A-19 A Comparative Study of Crack Propagation in Plates under Extension and Bending*

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Abstract

A comparative study of fatigue crack propagation in thin plates under fluctuating plane extension and cylindrical bending is made. As a basis of comparison an analysis based on a modified version of crack growth mechanism proposed by Schijve is used. Fully reversed bending tests on 2024-T3 and 7075-T6 aluminum alloys were performed and the results were compared with those existing for the same materials under plane extension.

1. Introduction

In the fatigue of bulky structures with no severe stress concentrations, the crack initiation phase usually forms the major portion of the fatigue life. During this phase, as a result of cyclic slip, the nucleation of fatigue cracks takes place and by the time these microcracks coalesce to form a dominant macrocrack, the remaining portion of the fatigue life is relatively very short. In this case generally the crack is part-through, the geometry is rather complex and the stress state in the neighborhood of the crack is three-dimensional and very complicated. In such structures the designer is guided by the results of studies leading to S - N type curves and their various modifications to take into account the multi-dimensional aspect of the stress-state.

On the other hand in structures composed of thin plates and shells the formation of a dominant macrocrack takes place relatively early in the fatigue life. Hence in such cases the propagation phase, i.e. the number of load cycles necessary for the fatigue crack to reach a critical length at which the structure may fail statically, represents the major portion of the total fatigue life. The dominant crack

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becomes a through crack shortly after its formation and, generally speaking, in the continuum treatment of the problem, a "two-dimensional idealization" is justified.

In thin plate and shell structures, usually the main component of loading is that which gives rise to meridional stresses. Hence justifiably the studies up to date dealing with the fatigue crack growth in thin plates have been confined to a wide plate with a central crack under repeated symmetric uniaxial loading [1-12]* In spite of some ambitious attempts to take into account all the relevant variables affecting the growth of fatigue cracks [13], the plausible explanations [14] are still partial and qualitative, and those with more thorough scope serve, at least for the moment, only to demonstrate the underlying complexity of the problem.

The dislocation-oriented theories attempting to explain all phases of fatigue mechanism are qualitative and, admittedly, far from rigorous [14,15,16]. From the designer's view point, since the crack growth is a field phenomenon and the variables available to him, such as the external disturbances, the geometry and the mechanical properties of the medium, are quantities which may lend themselves only to a field approach, for a quantitative analysis of the problem the continuum treatment is unavoidable. The fact that in some materials, notably in aluminum-copper alloys, crack extension occurs in every cycle as shown by fractographical studies of growth lines for macro-cracks [14], lends further heuristic credibility to the idea of treating the crack growth as a continuous phenomenon. Thus numerous continuum "models" have been proposed and reported in literature - each with its own experimental verification - for the fatigue crack propagation in thin sheets under one-dimensional plane loading [1-12].

In some sheet-stringer structures, in addition to meridional loads, the thin sheets are subjected to bending. In plane extension and cylindrical bending the crack initiation and growth mechanisms are expected to be the same. However, quantitatively the relationship between the respective crack growth rates under extension and bending is by no means clear and a mere substitution of the uniform bending stress on the plate surface for the uniform extensional stress away from the crack may result in considerable error in estimating the relative crack growth rates. Hence the primary objective of this study is to investigate the fatigue crack growth rates in thin plates under cylindrical bending and to search for a

quantitative relationship based on continuum considerations between the growth rates in bending and plane extension.

2. Theoretical Considerations

If we assume the mechanism recently proposed by Schijve for the fatigue crack propagation as valid [14], namely that the crack extension may be considered as a geometric consequence of dislocation movements, the crack growth rate for the cyclic loading may be expressed as

