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# DYNAMIC CRITERIA FOR CRACK NUCLEATION AND GROWTH*

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## ABSTRACT

A dynamical criterion for semi-brittle crack propagation is developed. A moving Griffith crack causes local displacements at rates proportional to its velocity. In the absence of plastic flow, these cause high local stresses which cause fracture. However, as the stresses develop they cause dislocation motion and multiplication which produces a certain plastic relaxation rate. This relaxation rate increases with time so the local stress tends to pass through a maximum which increases with the crack velocity and decreases with the local plastic strain-rate. Thus a critical velocity for crack propagation exists for which analytic and numerical values are given.

If a crack does not propagate in a semi-brittle way, it may still propagate when large plastic strains at its tip cause enough degradation of the local cohesion. A model for this is suggested.

Finally, a dynamical interpretation of the grain size effect is given.

## I. INTRODUCTION

Because of the observed rate dependence of fracture in elastic-plastic media (especially for semi-brittle behavior), it is clear that a static criterion for fracture is insufficient, and even a quasi-dynamic criterion is somewhat too indefinite to be satisfying. Also, the geometry of a crack is sufficiently complex to ensure that any accurate model will necessarily require numerical computations. Therefore, a simplified but analytic model will have value because it can give quick insight to the physical situation; and how it can be modified. Thus, the purpose of this discussion is to develop a simple, and therefore approximate, model of the conditions that must be met before a crack can grow in size in an elastic-plastic medium.

A primary criterion for fracture is needed in order to direct and bring the discussion to a conclusion. This criterion is taken to be that the local normal stress must reach a critical value (case of semi-brittle fracture); or that a combination of stress and dislocation density must exist such that the strength of the material becomes degraded to a relatively small stress level.

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The semi-brittle case will be discussed first. Here it is proposed that a critical stress criterion is needed in an elastic-plastic body because this has a unique result; namely, permanent separation of two parts of the body. Other criteria such as one based on energy alone, or one based on displacement, are not unique because various results can follow the achievement of the critical criterion.

The maximum stress that will be reached at small strains (the upper yield stress) will depend on a balance between the rate at which local displacements are imposed by the mechanical system, and the rate at which these displacements can be accommodated by plastic flow. If the latter effect is very small, the solid will behave elastically and will certainly break after a small amount of displacement has occurred. On the other hand, if the latter effect is large, the stresses caused by the imposed displacements will be quickly relaxed and the stress will not rise above a small and ineffective value.

From the above, it may be seen that the problem consists of two parts. First, estimation of the local displacement rate and the resulting stresses; and second, determination of the response of the elastic-plastic material. These two parts will be discussed separately in turn and then combined to form a criterion. Note that this approach is distinctly different from the usual one of starting with the distribution of elastic stresses, although these are implied by the assumed crack shape.

## II. DISPLACEMENT RATE AND STRESS AT CRACK TIP

Since the Griffith condition must be satisfied by any theory, we start with a critical Griffith crack and thereby determine the geometry of the problem. By definition in the semi-brittle case, fracture must occur following a small amount of displacement, so the shape will not change much prior to the development of the critical conditions.

Figure 1 shows the atomic displacements near the tip of a critical Griffith crack as calculated from Elliott's solution.(1) Since the local stress at the tip of a critical crack must always equal the cohesive stress, the local elastic strains and hence the displacements are relatively invariant towards changes of the crack length and depend mainly on the ratio of the surface energy to the elastic modulus. We can simplify the shape by linearizing it as shown in the figure. Then the overall shape can be described as shown in Figure 2.

The linearized crack is considered to be in a state of plane stress and is characterized by a length L, a depth H, and tip angles $\theta$. The length is determined by the applied stress through Griffith's equation:

$$L = \frac{2ES}{\pi(1 - \nu)\sigma_0^2} \qquad (1)$$





where E is Young's modulus, S is the specific surface energy, $\nu$ is Poisson's ratio, and $\sigma_0$ is the applied stress. The depth is related to L through the elasticity of the material;

$$H = \left(\frac{\sigma_0}{E}\right) L \qquad (2)$$

and the tip angle is nearly invariant (for a critical crack) with a magnitude of about 25°, as taken from Figure 1.

