24. Propagation of Cracks and Work Hardening

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ABSTRACT

The influence of plastic deformation on the propagation of a crack is discussed from a theoretical point of view. The interaction of the moving crack with dislocations already present in the material is considered, as well as the possible creation of new loops of dislocation by plastic relaxation. It is concluded that dislocations already present have very little effect on the propagation of the crack, even in heavily cold-worked materials. What matters essentially is whether plastic relaxation can occur, and, to a much smaller degree, whether the material is polycrystalline, making it necessary for the crack to cross grain boundaries.

Introduction

When under a large enough stress, a crystalline material can either shear plastically (by slipping or twinning), lengthen by fracture, or do both successively or simultaneously. It has been much emphasized recently [1–4] that a little plastic strain is often necessary to produce the stress concentrations that start cleavage. It is also known, however, that too much plastic deformation inhibits "brittle" fractures. Large strains, usually of the order of 50 to 100%, are then necessary to produce a "ductile" fracture. It is mainly this second aspect that is in view here, but the various factors involved are more easily studied in the case of cleavage first.

Fracture will occur when a certain equilibrium is reached between the driving force and the resistance encountered. The special resistance produced by work hardening will be discussed first, following which will be a discussion of the driving force to be expected in a work-hardened material. It will be emphasized that the exact nature of work hardening does not matter very much in ductile fracture, and finally, it will be

pointed out that the resistance offered by a work-hardened material to cleavage and the resistance to slipping or twinning are similar in nature, although somewhat different in strength.

Resistance Offered by Work Hardening to the Propagation of Cleavage

Two kinds of effects must be considered: surface effects (steps, blunting of the tip of the crack) and long-range elastic effects (interaction of the crack with dislocations already present, plastic relaxation). They will be analyzed by comparing cases where work hardening is absent to those where it precedes or is simultaneous with crack propagation.

Brittle Cleavage Without Stress Relaxation

It should first be recalled that the tip of a crack can be considered as a piled-up group of climbing dislocations. This analogy helps to clarify the study of the resistance offered in this case by the grown-in dislocations, and it especially aids in explaining the "river" markings.

Griffith crack as a piled-up group of climbing dislocations. It is self-evident that the production of a Griffith crack (Fig. 1) can be described in terms of climbing dislocations. It is produced by a group of n' dislocations climbing perpendicular to their Burgers vectors b' from region A to regions B and B' at the tip of the crack. There is, in fact, a continuous distribution of such dislocations with infinitesimal Burgers vectors b' so that

$$n'b' = h \tag{1}$$

where h is the distance between the sides S_1 and S_2 of the crack.

Since the length L of the crack is much greater than both the width h and the atomic dimensions, the state of strain and the elastic energy stored in the material are, not surprisingly, similar to those around a slip band produced by a piled-up group of gliding dislocations.[5] Thus most of the n' dislocations are concentrated at the tip of the crack. At equilibrium, their back stress compensates for the stress σ applied on the crack (Fig. 1a). At the center A of the crack, this gives

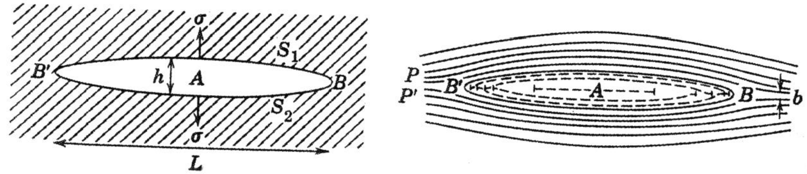
$$\sigma = \frac{\mu n' b'}{\alpha(1-\nu)L} = \frac{\mu h}{\alpha(1-\nu)L} \tag{2}$$

where μ is the elastic modulus, ν is Poisson's ratio, $\alpha = 1$ for a penny-shaped ellipsoidal crack,[6,7] and $\alpha = \pi/2$ for a long cylindrical one.[8]

The stress σ' at a distance r from the tip of the crack is

$$\sigma' \simeq \sigma \left(\frac{L}{r}\right)^{1/2} \tag{3}$$

if r is small compared with L but large compared with atomic dimensions. The force F per unit length acting on the leading dislocations at the tip of the crack is obtained by equating the work done on the leading dislocations when the crack lengthens to the work done by the applied



(a) (b)

Fig. 1. Griffith crack as a piled-up group of climbing dislocations: (*a*) the crack, (*b*) the distribution of dislocations with their (empty) supplementary half-planes.

stress σ. As most of the n' dislocations are at the tip of the crack, this gives, according to Eq. 3,

$$F = \beta\sigma h = \frac{\alpha\beta(1-\nu)L\sigma^2}{\mu} \tag{4}$$

where β is a numerical factor of the order of 1. Exact computation [8] gives $\beta = \frac{1}{2}$.

In a perfect crystal, a crack is produced when the atomic planes P and P' (Fig. 1*b*) have separated a distance approximately equal to their normal distance b, thus breaking their bonds. At the tip of the crack, *the leading group of dislocations with physical significance has therefore a total Burgers vector b.* The crack propagates when the driving force (Eq. 4) acting on it is larger than the resistance resulting from the production of surface energy γ_0 on the cleavage plane. This leads to the well-known Griffith criterion for propagation:

$$\sigma \geq \left[\frac{4\mu\gamma_0}{\alpha(1-\nu)L}\right]^{1/2} \tag{5}$$

with $\alpha \simeq 1$.

Elastic interaction with grown-in dislocations. Crystals, even when well annealed, are rarely perfect. The dislocations of their Frank net [9] will interact with the propagating crack. We assume here that they are fixed, leaving for later the question of plastic relaxation.

One type of interaction is due to the long-range stresses produced by the Frank net. This will be shown to be negligible. Dislocations *outside* the cleavage plane produce a randomly distributed internal stress [10] σ_i of average value zero, with wavelengths approximating the size l_F of the Frank net in the three dimensions and with an amplitude of the order of $\mu b/2\pi l_F$. By a well-known argument,[11] this stress is too weak to produce

any significant deviation of the tip of the crack, because the tip is equivalent to a dislocation with a very strong line tension $\mu(n'b')^2 = \mu h^2$. The tip of the crack remains straight, and it averages out these internal stresses, which exert no measurable frictional force.

Dislocations *piercing* the cleavage plane exert at short range a stress that varies inversely with the distance r and vanishes only if a dislocation is normal to the cleavage plane or if its Burgers vector is parallel to it. This stress varies in sign and strength with the nature and the orientation of the dislocation, but its average absolute value is of the order [10] of

$$\sigma_0 = \frac{\mu b}{10r} \tag{6}$$

Such a stress, acting on a crack length of at most r, produces a total force $\mu bh/10$. This is again negligible compared with the line tension μh^2. The tip of the crack, remaining straight, averages out this kind of interaction too.