$$\frac{\Delta a}{\Delta n} = \varphi m b \tag{1}$$

where a is the half crack length, n is the number of cycles, m is the total number of moving dislocations which could possibly contribute to crack extension, b is the magnitude of Burgers vector and the coefficient ϕ represents the fraction of the total number of dislocations effectively contributing to the crack extension (0< φ <1). Schijve actually envisions two different mechanisms. In the first (called the dislocation-absorption mechanism) the dislocations move towards the crack and flaw into the tip region and may be considered to be responsible for crack nucleation as well as crack growth at low rates. In the second (called the dislocation-generation mechanism), due to the presence of high shear stresses, the dislocations are "generated" at the tip region and may be responsible for the crack growth at high rates. As pointed out by Schijve both mechanisms may be active simultaneously the former being more dominant at low growth rates and the latter at high growth rates. Also both are essentially "sliding-off" mechanisms, in the first sliding-off starts at the interior and moves towards the tip, in the second it starts at the tip and moves towards the interior of the material.

Even though it is difficult to establish the functional relationships between the quantities m and φ needed for the estimation of crack growth rate on one hand and those which may readily be related to the continuum parameters on the other, for the sake of forming a basis of comparison between the extensional and bending cases one may argue that according to the mechanisms envisioned by Schijve, the dislocation movements will be concentrated in the plastic zone and those confined to a plane emanating from the tip will primarily be responsible for the creation of a new surface in a given cycle. Thus it may be assumed that m will be a function of a representative length, p, of the plastic zone and the magnitude of the plastic strains. However, on account of lack of reliable quantitative information about the plastic strains in plane extension and bending and considering the fact that p is dependent on the distribution of these strains, with the simplifying assumption of geometric similarity, it may

^{*} Numbers in brackets refer to references at end of paper

conversely be assumed that in a given material and geometry the magnitude of plastic strains will be dependent on the size of the plastic zone. Hence m may be expressed as a function of p only:

$$m = f_1(p) \tag{2}$$

Here departing from Schijve's line of argument we will assume that the coefficient φ will also depend on the magnitude of plastic strains. The reason for this is that the active dislocations are confined to the plastic zone where "stress" has essentially very little meaning and generally speaking the true measure of the severity of the forces compelling the dislocations to move is the magnitude of plastic strains. Following a similar reasoning as described above, φ may then also be assumed to be a function of p:

$$\varphi = f_2(p) \tag{3}$$

At this point all one can say about f_1 and f_2 is that they are monotonically increasing smooth functions and vanish at p = 0. Thus, within a given range of p, f_1 and f_2 may be approximated by appropriate power functions as follows

$$f_1(p) = A_1 p^{\alpha_1}, \quad f_2(p) = A_2 p^{\alpha_2}$$
 (4)

where A_1 , A_2 , α_1 , α_2 are positive constants. Combining the constants and considering the crack growth rate as a continuous process, (1) may then be written as

$$\frac{d\mathbf{a}}{d\mathbf{n}} = \mathbf{A} \mathbf{p}^{\alpha} \tag{5}$$

where A and a are positive constants. Here it should again be emphasized that the purpose of deriving (5) is not to add another "power law" to those which already exist, but to find a simple quantitative basis for the comparison of fatigue crack growth rates under various types of loading. In fact (5) and the arguments leading to it demonstrates the futility of searching for an all-encompassing "power law" applicable to all materials and loading ranges - as the nature of functions f1 and f2 may be different for different materials as well as for a given material with different microstructures and the constants α_1 and α_2 may vary with p rather drastically. In fact for greater plastic strain magnitudes or p. it is reasonable to expect that would also be greater. This partly explains the success of various "power laws" in their agreement with different experimental data.

If the representative length, p, is taken as the dimension of the plastic zone along the prolongation of the crack it may be estimated in various ways. The estimate given by Dugdale [17] appears to be fairly realistic and seems to agree with the available experimental results. The estimate is based on the removal of the stress singularity at the crack tip by introducing a rigid plastic strip ahead of the crack and, for a wide plate under plane stress, is found to be

$$p = [\sec(\frac{\pi\sigma^{\infty}}{2\sigma_{v}}) - 1] a$$
 (6)

where a is the half-crack length and σ_y is the yield stress. for small values of σ^{∞}/σ_y the first term approximation of (5) is

$$p = \frac{1}{2} \left(\frac{\pi \sigma^{\infty}}{2 \sigma_{y}} \right)^{2} a = \frac{1}{2} \left(\frac{\pi}{2 \sigma_{y}} \right)^{2} k^{2}$$
 (7)

where $k = \sigma^{\infty} \sqrt{a}$ is the stress intensity factor.