If the crack expands with velocity $V_c$, then the displacement rate $\dot{u}$ is:

$$\dot{u} = V_c \theta \qquad (3)$$

or if only an incipient crack is present in the form of a blocked glide band of length x and dislocation density $\rho$, then the displacement rate is:

$$\dot{u}_g = \dot{\varepsilon}_p x = b\rho V_d x \qquad (4)$$

where $\dot{\varepsilon}_p$ is the plastic strain rate, b is the Burgers displacement, and $V_d$ is the mean dislocation velocity.

Prior to fracture, the imposed local displacement, $\dot{u}t$ (where t = time) equals the sum of the local elastic and plastic displacements. The local elastic displacement is determined by the local stress $\sigma$, which varies with time as the instantaneous state of the material changes. It also depends on the size of the region over which the stress acts. This is some function of H in this case and will be taken to be 1/2 H as a first estimate. Thus, if the elastic and plastic strains are $\varepsilon = \sigma/E$ and $\varepsilon_p$, respectively; then the local displacement of the material will be:

$$\frac{H}{2}\left(\frac{\sigma}{E} + \varepsilon_p\right) \qquad (5)$$

(where E is Young's modulus) so the displacement balance becomes:

$$\dot{u}t = \frac{H}{2}\left(\frac{\sigma}{E} + \varepsilon_p\right) \qquad (6)$$

and combination of equations (1), (2), and (3) with equation (6) yields:

$$\sigma = \frac{\pi(1 - \nu)\ \sigma_0 V_c \theta E t}{S} - E\varepsilon_p \qquad (7)$$

so the local stress at a given time increases with the crack velocity and decreases with the amount of plastic strain that has taken place. Also, it may be noted that it does not depend explicitly on H.

## III. STRAIN-RATE EQUATION

In order to find the value of the local stress from equation (7), it is necessary to calculate the plastic strain after some time t, has elapsed, and so an expression for the strain-rate, $\dot{\varepsilon}_p$ is needed. The general equation is:

$$\dot{\varepsilon}_p = \phi b \rho v \tag{8}$$

where $\phi$ is a geometric factor, b is Burgers displacement, $\rho$ is the mobile dislocation density, and v is the mean dislocation velocity. Following Gillis and Gilman (2) this may be written in terms of stress and plastic strain:

$$\dot{\varepsilon}_p = b \cos \lambda \left(\rho_0 + M\varepsilon_p\right)[V^* e^{-2D/\sigma}] \tag{9}$$

where $\lambda$ is the angle between the mean active glide direction and the principal stress axis. The term in parentheses describes the increase of dislocation density that results from breeding; with $\rho_0$ being the initial dislocation lensity, and M a multiplication coefficient. The term in brackets describes the dependence of the mean dislocation velocity on stress; with V* being the limiting or terminal velocity at high stresses; D being the characteristic drag stress which determines the boundary between slow and fast dislocation motion; and $\sigma/2$ being the mean local shear stress. This rate equation applies only for small strains because strain-hardening is not included in it, but only small strains need to be considered in the semi-brittle fracture case.

## IV. AN APPROXIMATE ANALYTIC SOLUTION OF THE DYNAMICAL EQUATIONS

The stress equation (7) and the strain-rate equation (9) taken together describe the behavior of the material. As the moving crack imposes displacement on an element of material it becomes stressed according to the first term of equation (7). This stress, according to equation (9), produces a finite strain-rate which results in some plastic strain after a finite time has elapsed. This plastic strain has two effects: one is that it tends to reduce the stress as stated by equation (7), and the other is that it tends to increase the strain-rate as stated by equation (9). In the beginning of the process, the increasing displacement causes the stress to increase more rapidly than the plastic flow can relax it. Eventually, however, because of the increases in $\sigma$ and $\varepsilon_p$ in equation (9), the plastic strain-rate exceeds the specific displacement rate and then the stress begins to decrease. Thus a maximum exists in the curve of





stress as a function of time (the upper yield stress). If this maximum stress becomes large enough, it will equal the local fracture stress and hence cause fracture. If it is less than this, large amounts of plastic flow will occur before the stress again becomes large (via strain-hardening) and the crack will become plastically blunted and then will not be able to propagate. Our problem then is to solve equations (7) and (9) simultaneously in order to find the maximum local stress as a function of the crack velocity. From this, the critical velocity required for local fracture at the crack tip will be determined and hence the dynamic condition for growth of the Griffith crack.