Short-range interactions with grown-in dislocations. This is the interaction usually considered.[12] When the tip of the crack crosses a dislocation with Burgers vector not parallel to the cleavage plane, the screw component of the dislocation produces a step of height b (or a multiple of b) on each face of the cleavage (Fig. 2). These steps usually run normal to the tip of the crack, thus minimizing their length (Fig. 3*a*). Their tension γ is then usually larger than γ_0 in the cleavage plane because their orientation is not a simple crystallographic one.*

These steps increase the resistance to the propagation of the crack. For steps normal to the tip of the crack, Griffith's criterion becomes

$$\sigma \geq \left[\frac{4\mu\gamma_{\text{eff}}}{(1-\nu)L}\right]^{1/2} \tag{7}$$

with

$$\gamma_{\text{eff}} = \gamma_0 + \frac{\gamma b}{l} \tag{8}$$

where l is their average distance along the tip of the crack. With a reasonable value ($\gamma \simeq \mu b/5$), the traction ($\gamma b \simeq \mu b^2/5$) of a step is very

* These steps would follow a simple crystallographic direction only if the gain in surface tension could compensate for their increase in length. A glance at Fig. 3*b* shows that the condition is

$$\gamma_0' < \gamma \cos\phi$$

if the crystallographic direction considered has a surface tension γ_0' and makes an angle θ with the tip of the crack. Except in very anisotropic substances, γ_0' is unlikely to be much smaller than $2\gamma/3$, or perhaps even $\gamma/2$, so that ϕ must be near to the value $\pi/2$. An example of multiple steps with probably two crystallographic directions has been observed on calcite [13] with $\phi_1 \simeq \pi/4$ and $\phi_2 \simeq \pi/20$, but such a behavior seems exceptional.

small compared to the line tension μh^2 of the tip of the crack; the presence of the steps does not alter appreciably the general course of the crack. When the tip of the crack crosses a screw dislocation, there is a

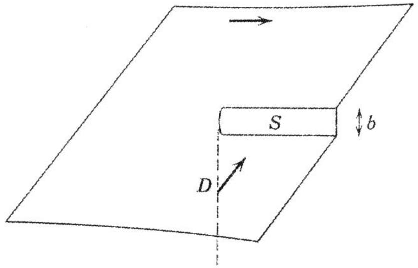


Fig. 2. Step S produced on a cleavage surface when the tip of the crack, moving from left to right, crosses a dislocation D with a screw component.

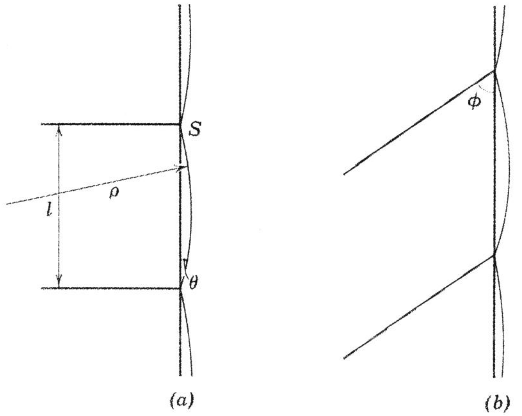


Fig. 3. Orientation of the steps relative to the tip of the crack: (*a*) general case, (*b*) special case of low surface tension.

further expenditure of energy to lengthen the dislocations of the crack by the height b of the forming step. For the leading dislocation, this is an energy expenditure of the order of that required by a jog [14,15] (thus of the order of $\mu b^3/10$) spent over a distance b; this leads to an effective tension γ_{eff} of the same order of magnitude as that obtained using Eq. 8. All these effects are therefore negligible.

Coalescence of steps. When a crack has propagated through distances that are large compared with the average distance l_F between the "trees" of the forest of screw dislocations, one could expect the density

of steps to be large and the resistance to cleavage appreciable. This is, however, not so, because the individual steps quickly coalesce. This process gives rise to many mutual annihilations of steps with opposite signs and to a few multiple steps that become optically visible. These are the "river" markings of brittle cleavages.

Rivers are possible because the course of a step is usually *far from straight*. The reason is that its course is set by the behavior of the *leading dislocation* with Burgers vector b at the tip of the crack. This dislocation has a "normal" line tension $T \simeq \mu b^2$; it can therefore progress from left to right (Fig. 3*a*) only if it bows out with a fairly large angle θ. The equilibrium of the tensions at point S gives:

$$\sin\theta = \frac{\gamma b}{T} \simeq \frac{\gamma}{\mu b} \tag{9}$$

A reasonable value ($\gamma \simeq \mu b/5$) gives $\sin\theta \simeq \theta \simeq \frac{1}{5}$ rad. The corresponding force F_0 applied on the leading dislocation is that which is necessary to produce the steps:

$$F_0 = \sigma h - 2\gamma_0 = 2\gamma\frac{b}{l} \simeq \frac{2\mu b}{5l}$$

When there are not yet too many steps ($l \geq l_F$), this is not a very large force; any perturbation might alter noticeably the course of the steps.

The forces associated with the internal stresses described earlier are of the same order of magnitude. They should make a step *oscillate* markedly around its general direction, which is normal to the tip of the crack.

When two neighboring steps are at a distance l' smaller than the average distance l between steps, they should quickly coalesce. This is shown in Fig. 4, where the steps are assumed, for simplicity, to be alternately at distances l' and l'' ($\neq l'$) such that $l' + l'' = 2l$. The equilibrium of the tensions at a point such as M gives

$$-\frac{dl'}{dx} = 2\cot\phi \simeq \pi - 2\phi = \theta' - \theta''$$

with

$$\frac{l'}{\sin\theta'} = \frac{l''}{\sin\theta''} = \frac{2T}{F_0} \simeq \frac{\mu b l}{\gamma}$$

The solution of the equation is

$$\frac{l - l'}{l} = \exp\left[-\frac{\gamma(x - x_0)}{\mu b l}\right] \tag{10}$$

Neighboring steps should coalesce when the tip of the crack has advanced by

$$x_0 = \frac{\mu b l}{\gamma}\ln\frac{2l}{\delta l'} \tag{11}$$

if $\delta l = (l'' - l')_{x=0}$ is the initial difference of distance between the steps. This will usually be an appreciable fraction of the average distance l,

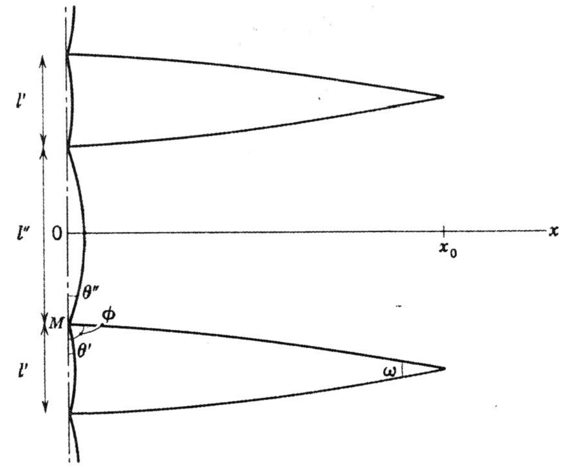


Fig. 4. Coalescence of nonequidistant steps.

and x_0 is then of the order of a few $\mu bl/\gamma \simeq 5l$. The two steps should meet at angle

$$\omega = -\left(\frac{dl'}{dx_0}\right) = \frac{\gamma}{\mu b} \simeq \theta \simeq \frac{1}{5} \tag{12}$$

Multiple steps. When two steps meet, they either form a double step of height $2b$ or annihilate, depending on whether their signs are the same or opposite. Double steps may also coalesce into higher ones by the same mechanism.