In the case of cylindrical bending the representative length of the plastic zone, p_b , may be estimated following an argument quite similar to that of Dugdale [17]. Here we will also assume that the stress state in plate remains elastic with the exception of a strip of material ahead of the crack, the response of which is rigid-plastic and a plastic hinge forms along the length p_b on the tension side of the plate. Assuming a crack length $2(a+p_b)$, the stresses at the crack tip due to uniform cylindrical bending at infinity with a surface stress σ_b and moments distributed over the portion of the crack surface $-(a+p_b)$ to -a and a to $(a+p_b)$, are respectively, given by

$$\sigma_{ij} = \frac{\sigma_o \sqrt{a + p_b}}{\sqrt{2r}} \frac{\delta}{h} f_{ij}(r, \theta)$$
 (8)

$$\sigma_{ij} = -\frac{\sigma_{y} f_{ij} (r, \theta)}{\pi \sqrt{2r} \sqrt{a+p_{b}}} \left[\int_{-(a+p_{b})}^{-a} \sqrt{\frac{a+p_{b}+x}{a+p_{b}-x}} dx + \int_{a}^{a+p_{b}} \sqrt{\frac{a+p_{b}+x}{a+p_{b}-x}} dx \right]$$
(9)

where δ is the distance from neutral plane, r, θ , are the polar coordinates at the crack tip and σ_y is the yield stress. Now if we use the condition that under the combination of

(8) and (9) the stress singularities at the points $-(a+p_b)$ and $(a+p_b)$ over the half-thickness from 0 to h will vanish we find

$$p_{b} = a \left[sec \left(\frac{\pi \sigma_{b}^{\infty}}{\mu \sigma_{y}} \right) - 1 \right]$$
 (10)

Again, for small values of $\sigma_b^{\infty}/\sigma_y$, first term approximation of (10) may be written as

$$p_b = \frac{1}{2} \left(\frac{\pi \sigma_b^{\infty}}{4\sigma_y}\right)^2 a = \frac{1}{2} \left(\frac{\pi}{4\sigma_y}\right)^2 k_b^2$$
 (11)

where $k_b = \sigma_b \sqrt{a}$ is the stress intensity factor in bending. According to this estimate the comparison of (6) and (10) shows that for the same characteristic plastic zone sizes in tension and bending we have $\sigma_b^{\infty} = 2\sigma^{\infty}$.

Substituting from (6) and (10) into (5) the crack growth rates may now be expressed as

$$\frac{da}{dn} = A a^{\alpha} \left[\sec \left(\frac{\pi \sigma^{\infty}}{2\sigma_{y}} \right) - 1 \right]^{\alpha} \qquad \text{(in tension)} \qquad (12)$$

$$\frac{da}{dn} = A_b \ a^{\alpha b} \left[\sec \left(\frac{\pi \sigma_b^{\infty}}{4 \sigma_y} \right) - 1 \right]^{\alpha b} \qquad \text{(in bending)} \qquad (13)$$

Or, for small values of $\sigma^{\infty}/\sigma_{y}$ and $\sigma^{\infty}/\sigma_{y}$ we may write

$$\frac{da}{dn} = A 2^{-\alpha} \left(\frac{\pi}{2\sigma_{V}}\right)^{2\alpha} k^{2\alpha} \qquad \text{(in tension)} \qquad (14)$$

$$\frac{da}{dn} = A_b 2^{-\alpha}b \left(\frac{\pi}{4\sigma_y}\right)^{2\alpha}b k_b^{2\alpha}b \qquad (in bending) \qquad (15)$$