For algebraic convenience we define the following non-dimensional quantities:

$$\text{stress} = \Sigma = \frac{\sigma}{2D} \tag{10}$$

$$\text{time} = T = \left[\frac{\pi(1-\nu)\sigma_0 V_c \theta}{S}\right] t \tag{11}$$

and the non-dimensional constants:

$$C_1 = \frac{Sv^* Mb \cos \lambda}{\pi(1-\nu)\sigma_0 V_c \theta} \tag{12}$$

$$C_2 = {}^{\rho_0}/_{M} \tag{13}$$

$$C_3 = {}^{2D}/_{E} \tag{14}$$

In terms of these quantities, equations (7) and (9) become:

$$C_3 \Sigma = T - \varepsilon_p \tag{15}$$

$$\underset{\cdot}{\varepsilon}_p = C_1 (C_2 + \varepsilon_p) e^{-1/\Sigma} \tag{16}$$

where the dot placed underneath refers to differentiation with respect to non-dimensional time.

No general analytic solution of this set of equations exists, but Gillis (3) has shown that a good approximate solution can be written at the upper yield point. This is the point where $\Sigma \to 0$ and $\Sigma$ has a maximum value, $\Sigma$ *. Letting $\Sigma$ be the reciprocal value of $\Sigma$*, the approximate solution is:

$$\Lambda(\Lambda + 1)\, e^{\Lambda}(\Lambda - \ln C_1C_2) = C_1C_3 + \Lambda(e^{\Lambda} - C_1C_2) \tag{17}$$

Since we are interested only in the small $\Lambda$ regime (large $\Sigma^*$), this can be further approximated by letting $e^{\Lambda} \to 1$ and $(\Lambda + 1) \to 1$; yielding the quadratic equation:

$$\Lambda^2 + \Lambda(C_1C_2 - 1 - \ln C_1C_2) - C_1C_3 = 0 \tag{18}$$

which has the solution:

$$\Lambda = \frac{1}{2}\left[-B \pm \sqrt{B^2 + 4C_1C_3}\right] \tag{19}$$

where:

$$B = C_1C_2 - 1 - \ln C_1C_2$$

and for this to be consistent with the assumption of small $\Lambda$, the term $4C_1C_3$ must be small compared with $B^2$ so that equation (19) may be written:

$$\Lambda \simeq \frac{1}{2}\left[-B \pm \left(B + \frac{2C_1C_3}{B}\right)\right] \tag{20}$$

and upon taking the positive root:

$$\Lambda \simeq \frac{C_1C_3}{B} \tag{21}$$

or:

$$\Sigma^* \simeq \frac{B}{C_1C_3}$$

$$\simeq \frac{C_1C_2 - 1 - \ln C_1C_2}{C_1C_3} \tag{22}$$

We shall consider the case of iron (low-carbon steel) near room temperature. Then the appropriate physical properties are as follows:

$b = 2.48 \times 10^{-8}$ cm

$G$ = Shear modulus $= 8 \times 10^{11}$ d/cm$^2$ $= 0.4$ E





$\nu = 1/3$; $\pi(1 - \nu) = 2.1$

$\cos\lambda = 0.707$

$v^* \simeq -4 \times 10^5$ cm/sec

$M \simeq 10^{11}/\text{cm}^2$

$\rho_o \simeq 2 \times 10^8/\text{cm}^2$

$D \simeq 2 \times 10^9$ d/cm$^2$ (corresponding to yield stress of $\sim 10^9$ d/cm$^2$)

