standard deviation.

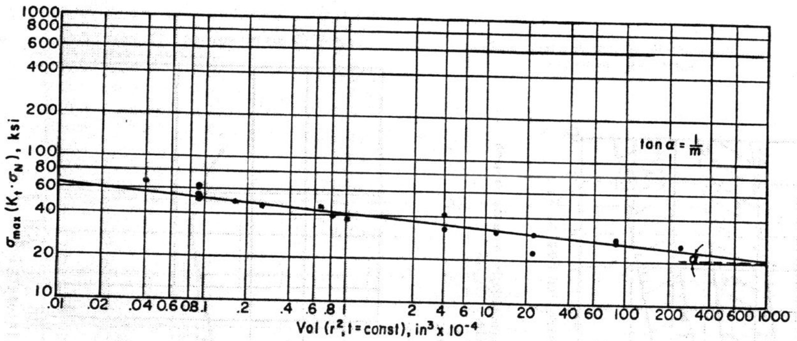

FIG. 10 Effect of critically stressed volume on c_{max} for Al 995 notch tensile specimens (t = 0.125)