Because of the similarity of fracture modes if we assume that $A = A_b$ and $\alpha = \alpha_b$, by combining the constants (14) and (15) may finally be written as

$$\frac{da}{dn} = B k^{2\alpha}$$
 (in tension) (16)

$$\frac{da}{dn} = 2^{-2\alpha} B k_b^{2\alpha} \qquad (in bending) \qquad (17)$$

In the case of fluctuating external loads with nonzero mean one more remark about the use of (12) through (17) is

necessary. Based on the simple argument leading to (4) it may be assumed that the calculation of total number of dislocations, m, should be based on maximum plastic zone size which could be estimated from (6) by substituting σ_{max}^{∞} for σ^{∞} . On the other hand it is expected that the factor φ would be influenced primarily by the range value of plastic strain fluctuations which, in turn, is dependent on σ_{max}^{∞} omin. This would indicate that the crack growth rate would depend on both maximum as well as the range values of the external loads. The experimental results concerning the dependence of da/dn on the mean value of the external loads are far from conclusive and some carefully controlled experiments are needed to clarify this point further. However, on the basis of shake-down considerations one may assume that the dominant load factor in the crack growth phenomenon will be the range of the external loads, $(\sigma_{\max}^{\infty} - \sigma_{\min}^{\infty})/2 = \Delta \sigma^{\infty}$, and hence, at least for comparison purposes in (12), (13), (16) and ($\frac{1}{2}$ 7) the quantities σ^{∞} , σ_b , k and k_b may be replaced by $\Delta \sigma^{\infty}$, $\Delta \sigma_b^{\infty}$, Δk and Δk_b , respectively.

3. Experimental Results

The test specimens consisted of 7075-T6 and 2024-T3 bare and clad commercial aluminum alloys (Table I). Triangular specimens cut from 12"x18" plates were used. In the plane of the crack the plate width was 8.625". The length of the artificial cut(consisting of 0.04" diameter hole, 0.015 in, wide saw cut and a tap with razor blade) was approximately 0.2 in. Before putting the cut on, the strain gages placed at various locations indicated that in the central trapezoidal portion of cantilever plates the strains were constant within 3%. The machine was a fully reversed constant displacement type running at 140 cpm. The static deflection tests with dead weights performed at various crack lengths showed that the stiffness of the plate and the strains away from the crack remained constant within a few percent up to plate width-to-crack length ratio (b/a) = 5. For larger crack lengths the bulging around the crack became significant enough to cause a reduction in the stiffness. Thus all tests were stopped at about 1.8 in. crack length.

The measurement of the tip-to-tip crack length was started after natural fatigue crack was formed and extended approximately 0.01 in. at each tip which roughly corresponds to the depth of triangular notch put by the razor blade. Crack length was measured by a fifty power traveling microscope by stopping the test on the peak of the tension cycle. The sensitivity of the microscope (*10-5 in)

is found to be higher than the accuracy within which the "crack tip" could be defined. The indefiniteness of the detection of the crack tip was partly due to the surface roughening in clad specimens. In the bare specimens there was no surface roughening, however some contraction in thickness at the crack tips was observable.

Some peculiarities of the crack tip in a thin plate under bending may be observed from Figure 1 which shows the photographs of the cross-section of the plate with a plane perpendicular to both the plate and the plane of the crack. Figure la indicates that the crack front is not a straight line. In fact, as also seen from Figure 2, which shows the fracture surface after static rupture, the crack front has a symmetrical V shape with its vertex pointed toward the middle of the crack. Occasionally the branching of the crack took place on the surface, usually one branch turned out to be the dominating through crack and the other eventually stopped growing. However on rare occasions, branching occured simultaneously on both surfaces, both being through cracks. In this case, too, after certain number of cycles one of the branches dominated and the crack grew macroscopically perpendicular to the direction of maximum bending stress. Figure 1d shows a section of such a branched crack.

The results of bending tests obtained for various stress levels and plate thicknesses are shown in Figures 3 and 4. The comparable results for the fully reversed extension tests are shown in Figures 5 and 6 [8]. Figures 7, 8, 9 show some of the results for the individual plates. The scatter bands covering 95 percent of the total population for a least square straight line fit and for $\alpha=2$ are also shown in the Figures 5 to 9.