$S = 1400$ ergs/cm$^2$

$\theta \simeq 25^\circ = 0.44$ radians

so if the applied stress is $10^{-3}G = 8 \times 10^8$ d/cm$^2$, the constants will have the values:

$$C_1 \simeq \frac{1.33 \times 10^3}{V_c}$$

$$C_2 \simeq 2 \times 10^{-3}$$

$$C_3 \simeq 2 \times 10^{-3}$$

and if the fracture criterion is taken to be that the local stress must reach $\simeq G/30$, then for fracture:

$$\sigma^* = 2D\Sigma^* \simeq G/30 \tag{23}$$

and:

$$\Sigma^* \simeq \frac{C_1C_2 - 1 - \ln C_1C_2}{C_1C_3} \simeq 6.7 \tag{24}$$

which can be solved by iteration for the critical crack velocity:

$$V_c^* \simeq 18 \text{ cm/sec}$$

and this appears to have the right order of magnitude, but no experimental results are available for comparison.

Since H is about $1.8 \times 10^{-6}$ cm from equations (1) and (2), the critical crack velocity above corresponds to a local strain-rate of about 8.7 x $10^6$/sec. Therefore, all of the dislocations that are involved are required to move at their terminal velocities; making this approximation an upper limit. In passing to the small $\Lambda$ regime we have eliminated the influence of the drag coefficient, D which is known to play an important role [by inference from the work of Stein and Low(4)]. Also, the choice of G/30 as the local fracture stress is probably an overestimate for a mild steel in which the initial cracks occur at defects such as ferrite-carbide interfaces [see McMahon and Cohen(5)] which appear to have strengths of the order of G/300. These deficiencies can be corrected by using the results of numerical calculations and this is the subject of the next section.

## V. NUMERICAL SOLUTIONS OF THE DYNAMICAL EQUATIONS

Here we simply adapt the results of Gillis and Gilman(2) [which have been extended by Gillis(6)] to the present situation. These results were obtained by numerical integration of equations (15) and (16) and are shown in Figure 3 where the non-dimensional yield stress is plotted as a function of the local displacement rate for various values of the non-dimensional drag stress. Also shown is an estimate of the cohesive stress which is taken to be about G/30 in shear or E/40 in tension. Therefore, in order for fracture to occur, the local displacement rate must exceed about 5 x $10^2$/sec.

Note that although the upper yield stress is nearly proportional to the drag stress at low displacement rates, it becomes nearly independent of the drag stress at the high rates needed to cause fracture. Since D is a measure of the yield stress, this means that the fracture criterion is relatively independent of the static yield stress (except that $\sigma_0$ cannot exceed the yield stress). The important flow parameters are therefore v* (the terminal dislocation velocity), and M (the multiplication coefficient). Of these, available data indicate that M exhibits the greatest range of values (from less than $10^8$/cm$^2$ to $10^{12}$/cm$^2$) and so may be the most influential as the author has emphasized previously.(7)

## VI. CASE OF PLANE STRAIN

Although it will not be discussed in detail here, when a state of plane strain prevails (as it will near an interior crack) stress relaxation through plastic flow will be severely inhibited, and crack growth facilitated. This is because the normal stresses cannot be fully relaxed in this case no matter how great the flow rate. Only the shear part of the stress tensor can be relaxed leaving appreciable hydrostatic tension. Numerical methods are required for the study of this situation.

## VII. DEGRADATION OF COHESION AT LARGE STRAINS

If the stress does not rise high enough to cause fracture at a crack tip immediately after a load is applied, ductile fracture may





occur later because the material is weakened in local regions as a result of plastic strain.