It is of interest to analyze the behavior of the tip of a crack when it is driving such a multiple step of height mb ($m > 1$). The main point is that the leading dislocation of Burgers vector b arrives at the step at points M and N in different cleavage planes (Fig. 5a). The two parts are normally* connected by a part MN which runs across the forming step, thus forming two elementary steps of height $b/2$ along the edge MN. The subsequent dislocations in the crack widen these two steps until they meet to form the final one with height mb. The equilibrium conditions for the leading dislocation in MN are the same as for simple steps. *Whatever their height, multiple steps should therefore oscillate and coalesce in essentially the same way.* This is borne out by a discussion of the river patterns.

* Another possibility would be for the dislocation to bend over a certain length along the edges $MM'N'N$ of the incipient step (Fig. 5b). The force pulling back the tip of the crack on M and N is of the same order of magnitude as that in Fig. 5a.

River patterns. Two types of river patterns may be distinguished: those formed when a cleavage develops in a crystal, and those formed after cleavage has gone through a grain boundary.

After having gone through a grain boundary, a cleavage has a high density of similar elementary steps that are the result of the crossing of the parallel dislocations of the boundary. These coalesce into multiple

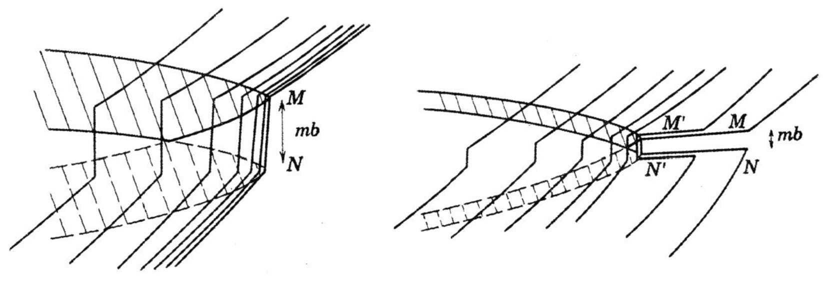


Fig. 5. Two possible behaviors of the dislocations at the tip of a crack where it meets a step of multiple height mb: (a) large step ($m \geq 10$); (b) small step ($m \leq 10$).

steps of increasing height in a characteristic manner [14–16] (Fig. 6). In agreement with the aforementioned conclusions, all the steps are inclined in the same way: Neighboring steps usually run a distance 5 to 10 times their initial distance before coalescing; their courses oscillate somewhat, but they often present a slight curvature towards each other, meeting at a fairly large angle, usually about $\frac{1}{5}$. This confirms our general picture and the estimate, $\gamma \simeq \mu b/5$, for the surface tension along a step. Finally, the height of the steps increases proportionately to the distance from the grain boundary. This height is approximately equal to the distance between steps and may reach large values.

When *developing through a crystal* where it was initiated, a cleavage has less well-marked rivers. These only begin to be optically visible some distance from the origin of the crack. They are made up of steps of both signs, they are somewhat further apart, they coalesce less quickly, and new ones continually form so that their average distance remains fairly constant, of the order of a few microns. Finally, their height increases much less rapidly with distance; some steps actually decrease in height or even stop [14] (point M, Fig. 7). These characteristics are those to be expected from the coalescence of steps of both signs starting from the randomly distributed dislocations P of the Frank net. The short, lateral branches such as PQ (Fig. 7) are not usually visible, because they are made by elementary steps. For no new river to start in between, the distance between two rivers must be of the order of the size l_F of the Frank net.

Each elementary step then coalesces with the nearest river, at a distance of the order of l_F; the average distance between successive meeting points Q, Q', Q'' on a river is thus of the order of l_F. As the elementary tributaries have random signs, the height of the step should increase on

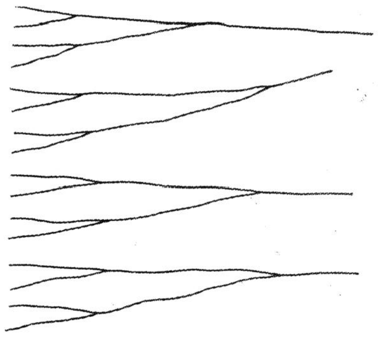


Fig. 6. Coalescence of steps resulting from a low-angle boundary (crack moving from left to right).

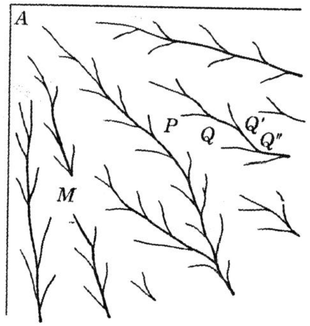


Fig. 7. River patterns within a crystal. The crack started from corner A; the lateral branches such as PQ are usually not visible.

the average as the square root of its total length, which is approximately the crack length L. Therefore

$$mb \simeq \left(\frac{L}{l_F}\right)^{\frac{1}{2}} b \tag{13}$$

Typical values of the constants, $l_F \simeq 10^{-3}$ cm and $L < 1$ cm, give $m \leq 30$ and $mb \ll l_F$. Thus the height of the steps remains small compared with their distance. This is in agreement with observations on LiF.[14]

The effective surface tension appearing in the corresponding Griffith criterion (Eq. 7) is thus practically the same as γ_0 *for a perfect crystal:*

$$\gamma_{\text{eff}} = \gamma_0 + \frac{m\gamma b}{2l_F} \simeq \gamma_0$$

Therefore, the coalescence of steps is a very powerful mechanism for eliminating steps.

Brittle Cleavage with Stress Relaxation

This is a fundamental kind of problem where work hardening plays a role in cleavage.

Around the tip of a crack propagating in a brittle material, stresses

are very high: At a distance r from the tip, small compared with the length L of the crack, they are, according to Eqs. 3 and 5, of the order of

$$\sigma' \simeq \sigma \left(\frac{L}{r}\right)^{\frac{1}{2}} \geq \left[\frac{4\mu\gamma_0}{\alpha(1-\nu)r}\right]^{\frac{1}{2}} \simeq \mu \left(\frac{b}{2r}\right)^{\frac{1}{2}} \tag{14}$$

In this expression, a reasonable value ($\gamma_0 \simeq \mu b/10$) has been taken for the surface tension γ_0 in the cleavage plane.

These very high stresses might induce some plastic deformation near the tip of the crack if they are applied for a long enough time on the same volume of material, that is, if the crack moves slowly enough. Various experiments have now shown that there is a critical velocity of propagation of the crack below which plastic relaxation occurs.

Critical velocity of crack propagation. Plastic relaxation can occur by making neighboring dislocations move or even by creating new ones in regions of perfect crystal. These two processes lead to two somewhat different criteria, which have been studied theoretically by Stroh [17] and Gilman,[18] respectively.