4. Discussion of the Results

Since the stress intensity factor, $k = \sigma^{\infty} \sqrt{a}$, is the simplest and the most appropriate single variable representing both the external load and the geometry of the problem, in Figures 3 through 9 the crack growth rate is plotted as a function of k. Because of the zero mean stress, we have $k = k_{max} = \Delta k$, for both extension and bending. However, apart from the material constants, since da/dn is a function of k alone only for small values of $\sigma^{\infty}/\sigma_{y}$, for relatively large values of $\sigma^{\infty}/\sigma_{y}$, the choice of k as the independent variable may not be appropriate. To study the limitations of this choice we express for example (12) in the following form

$$\frac{\mathrm{d}a}{\mathrm{d}n} = B k^{2\alpha} \beta^{\alpha} \tag{18}$$

where from (12), (14) and (16), it is seen that the coefficient β is given by

$$\beta = 2 \left[\sec \left(\frac{\pi \sigma^{\infty}}{2 \sigma_{y}} \right) - 1 \right] / \left(\frac{\pi \sigma^{\infty}}{2 \sigma_{y}} \right)^{2}$$
 (19)

Similarly, for bending

$$\frac{\mathrm{d}a}{\mathrm{d}n} = 2^{-2\alpha} \, \mathbf{B} \, \mathbf{k}_{\mathbf{b}}^{2\alpha} \, \beta_{\mathbf{b}}^{\alpha} \tag{20}$$

$$\beta_{b} = 2 \left[\sec \left(\frac{\pi \sigma_{b}^{\infty}}{4 \sigma_{y}} \right) - 1 \right] / \left(\frac{\pi \sigma_{b}^{\infty}}{4 \sigma_{y}} \right)^{2}$$
 (21)

Note that β and β_b are functions of stress ratios, σ^{∞}/σ_y and $\sigma_b^{\infty}/\sigma_v$, only. That is, for a fixed stress state at infinity da/dn would depend mainly on the stress intensity factor. Equations (19) and (21) along with log108 are plotted in Figure 10. For bending, \$b varies between 1 and 1.35 as σ_b^∞ goes from zero to σ_y . On the other hand in extension β goes to infinity as σ^∞ approaches σ_y . Here it should be added that the stress ratios given in Table I are based on yield stresses (0.2% offset) in tension obtained from a single loading test. Since the yield stress increases considerably after the first few cycles, in the calculation of β a value of σ_V higher than that indicated in Table I should be used. For example, in 2024-T6 aluminum alloy or increases by approximately 40% after the first cycle and remains reasonably constant thereafter [14]. This would also indicate that the variation in σ_v obtained from a single loading of virgin materials of essentially same composition would not have much influence on da/dn. The reason for this may be that in such materials the limiting values of the vield stresses, which are attained after very small number of cycles, would be approximately the same.

If the assumptions leading to (12) is valid, (18) indicates that of the two plates with equal stress intensity factors the one with a higher stress state at infinity will have higher crack growth rate. In a given range of k for which α may be assumed to be constant, this would mean a parallel shift in the logarithmic plot of da/dn vs. k. On the other hand, as pointed out above in the discussion preceding (5), α would be expected to increase with increasing plastic zone size, that is with $k^2\beta$. Hence crack growth

data obtained from various specimens tested under different stresses at infinity and approximately same initial and final crack lengths would be qualitatively similar to that shown in Figure 11. For the ranges of the experimental data shown in Figures 3 through 9 since the shift, $\alpha \log \beta$, in the logarithmic plot, da/dn vs. k, is of the same order of magnitude as the natural scatter of the data, it is difficult to verify (18) and (20) quantitatively. However, a general qualitative trend similar to Figure 12 is easily observable from the data.

