Crack onset assessment near the sharp material inclusion tip by means of modified maximum tangential stress criterion

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ABSTRACT. In the case of particle reinforced composites, where the particles are in a form of sharp material inclusions, singular stress concentration exists on each tip of each inclusion. This is due to the geometric and material discontinuities between matrix and particle. These points of stress concentration are susceptible of crack initiation and thus often responsible for failure of the whole structure. The modified maximum tangential stress criterion is employed in order to predict crack onset conditions.

KEYWORDS. Sharp Material Inclusion; Singular and Non-singular Stress Terms; Generalized Fracture Mechanics.

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INTRODUCTION

Composites find application in a variety of engineering structures. Overall excellent properties, which can only be achieved by combination of two or more homogenous materials, bring solution for high demands given by contemporary advanced technologies. However, the very nature of composites also causes a difficulty in their assessment in terms of fracture mechanics. Common composite types, i.e. fibre reinforced composites or particle reinforced composites give rise to points of singular stress concentration. These points are found in loci where fibre ends are embedded in matrix or at each tip of each sharp particle and are referred as General Singular Stress Concentrators (GSSCs). This paper deals with the case of particle embedded in matrix which is in a form of Sharp Material Inclusion (SMI). The strength of singularity in this case is different and lower than in the case of a crack. However, the GSSCs are points often susceptible for crack initiation and thus they can be responsible for failure of the whole structure. A thorough understanding of crack initiation conditions in GSSC leads to improved estimation of the critical failure load. Stress, energy or coupled criteria are commonly used in fracture mechanics analyses of many GSSC types such as sharp notches. These criteria can be employed in order to predict crack onset conditions to the case of SMI. The GSSC characterized by weak singularity often require consideration of higher order non-singular terms of the asymptotic stress series to describe the stress state and thus to receive consistent critical load predictions. Suggestion of modified maximum tangential stress stability criterion for the SMI with consideration of non-singular terms is the main objective of this paper.

FRACTURE MECHANICS OF SHARP MATERIAL INCLUSION

The SMI is modelled in 2D as a special case of multi-material junction, the bi-material junction [1, 2] as shown in Fig. 1. Both material regions are considered to consist of linear elastic material and fully described by Young's moduli E_1 and E_2 and Poisson's ratios ν_1 and ν_2. The geometry is characterized by the opening angle α. Perfect bonding (traction and displacement continuity) is assumed at both interfaces Γ_0 and Γ_1. The problem is considered either in a state of plane stress or plane strain. The stress field in the vicinity of bi-material junction tip, i.e. when $r \rightarrow 0$, is described by following asymptotic series:

$$\sigma_{ij} = \frac{H_1}{\sqrt{2\pi}} r^{\lambda_1 - 1} f_{ij1m}(\theta) + \frac{H_2}{\sqrt{2\pi}} r^{\lambda_2 - 1} f_{ij2m}(\theta) + \frac{H_3}{\sqrt{2\pi}} r^{\lambda_3 - 1} f_{ij3m}(\theta) + \ldots \quad (1)$$

where H_k is the kth Generalized Stress Intensity Factor (GSIF) which corresponds to the kth eigenvalue λ_k, that forms the stress singularity exponent $(1 - \lambda_k)$. The terms of the series can be either singular, when $0 < \Re(\lambda_k) < 1$ or non-singular when $1 < \Re(\lambda_k)$. As $r \rightarrow 0$ the singular terms become unbounded, while the non-singular terms vanish. $f_{ijkm}(\theta)$ is the dimensionless angular eigenfunction constructed for ijth component of the stress tensor, kth eigenvalue and mth material as in [2, 10]. Note that the series above is in sake of simplicity written for real eigenvalues and real GSIFs. Such form of the series provides satisfactory description for most of the cases.

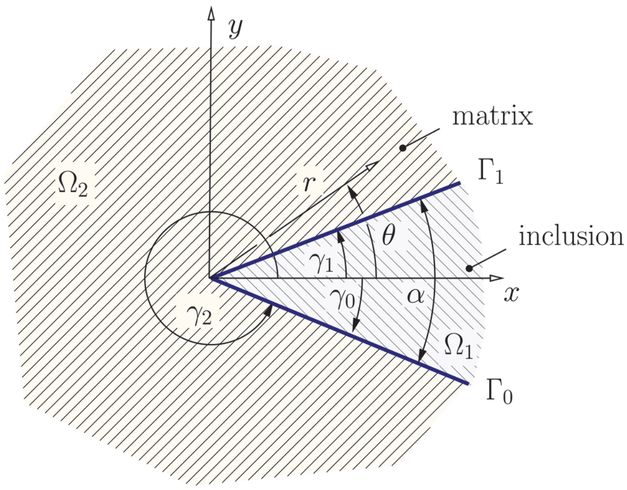

Figure 1: Sharp material inclusion model.