(*a*) *Activation of neighboring sources.* These might be Frank-Read [19] sources, that is, segments of dislocation lines more or less fixed at their extremities, or small loops of dislocations produced by quenching.[20,21]

In *ductile* materials, the stress acting on the nearest Frank-Read sources (at distances $r \simeq l_F$) is of the order of $\mu(b/2l_F)^{\frac{1}{2}}$, according to Eq. 14. It is much larger than the elastic limit, which is of the order of $\mu b/10l_F$ (see Eq. 27). Plastic relaxation will begin when this stress acts for a long time compared with the time necessary for a loop to be emitted $(c/l_F)^{-1}$ (where c is the sound velocity).[22] The critical velocity of crack propagation is thus

$$v \simeq l_F \left(\frac{c}{l_F}\right) = c$$

As this is the maximum possible velocity for the crack, *plastic relaxation should always occur in ductile materials* (except perhaps in the relativistic region $v \simeq c$ where Eq. 14 does not apply).

In *brittle* materials, grown-in dislocations are only able to bend and emit loops under the fairly high stress necessary to move them against a strong Peierls-Nabarro force or free them from pinning by precipitates or impurity atoms. These processes are thermally activated with an activation energy U which usually depends markedly on the applied stress $\sigma_c \simeq \mu(b/2l_F)^{\frac{1}{2}}$. (For fine precipitates and Cottrell clouds at very low temperatures,[23,24] U varies linearly with σ_c; for Cottrell clouds at higher temperatures,[25] U varies as σ_c^{-1}.) As a result, the rate of emission of loops and thus the critical velocity v, which vary as $\exp(-U/kT)$, will depend markedly on the exact size of the Frank net. They should thus *vary from sample to sample.* Cases of plastic relaxation, probably of this type, in iron have been reported by Low.[16]

Furthermore, many of these materials exhibit, in slow homogeneous tensile tests, elastic limits much larger than any likely value of $\sigma_c \simeq \mu(b/2l_F)^{1/2}$. For these materials, plastic relaxation could only occur by a second mechanism which will now be described.

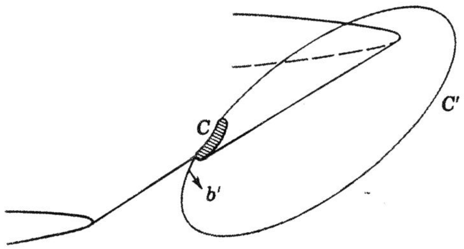


Fig. 8. Relaxing loop of dislocation at the tip of a crack.

(*b*) *Creation of loops in perfect crystals.* This process has been suggested by Gilman to explain the creation of many loops around the tips of cracks in regions of LiF crystals that are very far from any visible Frank-Read source,[18] and the process does seem the most likely possibility in that case, although some neighboring invisible Frank-Read sources or some quenched-in dislocation loops might have been present.

It will be shown here that the stresses around a crack are barely sufficient for such a process. As a consequence, *it probably occurs in only a restricted number of materials. When it occurs, the slip nucleates from the crack itself:* A step C appears on the surface of the crack and is compensated by the development of a "punching" dislocation C' on a cylinder passing along C and parallel to the Burgers vector of the slip (Fig. 8).

Consider a loop of diameter D under a shear stress σ. The critical value of D, above which the loop expands, is obtained by minimizing the total energy:[18,26]

$$U(D) = \frac{\mu b^2 \pi D}{4\pi K} \ln \frac{D}{b_0} - \sigma b \frac{\pi D^2}{4}$$

thus

$$\frac{dU}{dD_c} = \frac{\mu b^2}{4K} \ln \frac{eD_c}{b_0} - \frac{\pi \sigma b D_c}{2} = 0$$

Then

$$U(D_c) = \frac{\mu b^2 D_c}{8K} \ln \frac{D_c}{e b_0} \tag{15}$$

where $K \simeq 1$, e is the base of natural logarithms, and b_0 is the usual parameter related to the core energy.

The stresses at the tip of the crack are given by Eq. 14 with r at least of the order of D_c. Thus

$$\left(\frac{2D_c}{b}\right)^{1/2} = \frac{1}{K\pi} \ln \frac{eD_c}{b_0} \tag{16}$$

It is easy to see that such an equation can have a solution only if the parameter b_0 is much smaller than b. This is rarely the case, as estimated [22] b_0 values are usually larger than $b/2$.

If there is a solution to Eq. 16, it is of atomic dimensions at most, $D_c \simeq b$, so the loop is really punched from the surface of the crack. The corresponding activation energy $U(D_c)$ is a very small fraction of μb^3, thus a small fraction of 1 ev. Loops can be produced only if the velocity of the crack is such that

$$v < c \exp\left[-\frac{U(D_c)}{kT}\right] \tag{17}$$

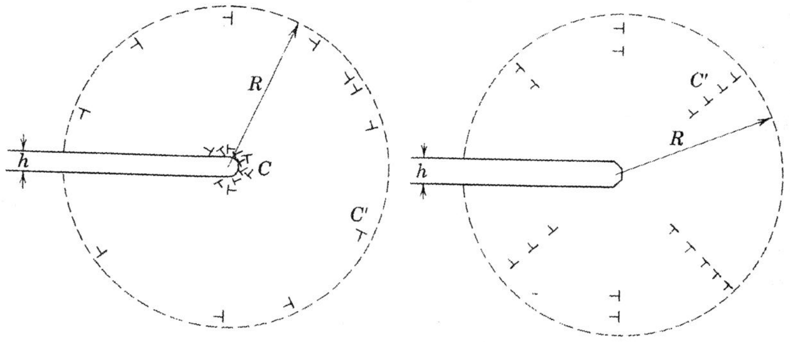
where c is, as before, the velocity of sound.

A critical velocity under which plastic relaxation occurs is indeed observed in LiF; this is $v \simeq 10^{-2}c$ at room temperature, leading to an activation energy $U \simeq 0.1$ ev. However, small loops are still formed at 77°K but do not expand as they do at room temperature. As pointed out by Gilman, this might indicate that the critical velocity observed is due to a Peierls-Nabarro force.[18] The activation energy $U(D_c)$ for the formation of loops should be still smaller ($U \lesssim 0.025$ ev).

Extension of the plastic relaxation. From the preceding section, it is clear that the mechanism and amount of plastic relaxation depend very much on the material and on the velocity of the crack. We shall now consider a fairly *ductile* material with a *slowly* moving crack.

If the material has *many possible slip systems*, there will be a cylindrical region of fairly big radius R around the tip of the crack where the elastic stresses of the crack have been almost completely relaxed. In terms of dislocations, the Frank-Read sources of this region have emitted loops, some part C of which have come to the immediate neighborhood of the tip of the crack in such a way that their total Burgers vector just *compensates* that $h = n'b'$ of the crack. The stress relaxation produced makes the piling-up of the dislocations of the crack easier, which thus blunts the tip of the crack; the crack now has parallel edges, except very near the tips. The parts C' of the same loops farther from the crack must have a total Burgers vector equal to h (Fig. 9). Each of the parts C' does not feel the elastic stress of the crack, which has been relaxed, but only the stress resulting from the other similar parts C'. They are repelled from the crack to a distance R so that their mutual repulsion is equal to the elastic limit σ_c of the material.

In this somewhat oversimplified model, the plastic relaxation replaces the large dislocation at the tip of the crack with a fairly continuous distribution of dislocations along a cylinder of radius R with the same total



(a) (b)

Fig. 9. Two mechanisms of plastic relaxation around the tip of a crack.