A summary of experimental results appears in table I. However reluctant one may be to simplify the phenomenon by using power functions for da/dn, at least for the range of growth rates from 10^{-6} to 10^{-3} in./cycle, the data itself suggests such simplification. Hence writing

$$\frac{\mathrm{d}a}{\mathrm{d}n} = B k^2 \alpha \tag{22}$$

B and α can be determined from a best straight line fit in least square sense, where k is the stress intensity factor (in extension or bending). The values of B and 2α for plates with various thicknesses subjected to reversed bending and extension [8] are shown on columns 5 and 6. The fact that the correlation coefficient, r, (column 7) is so close to unity implies that for the present data an assumption such as (22) is not unrealistic. However, the values found for 2α indicate that there is no simple power function (or a single value of α) applicable to both materials for both loading conditions.

On the other hand to compare the crack growth rates under bending and extension one may "select" a value for α and compare the values of B obtained from the straight lines going through the center of gravity of the data. Thus, selecting $\alpha=2$, we may write

$$(\frac{da}{dn})_t = (B_1)_t k^4$$
, $(\frac{da}{dn})_b = (B_1)_b k_b^4 = (B_1)_t (\lambda k_b)^4$ (23)

where k, and $k_{\rm b}$ are the stress intensity factors in extension and bending respectively. Column 8 shows the values of $B_{\rm l}$ obtained from the data. Writing (16) and (17) as

$$\left(\frac{\mathrm{d}a}{\mathrm{d}n}\right) = B k^{2\alpha}, \quad \left(\frac{\mathrm{d}a}{\mathrm{d}n}\right)_b = B(0.5k_b)^{2\alpha}$$
 (24)

it is seen that the theoretical value of λ which appear in (23) is 0.5, whereas its experimental values are given on column 9. The agreement seems to be fairly good implying

that in the absence of experimental data, for a quick estimate of crack growth rate in plates under cylindrical bending one may use the extension data by merely replacing k by kb/2. Selecting a value for α other than 2, the present data indicate that, for the same stress intensity factor in extension and bending of thin plates, the relation

$$\left(\frac{\mathrm{da}}{\mathrm{dn}}\right)_{b} = 2^{-2\alpha} \left(\frac{\mathrm{da}}{\mathrm{dn}}\right)_{t} \tag{25}$$

is approximately valid and may be used to estimate the crack growth rate in bending.

Concerning the value of λ defined in (23) it should be pointed out that the theoretical value of λ being 0.5 is based on, among other simplifications, the fact that the plate is "thin", the elastic stress distribution is linear in δ , the distance from the mid-plane, and a plastic hinge develops ahead of the crack tip. However, as the plate thickness increases none of these assumptions will be valid, the nonlinearity of the stress distribution in δ will become more significant [18,19] and in limit the stress state will approach that of plane strain. Hence, it would be expected that the value of λ will increase with the increasing plate thickness*. On the other hand for relatively thin plates, this variation in λ is somewhat compensated by the fact that as a result of the plastic hinge never fully developing the theoretical value of λ for "thin" plates would be actually less than 0.5.

To check the validity of the assumption $A=A_b$, $\alpha=\alpha_b$ in going from equations (14) and (15) to (16) and (17), in Figures 12 and 13 the crack growth rates are plotted against the plastic zone size. Figure 13 shows the result of extensional tests with the scatter band of bending results superimposed on it. Even though the result seems to be satisfactory, mainly due to the limited nature of the extensional data it is impossible to make any firm statement about the validity of the assumption.

^{*} In plane strain the plastic zone size is approximately one half of that for plane stress; thus for this case the value of λ would be approximately $1/\sqrt{2}$.