In order to describe the behavior at large strains, the strain-rate equation (9) must be modified. This is done by adding a factor:

$$e^{-\psi\varepsilon/\sigma} \qquad (25)$$

which can be thought of either: as the fraction of the total dislocation population that remains mobile after a strain $\varepsilon$ has occurred, or as an increase in the drag stress by an amount $\psi\varepsilon$. The rate equation then becomes:

$$\dot{\varepsilon}_p = b\,(\rho_0 + M\varepsilon_p)\,v^* e^{-(D + \psi\varepsilon_p)/\sigma} \qquad (26)$$

There is considerable evidence that the internal damage of crystals which leads to strain-hardening, also degrades the intrinsic strength of a material. This degradation of strength at large strains can probably be associated with the concentration of edge dislocation dipoles in the structure.(8) We are not concerned here with the detailed mechanism of degradation, but with obtaining a suitable analytic description which can be compared with experimental measurements. If degradation of cohesion is caused by the accumulation of dipoles, then a measure of it should be the concentration of immobilized dislocations in a structure. Now equation (25) gives the fraction of the total dislocation population that is mobile after a strain $\varepsilon$ so, the expression:

$$1 - e^{-\psi\varepsilon_p/\sigma_s} \qquad (27)$$

gives the immobile fraction of the total population; where the subscript on the stress emphasizes that this is the shear stress.

If we let $\rho_I$ be the density of immobile dislocations then a first approximation to the local fracture stress might be:

$$\sigma_f = \sigma_{coh}\,(1 - W\rho_I) \qquad (28)$$

where W is a coefficient, and when $\rho_I = 1/W$ the local strength drops to zero. In terms of plastic strain, the immobile dislocation density will be given by:

$$\rho_I = (\rho_0 + M\varepsilon_p)(1 - e^{-\psi\varepsilon_p/\sigma_s}) \qquad (29)$$

so the local fracture stress is:

$$\sigma_f = \sigma_{coh}\,[1 - W(\rho_0 + M\varepsilon_p)(1 - e^{-\psi\varepsilon_p/\sigma_s})] \qquad (30)$$

and it gradually decreases to zero with increasing plastic strain. Also, it may be noted that it depends on the value of the local shear stress.

## VIII. DYNAMICAL INTERPRETATION OF THE GRAIN SIZE EFFECT

In polycrystals the fracture stress is inversely proportional to the square root of the grain size[9]; and for large grain sizes fracture follows yielding after very little plastic strain; whereas for small grain sizes fracture does not occur immediately after yielding even though microcracks as large as the grain size appear upon yielding. These observations have often been interpreted in terms of the stress concentrations that dislocations can produce at the ends of glide bands. However, as was mentioned here in the introduction, stress alone is not a criterion for fracture because it may produce flow instead. The author pointed this out sometime ago[10] and proposed an alternative interpretation of the grain size effect which will be restated here.

A dynamical interpretation that is consistent with all of the experimental results of grain size studies can be based on the fact that a moving crack will be slowed down considerably upon crossing a grain boundary. If the misorientation of the cleavage plane in the next grain is small, then the crack can cross the boundary, but will necessarily acquire many cleavage steps. If the misorientation is large, the crack will have to propagate discontinuously by starting a new crack in the succeeding crystal. In either case there will be a sudden change in the energy absorption as the crack crosses the boundary and this will reduce the crack velocity. If the velocity is reduced to a value below the critical propagation velocity, then the fracture process will stop and will not continue until the stress is raised and a new crack starts. On the other hand, if the crack is moving fast enough, it will have enough kinetic energy to bridge the grain boundary and keep moving at a reduced velocity. Thus, it is proposed that there are two critical crack velocities in semi-brittle polycrystals. First, a crack must have a certain velocity in order to propagate through an individual grain, and second, it must possess a somewhat higher velocity in order to break out of the grain and cause fracture of the aggregate.

This interpretation is not concerned with the mechanism by which a crack is nucleated. It simply imposes the critical velocity condition in addition to the Griffith condition on the nucleation process. The picture is, that, in a manner that remains obscure, plastic glide nucleates cracks, these then grow until they reach grain boundaries, where they either stop or continue propagating, depending on whether or not their velocity exceeds a critical value.