Burgers vector. The stress on each of these dislocations is of the order of $\mu h/R$.* Thus,

$$\frac{R}{h} \simeq \frac{\mu}{\sigma_c} \tag{18}$$

Certain refinements of the model are now considered:

1. Incomplete plastic relaxation occurs if only a few slip (or twin) systems are possible in the material. However, the preceding analysis applies to that part of the stresses that can be relaxed.

2. If the loops are punched in the perfect crystal, they occur at the surface of the crack. The parts C of the loops should then appear as regularly spaced steps on the tip of the crack, which should take sharp edges, with radii of curvature of atomic dimensions. The parts C' of the loops should be piled up somewhat anisotropically along the glide planes issuing from the tip of the crack (Fig. 10). If the loops are emitted by neighboring Frank-Read sources, the distribution of the parts C' should be more isotropic, as in Fig. 9. The parts C of the loops should be less regularly spaced along the crack, and they should relax the stresses at a somewhat larger distance, of the order of the size l_F of the Frank net. This is also

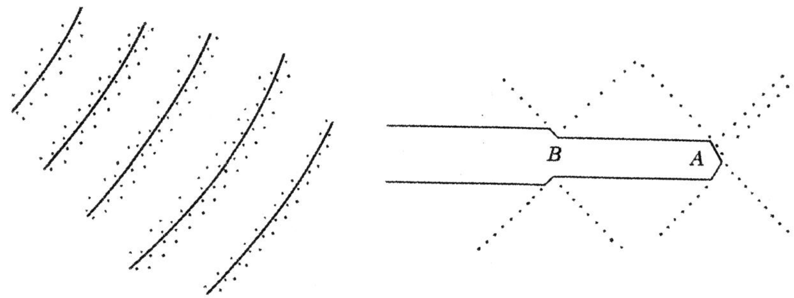
* There are about h/b dislocations on the cylinder, at distances $\epsilon R \simeq 2\pi R b/h$. The stress they exert on one of them is

$$\sum_m \frac{\mu b}{4\pi R \sin(m\epsilon/2)} \simeq \int_\epsilon^{2\pi-\epsilon} \frac{\mu b}{4\pi R \sin(\theta/2)} \left(\frac{h}{2\pi}\right) d\theta = \frac{2\mu h}{\pi^2 R} \ln \frac{2h}{\pi b}$$

For $10^3 < h/b < 10^6$, this expression is of the order of $\mu h/R$.

the size of the region where the blunting should occur and should thus be the average radius of curvature at the head of the relaxed crack.

3. The model neglects the parts of the dislocation loops that lie in the plastic region between the parts C near the axis and the parts C'



(a) (b)

Fig. 10. Edges and zones of plastic relaxation at successive positions of rest of the tip of a crack: (a) top view, (b) cross section.

near the outside limit. It also neglects the fact that, when emitted in great numbers, these loops must intersect or pile against each other. The resulting work hardening prevents some of the loops C' from reaching the surface of the cylinder. A better value for σ_c than the elastic limit would thus be an indentation hardness. With $\sigma_c \simeq 10^{-3}\mu$ in ordinary materials, Eq. 18 then gives

$$R \simeq 10^3 h \tag{19}$$

4. When plastic relaxation occurs, the width h of the crack varies, depending on the value of R compared with the size Λ of the crystal and the length L of the crack. In this connection, one has to distinguish between "brittle" and "ductile" cracks: The former merely blunt, the latter also widen under the applied stress.

(*a*) *"Brittle" cracks.* If $R \ll \Lambda$ and L, then the back stresses acting on the middle of the crack are practically unaltered by the plastic relaxation because this does not change the total Burgers vector h in the cylinders of radius R; therefore the *plastic relaxation does not change the width h of the crack*, it makes the crack only somewhat blunter and diminishes the stress concentrations by transferring them from the axis to the surface of the plastic cylinder. Equation 3 is still valid and gives, with Eq. 18, for the size of the cylinders:

$$R \simeq \frac{\sigma}{\sigma_c} L \simeq 10^3 \frac{\sigma}{\mu} L \tag{20}$$

From this equation, it follows that the case considered here only applies to fairly brittle materials or to fatigue, when a crack propagates at stresses σ well below the plastic limit σ_c. The specimen must also be thick compared with the length of the crack.

(*b*) *"Ductile" cracks.* If $R \simeq L$ (ductile materials, where σ is not much smaller than σ_c) or $R \gtrsim \Lambda$ (*thin* specimens), plastic relaxation very much reduces the back stress on the middle of the crack and *can then widen the crack considerably*. The tip of the crack is again somewhat blunted.

Stability of the plastic relaxation. Before plastic deformation sets in, the width h of a crack is proportional to the applied stress σ, and it is so small as to make cracks difficult to observe (from Eq. 3, $h/L \simeq \sigma/\mu$, usually $<10^{-3}$).

Once plastic relaxation has occurred, the crack preserves a width almost equal to that reached under the applied stress, even when the stress is removed; *plastic blunting of the tips stabilizes a crack*. The reason is that the relaxing dislocation loops C' build up, together with the tip of the crack, a fairly stable system, and this remains practically unaltered when the applied stress is removed. There is little tendency for the dislocations piled up at the tip of the crack to retract towards the left (Fig. 9), because they are immediately called back by the stresses resulting from the plastic loops. For the crack to be destroyed, one would need an applied stress opposite in sign and at least equal to that which has created it. Use can be made of this characteristic stability of plastic phenomena to check these conclusions. Two examples will be given:

(*a*) *Ductile cracks in iron.* It has been observed [27,28] that at low temperature polycrystalline iron with a fine enough grain fractures only after extensive plastic deformation. At small strains, cracks are already observed across many grains, but they do not seem to cross grain boundaries, probably because the stress concentrations at the tips of the cracks are too small to force the crack to propagate from grain to grain.[22,23] These cracks are obviously, as they should be, of the ductile type defined above: They have parallel edges and blunted tips, and a heavy deformation is visible near the tips. Finally, their width appears to be a few tenths of their length, although the stresses applied were at most of the order of $10^{-2}\mu$.

(*b*) *Brittle fracture in lithium fluoride.* A good example of the type of plastic relaxation pictured in Fig. 10 is given in one of Gilman's papers [29] on LiF. As far as can be seen from heavy etching, the two edges of the crack are parallel down to a blunted tip. Dislocation loops around the tip concentrate on the four slip planes going through the tip. This is in agreement with Gilman's contention that they are issued from the tip and not from neighboring Frank-Read sources. The number of etch pits should give the number of loops C' and thus an order of magnitude of the width of the crack. It seems that there are about 200 etch pits, corresponding probably to a total Burgers vector $h \simeq 100b \simeq 5 \times 10^{-6}$ cm. As the length L of the crack is much larger than the plastic region, Eq. 3 gives the stress that was applied to propagate the crack. With $L \simeq$ a few millimeters, this gives reasonable stress $\sigma \simeq 5 \times 10^7$ dynes/cm^2. According to Eq. 19 and with $\sigma_c \simeq 2 \times 10^4$ dynes/cm^2 for the elastic limit, the size of the plastic region should be $R \simeq 2 \times 10^{-2}$ cm, a good agreement with experiment. As Eq. 19 was established for an isotropic distribution of dislocation loops of the type pictured in Fig. 8, no more than an order-of-magnitude agreement is to be expected.