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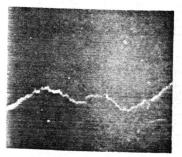
6	λ (α=2)	0.460	0.474	0.479	0.493	0.500	0,502				
8	B ₁ x10 ² 2 (in? lb- ⁴)	5.8	6.5	6.7	7.1	7.6	7.7	7,61	130		120
7	ı	.87	96.	.99	.97	16.	.99		.98		66.
9	2α	3.06	3,21	2,83	4.43	4.35	3.99	,	3.68		3.84
5	B (in., lb.)		1.76×10 ⁻¹⁸	7.60×10-17	1.04×10-23	2.17x10 ⁻²³	7.98x10-22	- 20	8.24×10		4.19×10 ⁻²⁰
ħ	Range of σ'/σ_y	0.346 - 0.475 5.25×10 ⁻¹⁸	0.446 - 0.826 1.76x10-18	0.495 - 0.672 7.60x10-17	0.232 - 0.668 1.04x10-23	0.289 - 0.785 2.17x10 ⁻²³	0.358 - 0.693 7.98x10-22		0.081 - 0.402 8.24x10 - 3		0.113 - 0.565 4.19×10 ⁻²⁰
3	Thickness (in.)	0.050	0.100	0,120	0.080	0.100	0.125	0.081		0.081	
2	No. of Plates	ĸ	æ	7	7	8	9		m		5
1		7075-16	Bare & Clad	(Bending)	2024-T3	Bare & Clad	(Bending)	7075-T6	Bare & Clad (ref. 8)	2024-T3	Bare & Clad (ref. 8) (Extension)



a) 2h = 0.100 in.



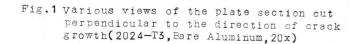
b) 2h = 0.100 in.



c) 2h = 0.125 in.



d) 2h = 0080 in.



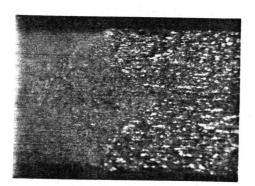
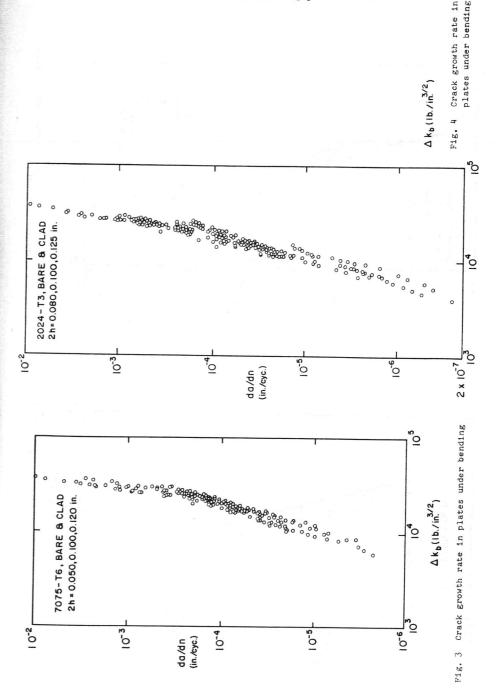
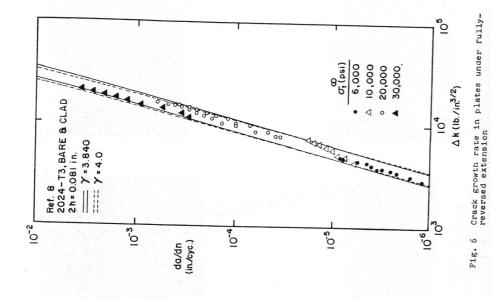
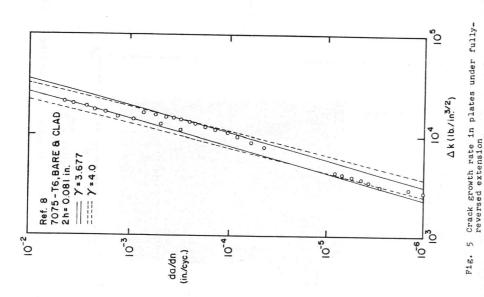