The velocity of a crack, as shown by Dulaney and Brace[9] is given by:

$$V_c = V_t \; (1 - L_0/L) \tag{31}$$





where $V_T$ is the terminal velocity, L is the instantaneous length, and $L_0$ is the critical Griffith length as given by equation (1) for the plane stress case. The velocity of a crack when it reaches a grain boundary (where $L = \Delta$ = grain diameter) will depend on the ratio $L_0/\Delta$ which is:

$$L_0/\Delta = K/\sigma_0^{\,2}\Delta \tag{32}$$

Thus $\sigma^2{}_0\Delta$ must have a critical value in order for the crack velocity to have a critical value, and the fracture condition for an aggregate becomes:

$$\sigma_f \sim (1/\Delta)^{1/2} \tag{33}$$

## IX. SUMMARY

It is pointed out that except for reversible conditions, a static criterion for crack nucleation or growth is inadequate because the concentrated stresses at the crack tip will relax with time.

Plastic flow is the most important mechanism which can cause rapid stress relaxation so its effects are considered in some detail with the aid of a simplified model. In this model a moving crack causes local displacements at a rate proportional to its velocity. These displacements result in local stresses which rise to values high enough to cause fracture in the absence of plastic flow. However, the stresses cause dislocation motion and multiplication which produces a certain plastic strain-rate. With time this causes plastic displacements tending to relax the local stresses caused by the moving crack. The local stress thus passes through a maximum value that increases with crack velocity and decreases with strain-rate. The occurrence of crack propagation, or of plastic blunting, thus depends on the crack velocity and a critical velocity exists above which propagation occurs and below which blunting occurs. Analytic and numerical estimates of this critical velocity are presented.

If a crack does not propagate when it is first loaded (or created in a stress field) it may become unstable later when the plastic strains at its tip reach critical values. That is, it may occur after large strains have caused the local cohesion of the material to become degraded by a high concentration of stored defects such as dislocation dipoles. An analytic description of this process is suggested.

Finally, it is shown that the grain size effect observed for polycrystalline aggregates can be interpreted best in terms of the dynamics of crack propagation.

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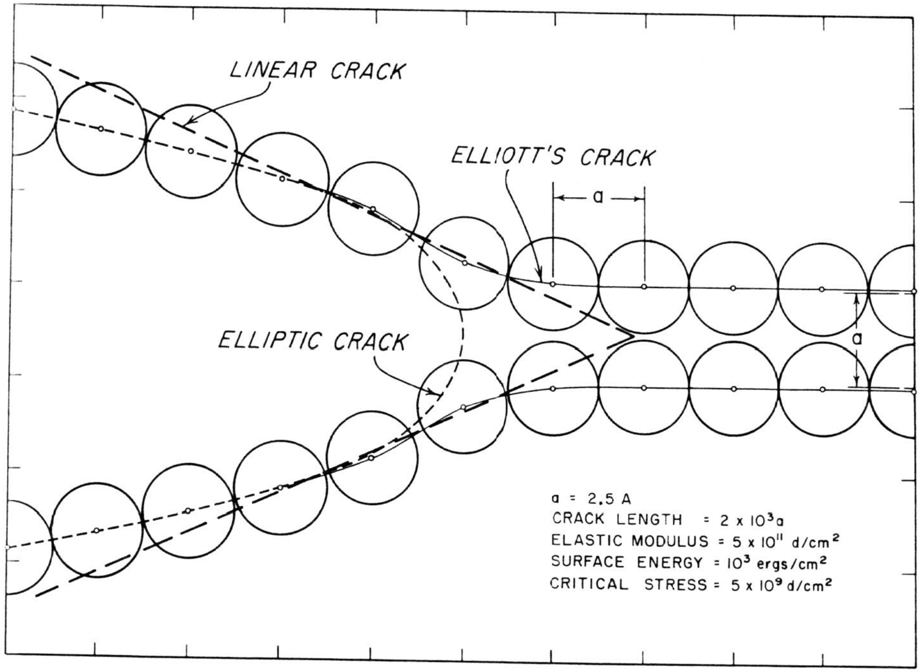