Blocking of cracks by plastic relaxation. The plastic relaxation, by stabilizing the tip of a crack, opposes its further displacement in the forward as well as in the backward direction. For velocities smaller than the critical ones described on p. 509, the crack stops suddenly and starts again only under a higher applied stress.[18] To estimate this "frictional" force, one must distinguish various factors.

(*a*) *Plastic work and blunting of the crack.* Plastic relaxation changes a large piled-up group of dislocations in a crack into a more continuous distribution over a region of size R. The plastic work W thus done is equal to the decrease in elastic energy stored: [22]

$$W \simeq \frac{\mu h^2}{4\pi K} \ln \frac{2R}{h} \tag{21}$$

As soon as R becomes large compared with the width h of the thin elastic crack, this energy becomes a large fraction of the total elastic energy stored. *Once the crack has started to relax, it will be energetically more favorable for the crack to stop and to relax completely. Once it has stopped, a small increase in stress, if slowly applied, will widen it plastically in preference to making it start again.*

*The crack might start to move again if a large enough increase in stress is suddenly applied.** This will pile up new crack dislocations at the tip. If they arrive quickly enough, they will build up a large enough stress concentration to induce the tip of the crack to move before plastic relaxation sets in. The *increase* in stress $\Delta\sigma$ must obey a Griffith criterion of the form of Eq. 7, where

$$\gamma_{\text{eff}} \simeq \gamma_0 \frac{\rho}{b}$$

* Another *a priori* possibility would be for the hardening produced by the initial plastic relaxation to be large enough to prevent any further relaxation. However, this is impossible, as has already been discussed.

if ρ is the radius of curvature of the blunted crack. This equation leads to quite different conditions, depending on which relaxation mechanism is operative.

In the usual case, where relaxation probably occurs through activation of neighboring sources, the crack is so blunt that no further propagation should be possible ($\rho \simeq l_F \geq 10^4 b$). When relaxation occurs by punching dislocations from the crack into the perfect crystal, the edges of the crack should be so sharp that propagation should be possible under a small stress increment $\Delta\sigma$ (ρ = a few b).

Only the new crack dislocations move forward; they leave behind the "old" crack dislocations of the blunted tip as "ridges" on the cleavage planes, stabilized by a surrounding plastic region. Such parallel ridges, with their surrounding clouds of dislocation loops, are a feature of cleavages where plastic relaxation has occurred periodically.[16,18,29] Figure 10*a* pictures schematically these ridges as observed on the cleavage planes. Figure 10*b* shows the cross section of a crack that has stopped (position A); etch pits resulting from dislocations and the change of width can be seen at a previous position (B).

(*b*) *Cleavage steps.* The cloud of dislocations in the plastic region gives rise to many steps when crossed by the crack. This is, however, not a very important factor in the propagation of the crack.

The density of this cloud can be estimated by the following argument: Except in the very special case where the tip of the crack runs parallel to the intersection of several slip planes, each loop in the plastic region is in a plane making a large angle with the crack. It relaxes the stresses of a crack dislocation only over the distance $\Lambda \simeq h$, where it combines with the crack to blunt it (Fig. 8). *The total number of loops C necessary* for complete relaxation is thus $h/b \times b/\Lambda$, which is about one *per distance b along the crack.* The corresponding density of loops C' piercing the cleavage plane in the plastic region of size R is

$$\rho_{c'} \simeq \frac{1}{2bR} \tag{22}$$

In Gilman's observations on LiF, $R \simeq 10^{-2}$ cm; the density predicted is $\simeq 10^9/\text{cm}^2$, somewhat larger than that observed (10^7 to $10^8/\text{cm}^2$). This is due, at least partly, to some of the loops having "popped" out of the crystal after the cleavage had developed.[29]

An estimate of the corresponding frictional stress $\Delta\sigma$ can then be obtained. If each loop C gave rise to a step *normal* to the tip of the crack when it moved forward, $\Delta\sigma$ would be given by Griffith's criterion of the form of Eqs. 7 and 8, with $l \simeq b$. Since $\gamma \simeq 2\gamma_0$, *$\Delta\sigma$ should be about twice the initial stress σ.* When the crack has moved a distance large compared

with R, these steps should have disappeared by recombination with steps of opposite sign issued from the corresponding loops C'. Another possibility is for all the steps C to coalesce into a ridge *parallel* to the tip of the crack (Fig. 10*a*). Steps normal to the crack are then only created by the loops C'. They are all of the same sign, and thus they should form rivers, such as those of Fig. 6, when the crack has moved a distance large compared with R. The total height of the steps would then be the same as in the first case when the crack has just started moving. Both cases lead therefore to the same value of $\Delta\sigma$.

Inspection of step patterns shows that both cases occur, as do intermediate ones[16,29] for which the stress level $\Delta\sigma$ should be somewhat smaller.

From the discussion, it follows that, once fully blunted by plastic relaxation, a crack is difficult to propagate again except under a high enough strain rate and in a fairly brittle material.

Ductile Cleavage

The preceding discussion helps to explain the propagation of cleavage in a material previously work hardened.

Plastic relaxation. Since the stresses of Eq. 2 around an unblunted crack are much larger than any likely hardness resulting from work hardening, a stationary crack is surely blunted by plastic relaxation even in a very highly work-hardened material. There will therefore be a *critical velocity* below which the crack blunts plastically and stops. By the argument given earlier, which is still valid, this velocity is *near to that of sound*, except in fairly brittle materials (aged impure materials, crystals with a high Peierls-Nabarro force), where the critical velocity will be somewhat lower. Thus only fast cracks are mobile.

Appearance of fast cracks. The creation of steps should lead to the type of river pattern pictured in Fig. 7, containing parallel multiple steps of both signs. Their average distance is of the order of a few times the distance l_F between dislocations in the material, and their average height mb increases slowly with the size L of the crack, according to Eq. 13.

For highly work-hardened materials, $l_F \simeq 10^{-6}$ cm. Individual rivers should then be hardly visible. The cleavage surfaces should be fairly smooth and parallel to a crystallographic plane over a distance L such that $mb \ll l_F$. Thus L should be smaller than a critical distance

$$L_c = \frac{l_F{}^3}{b^2} \simeq 10^{-3} \text{ cm} \tag{23}$$

At larger distances, the crack should become progressively noncrystallographic.

These predictions are in good agreement with detailed observations on ductile cracks occurring around small precipitates in austenitic or ferritic steels.[30] At the striction, a large number of such cracks appear across a section of the sample. Observed with the electron microscope, the cleavages are fairly smooth and near to a crystallographic plane in the central part of each crack at distances of a few microns from its precipitate. At larger distances, the cracks become completely noncrystallographic. These are the so-called "fibrous" cracks observed in a number of ductile fractures.[30,31]

Resistance to the propagation of fast cracks. As for brittle cracks, and for the same reasons, the resistance owing to the internal stresses is negligible. Two other factors might be considered: step formation and, eventually, grain size.

(*a*) *Steps.* The creation of steps of height mb and distance of the order of $2l_F$ leads to a Griffith criterion of the type of Eq. 7, with an effective surface tension given by

$$\gamma_{\text{eff}} \simeq \gamma_0 + \gamma \frac{mb}{2l_F} \simeq \gamma_0 + \frac{\gamma}{2}\left(\frac{L}{L_c}\right)^{1/2} \tag{24}$$

where $L_c \simeq l_F{}^3/b^2$ is the critical distance of Eq. 23 at which the cleavage becomes strongly noncrystallographic.