Fig. 2 Fracture surface after static rupture







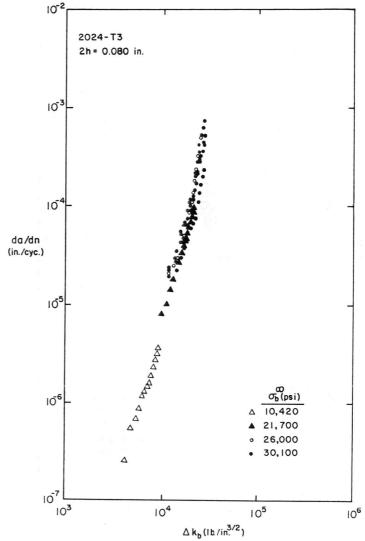


Fig. 7 Bending results for plates with constant thickness

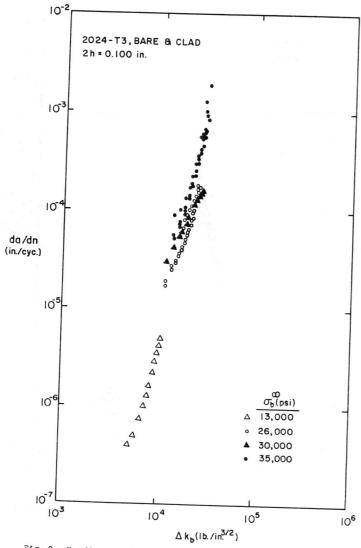
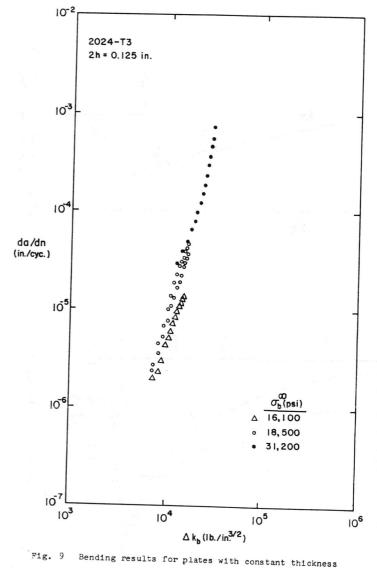
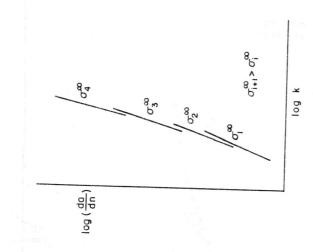


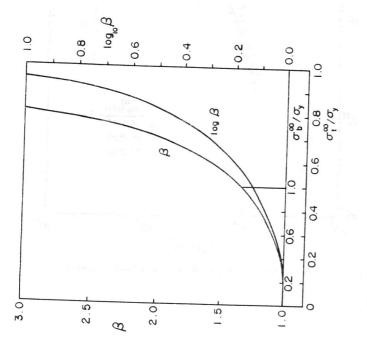
Fig. 8 Bending results for plates with constant thickness



10-2



. 11 Variation of growth rate with the stress level



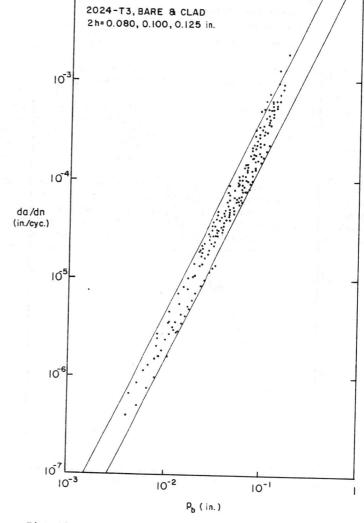


Fig. 12 Crack growth rate vs plastic zone size (bending)

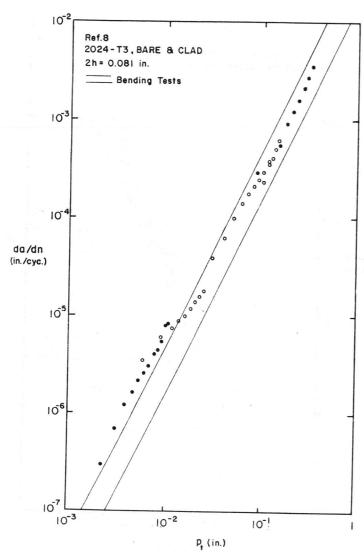


Fig. 13 Crack growth rate vs plastic zone size (extension)