**Figure 1**

**Atomic Displacements Near the Tip of a Critical Griffith Crack According to Elliott's Solution**

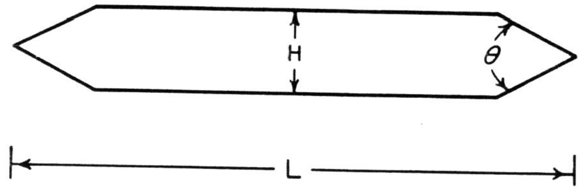


**Figure 2**

**Linearized Critical Crack of Length L, Depth H, and Tip angles θ.**

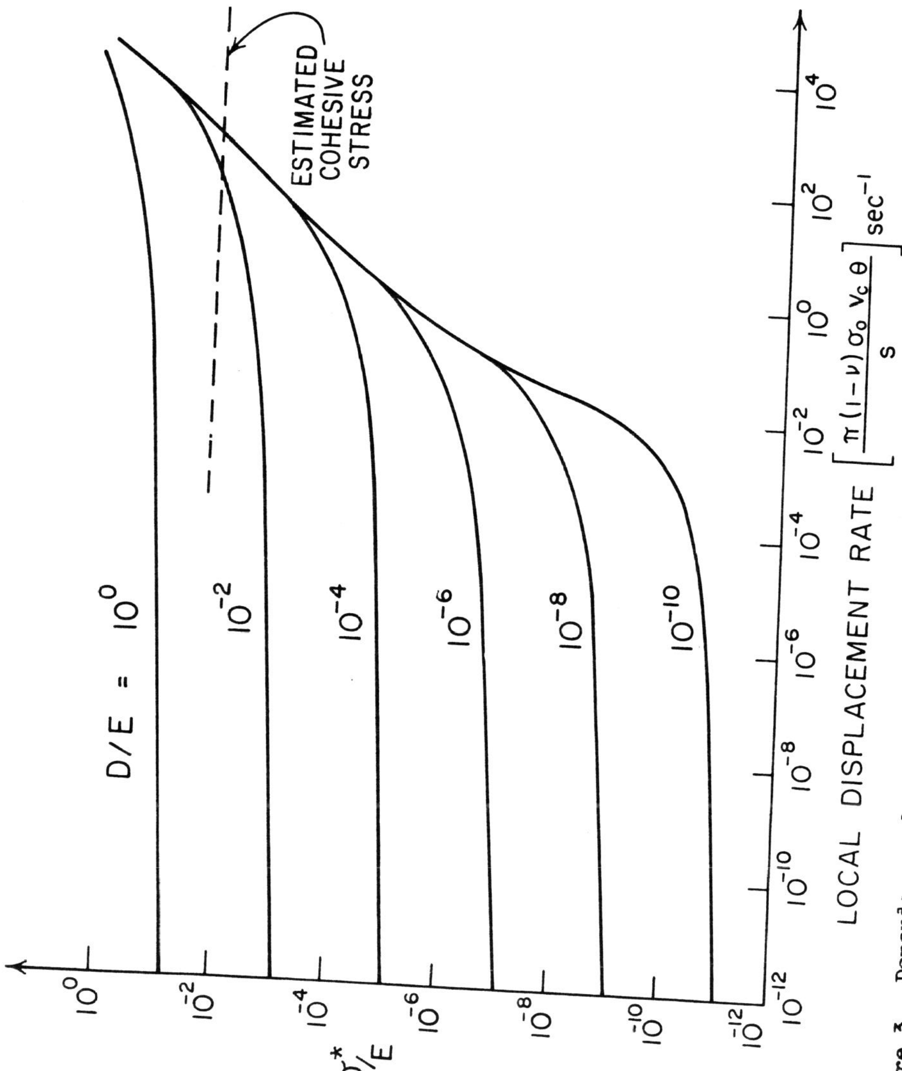


Figure 3 Dependence of the Local Upper Yield Stress on Displacement Rate and Characteristic Drag Stress (constants were: $C_1 = 1.36 \times 10^8$; $C_2 = 10^{-6}$).