Since $\gamma \simeq 2\gamma_0$, it is seen that the stress necessary to propagate the crack should increase with the length L of the crack; it should become notably larger than in a perfect crystal for a crack much larger than the critical length L_c, thus a few microns in strongly work-hardened materials.

(*b*) *Grain size.* Equation 24 is valid within a grain. When a cleavage crosses a grain boundary, elementary steps all of the same sign appear along the line of the boundary. As they are of atomic height ($m = 1$) and at atomic distance from each other ($l \simeq b$), they do not cancel the higher and less numerous steps present in the previous grain. They lead therefore to a large and sudden increase in effective surface tension, of the order of $\gamma mb/l \simeq \gamma$. Thus for $L \geq$ the grain size D,

$$\gamma_{\text{eff}} \simeq \gamma_0 + \gamma\left[1 + \frac{1}{2}\left(\frac{L}{L_c}\right)^{1/2}\right] \tag{25}$$

For *fine polycrystals*, the stress necessary for propagating a crack should thus be *independent of work hardening*

$$\sigma \simeq 2\left[\frac{4\mu(\gamma_0 + \gamma)}{D}\right]^{1/2} \tag{26}$$

This equation should be valid for grain sizes smaller than the critical length L_c defined by Eq. 23, thus smaller than 10^{-3} cm. The factor 2 comes from the fact that the stress axis is not normal to the many little cracks that have to be developed.

Such a law has indeed been observed by Low[27] on low-carbon steels at low temperature, when the grain size is small enough for the fracture to be ductile. From the slope of σ vs. $D^{-1/2}$, one can deduce an approximate value of the surface tension of the steps,[23]

$$\gamma \simeq 2\gamma_0$$

This is the value of γ used in this discussion. It must be pointed out that other measurements on the ductile fracture of low-carbon steels at low temperature lead to a relation of the type[32]

$$\sigma = \sigma_0 + \text{const } D^{-1/2}$$

where σ_0 is a large constant term. This is analogous to the relations observed for brittle fractures and yield points. It suggests that in those experiments σ is the stress necessary to pile up enough dislocations on grain boundaries to nucleate cracks.[17]

Stresses Applied to Cracks During Work Hardening

Now that the conditions necessary for the *propagation* of cracks have been defined, it seems desirable to show that they can rarely be obtained during a homogeneous work hardening. The *nucleation* of the cracks, for which stress concentrations of some kind are known to be necessary, is out of the scope of this chapter. It is not therefore the high local stress concentrations around piled-up groups of dislocations that are of interest here, but the average internal stresses produced by work hardening and their relation to the density of dislocations.

Work Hardening as a Function of the Density of Dislocations

It will be recalled that, in many pure metals when the temperature is low enough to suppress diffusion phenomena, the stress necessary to produce a given strain ϵ at a strain rate $\dot{\epsilon}$ is made up of two parts[11] (Fig. 11):

(*a*) A part σ_0 which decreases slowly with increasing temperature in the same way as the elastic constants ($\sigma_0/\mu = const$).

(*b*) A part σ_T which decreases more rapidly with increasing temperature and disappears above a critical temperature T_c. The part σ_0 increases with the strain ϵ but does not vary with $\dot{\epsilon}$; σ_T increases with ϵ and $\dot{\epsilon}$. For given T and $\dot{\epsilon}$, σ_T/σ_0 remains constant when ϵ varies.[33]

It is thought that σ_T is due to *jog formation*, occurring when the slipping dislocations cut through the "trees" of the forest of screw dislocations, piercing their glide plane.[34,35] The formation of jogs requires movements over distances of atomic dimensions and thus can be helped by thermal agitation. The fact that σ_0/μ does not vary with temperature shows that it corresponds to a hardening varying only over distances that are large compared with atomic dimensions. As pointed out by Hirsch,[36] the constancy of σ_T/σ_0 with varying ϵ suggests that σ_0 is due to *long-range elastic interactions between the slipping dislocations and the trees.*

The possible hardening from such an interaction is somewhat difficult to estimate. It requires some averaging over the possible natures, orien-

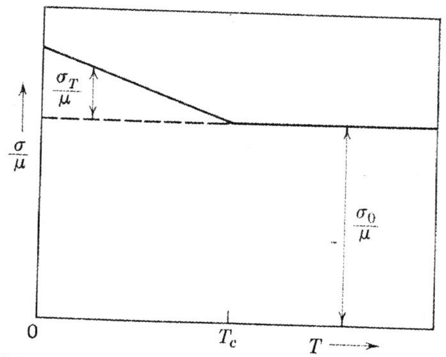


Fig. 11. Variation of the flow stress with temperature in a work-hardened metal.

tations, and lengths of the trees. Reactions can take place between moving dislocations and trees if they attract each other. Finally, there is still some discussion as to whether the kinetic energy of the moving dislocations might help them to overcome these long-range stresses. Order-of-magnitude estimates lead to [10,36,37]

$$\sigma_0 \simeq \frac{\mu b}{\delta l_F} \tag{27}$$

where l_F^{-2} is the dislocation density and δ a numerical factor of the order of 10.

The theory of jog formation leads to an equation [34]

$$\dot{\epsilon} = \text{const} \exp\left(-\frac{2U_c - \sigma_T b d l_F}{kT}\right)$$

where U_c is the energy of a jog, d is the width of a dislocation, and where the constant depends on the geometry of the dislocation arrays and other factors. From this it follows that

$$\sigma_T = \frac{2U_c}{bdl_F}\left(1 - \frac{T}{T_c}\right) \tag{28}$$

where the critical temperature T_c varies logarithmically with $\dot{\epsilon}$, and

$$\frac{d\sigma}{d(\ln \dot{\epsilon})} = \frac{kT}{bdl_F} \tag{29}$$

Equations 27 and 28 show that, at given temperature and strain rate, σ_0 and σ_T increase in the same way with increasing strain; this is in agreement with experiments. As the jog energy [22] is such that $2U_c \simeq \mu b^2 d/15$, it is seen that σ_T should be at most of the same order of magnitude as σ_0, and this only at low temperatures or very high strain rates. From Eqs. 27 and 29, it follows that the ratio

$$\frac{\dot{\epsilon}\, d\sigma}{\sigma_0\, d\dot{\epsilon}} = \delta \frac{kT}{\mu b^2 d} \tag{30}$$

should be independent of strain. These points are indeed observed [38,39] and lead, in metals where the dislocation width d is known, to values of δ of the order of 10. Finally, direct measurements of dislocation densities in work-hardened materials have shown that Eq. 27 is approximately followed.[36]

In conclusion, the hardness of a work-hardened material seems to be due mainly to the long-range elastic interactions of its dislocations and, therefore, to be inversely proportional to their average distance l_F (Eq. 27), as first pointed out by Taylor.[40]

Size of a Griffith Crack in a Work-Hardened Material

It will now be shown that the Griffith criterion for a fast crack is practically the same in work-hardened and perfect crystals, regardless of the degree of work hardening. Thus the difficulty in developing a ductile fracture does *not* come from step formation on the cleavage faces, contrary to what is often stated.

In regions where there are no stress concentrations, the stress applied to a crack is of the order of the external stress, thus, of the hardness. Equating the hardness σ_0 of Eq. 27 to the stress σ of the Griffith criterion, Eq. 7, and using the effective surface tension, Eq. 29, of work-hardened materials, one obtains easily for the length L the condition

$$L \geq \frac{4\delta^2 l_F^2}{\mu b^2} \gamma_0 \left[1 + \left(\frac{Lb^2}{l_F^3}\right)^{1/2}\right]$$

where the term in $L^{1/2}$ comes from the resistance of the steps. A discussion of this equation of the second order in $L^{1/2}$ shows that the term in $L^{1/2}$ is negligible if $l_F > \delta^2 \gamma_0/\mu b \simeq 10b$. This condition is fulfilled even in heavily cold-worked materials, where $l_F \leq 10^{-6}$ cm $\simeq 30b$.

In conclusion, the Griffith criterion for fast cracks in work-hardened materials reads

$$L \geq \frac{4\delta^2 l_F^2 \gamma_0}{\mu b^2} \simeq 40 \frac{l_F^2}{b} \tag{31}$$

In heavily cold-worked and thus hard materials, this is not very large: $l_F \geq 30b$ leads to $L \geq 3 \times 10^4 b \simeq 10^{-3}$ cm.

Plastic Relaxation

Griffith cracks with $L \simeq 10^{-3}$ cm are easily obtained in brittle materials in regions of some stress concentration: at the grips, where a slip band or a twin meets the surface, at a grain boundary, or at another slip band or twin.

In materials *with only a few slip systems*, the same types of stress concentrations are only partially relaxed by slip. They can initiate cracks, which are not much blunted by plastic relaxation, especially in cases such as zinc, where the cleavage plane is also the easy slip plane. These cracks can therefore grow slowly during work hardening until the applied stress is large enough for them to develop from the regions of stress concentration. Monocrystals and polycrystals of such materials are indeed known to fracture in this way after various degrees of work hardening.

Such brittle cracks do *not* develop in work-hardened materials with many slip systems, at least not unless their dislocations have been pinned down by aging after or during work hardening. The reason is that any large local stress concentration is plastically relaxed and that any incipient crack is plastically blunted in the way already described. Once blunted, a crack can propagate again only under a large applied stress and after enough work hardening such that, according to the Griffith criterion, $\gamma_{\text{eff}} = \gamma_0(l_F/b)$ is not too large; thus after heavy work hardening, $l_F \simeq 10^2$ to $10^3 b$.

It is indeed well known that in cubic *polycrystalline* metals ductile fracture occurs only after the onset of necking, during which the stress concentrations increase fast enough. The cracks do not develop from the regions of stress concentration but multiply until the effective section of the material is so small that the material breaks by shear. This is the "fibrous" fracture referred to in the section on ductile cleavage.

Cubic *monocrystalline* metals also break only after the onset of necking, but they break often by shearing to a point and without any cleavage.[17]

In conclusion, it is the plastic relaxation, and not the formation of steps, that hinders the cleavage of work-hardened materials.

Application to Fatigue Cracks

It seems that fatigue cracks often start from small surface cracks probably caused by the emergence of two parallel slip bands of opposite signs [41] on the surface of very strongly deformed grains.

From the preceding discussion, work hardening does not alter significantly Griffith's criterion as stated in Eq. 5: The crack can start developing under an applied stress σ if $L \geq 4\mu\gamma_0/\sigma^2$, that is, when L is a few 10^{-3} cm for the usual values of σ. Once started, the crack must travel fast or be stopped by plastic relaxation. Once stopped, it blunts and should start again only when work hardening at the tip of the crack has decreased the size l_F of the dislocation network sufficiently. It is more likely to branch at a point where, for instance, the crack has been cut by a slip band developed in later cycles. All these aspects indeed seem characteristic of fatigue cracks.

Comparison Between Cleavage, Slip, and Twinning

A cleavage crack, a slip band, and a twin lamella [22] can all be described as a suitable piling-up of dislocations; the state of strain and stress around them varies similarly with distance and orientation. They give rise to the same possibility of plastic relaxation, with a "cloud" of relaxing dislocation loops and a blunting which stabilizes them.

In a work-hardened material, resistance occurs in all three cases from the long-range internal stresses and from the short-range interactions with the dislocations piercing their plane. However, differences arise in the detailed mechanisms involved.

Long-Range Internal Stresses

As pointed out earlier, these stresses are too small and too short in wavelength to give rise to any significant resistance to a piled-up group of dislocations; but they do, however, hinder the slipping of new dislocations emitted by a Frank-Read source during the formation of a slip band or a twin. These dislocations are well separated before they join the piled-up group, so they do not help each other very much in their movement. Thus, in work-hardened materials, the *thickening* of these two types of bands is usually stopped by the internal stresses resulting from the cold-work. *No* such resistance to thickening occurs for cleavage cracks. This is because the cracks can be seen as a continuous distribution of infinitesimal dislocations whose mutual repulsions help them to overcome the internal stresses, even when they are far from the tip of the crack.

Jogs and Steps

In slip, the geometrical equivalent of the crystallographic cleavage steps of Fig. 3*b* is the slip of a jog along its glide plane. The equivalent of a noncrystallographic jog (Fig. 3*a*) is the climb of a jog together with the creation of a row of vacancies or interstitials. It is usually thought that, for a jog, slip requires much less energy than climb.[22] Thus, once formed, jogs should slip without increasing the resistance very much, but their formation produces a noticeable and temperature-dependent resistance. This is contrary to the cleavage steps, which provide only a small temperature-independent resistance to the tip of a crack.

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DISCUSSION

F. A. McClintock, *Massachusetts Institute of Technology*. It is interesting to contrast some of the results obtained by Friedel from dislocation theory with results obtained from plasticity theory applied to a continuum and with observations on a macroscopic scale. To be sure, agreement is not necessarily expected, for one can no more apply the equations for the motion of individual dislocations to regions large compared to the length of slip lines than one can apply classical plasticity to the processes that dominate in regions of the order of the length of slip lines.

As regards the extent of the plastic zone, Friedel's Eq. 20 indicates that the radius of the plastic zone should vary linearly with the applied stress level. In the case of longitudinal shear, however, it has been found that the radius varies as the square of the applied stress at low levels of stress, and even more rapidly as the yield point is approached.[1]

In cracks subjected to longitudinal shear, blunting would not be expected. Instead, plasticity analysis and experiments on 7075 T-6 aluminum alloy [2] both show that cracks are initially stable and grow slowly with increasing stress, later becoming unstable and accelerating. In the case of cracks under tension, an elastic-plastic analysis is not available, but experiments on aluminum foil,[1] as well as unpublished tests on the 7075 T-6 aluminum alloy, show that here also cracks can at first grow slowly and steadily, remaining sharp as far as can be judged by a low-power microscope and becoming unstable only later on. Thus in some cases, at least, cracks will not necessarily become blunted and stop when their velocity becomes appreciably smaller than the sound velocity. On the contrary, plastic deformation around a crack may sometimes lead to a slow, stable fracture under increasing load, which becomes unstable only after a certain stage has been reached.

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