In majority of the fracture mechanics analyses of GSSCs only the singular terms are used for description of stress field [3, 4] and following determination of crack onset conditions. In [5, 6] the effect of first non-singular stress term on stress distribution in case of sharp V-notch is studied. The effect of the first non-singular term in case of bi-material notches is studied in [7, 8]. In [9] Klusák et al. have shown the significance of consideration of the first non-singular term in the case of sharp bi-material orthotropic plate. A study which has shown the effect of non-singular terms in the cases of SMI has been conducted in [10]. In order to identify singular and non-singular terms for given configuration the dependence of the eigenvalues λ_1, λ_2 and λ_3 of chosen geometric SMI cases on Young's moduli ratio E_1/E_2 is presented in Fig. 2. Here, material region 1 and Young's modulus E_1 belongs to inclusion and the region 2 with modulus E_2 corresponds to the matrix. In the case of rectangular SMI characterized by $\alpha = 90°$ two singular and one non-singular terms for all E_1/E_2 ratios are found. In the case of sharper SMI with $\alpha = 60°$ there are two singular terms together with one non-singular terms for cases where inclusion is more compliant than matrix, i.e. $E_1/E_2 < 1$ and one singular term together with two non-singular terms for cases of inclusion stiffer than matrix, i.e. $E_1/E_2 > 1$. The SMI with a blunt opening angle of $\alpha = 120°$ shows reverse trend, as the cases of more compliant inclusion have one singular term together with two non-singular terms and cases with stiffer inclusion have two singular together with one non-singular term.

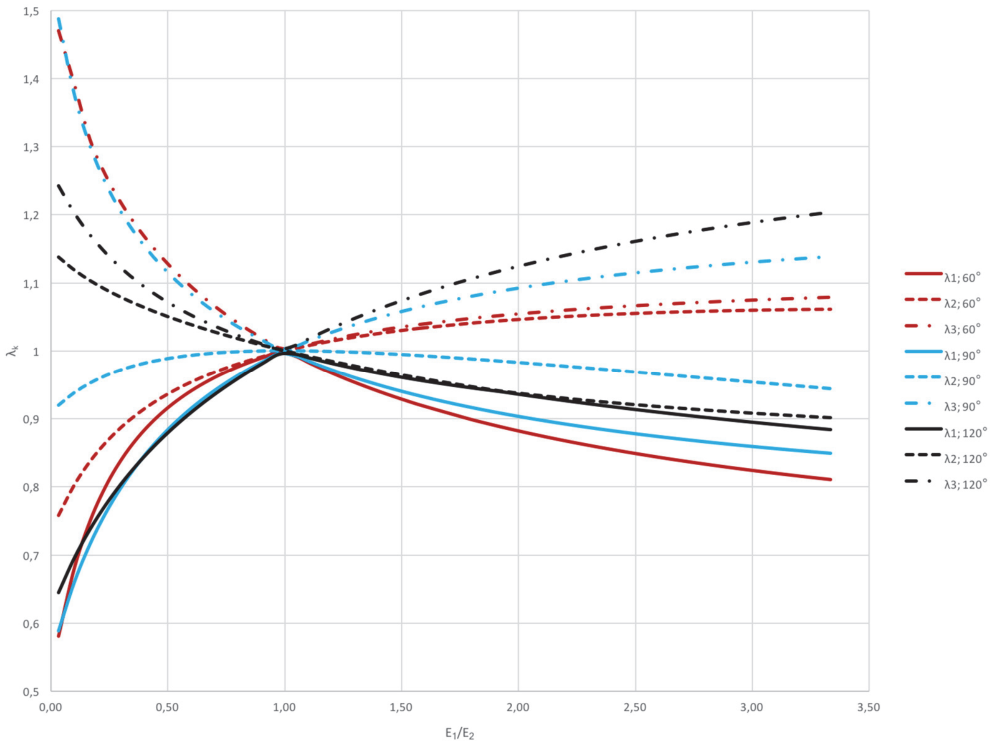

Figure 2: Dependence of λ_1, λ_2 and λ_3 on E_1/E_2 shown for 3 SMI geometries α = 60°, 90°, 120°.

To predict crack initiation conditions, a modified maximum tangential stress criterion is used. The tangential stress depends on radial distance from the singular concentrator tip. To mitigate the radial distance dependence the mean value can be calculated as:

$$\bar{\sigma}_{\theta\theta m}(\theta) = \frac{1}{d}\int_0^d \sigma_{\theta\theta}(r,\theta)\,\mathrm{d}r \tag{2}$$

The parameter d is related to the fracture mechanism, e.g. in case of cleavage fracture it can be set as $d = 2$ - $5 \times$ grain size of the material [13]. The criterion states that the crack will initiate in the direction $\theta_{0,m}$ of maximum value of mean tangential stress. The extreme value is found as:

$$\left(\frac{\partial\bar{\sigma}_{\theta\theta}}{\partial\theta}\right)_{\theta_{0,m}} = 0; \quad \left(\frac{\partial^2\bar{\sigma}_{\theta\theta}}{\partial\theta^2}\right)_{\theta_{0,m}} < 0 \tag{3}$$

Let's consider the first 3 terms (singular and non-singular) of the stress series (1). By averaging it over specific distance d as in (2) and substituting it into Eq. (3) we obtain:

$$H_1\frac{d^{\lambda_1}}{\lambda_1}\frac{\partial f_{\theta\theta 1m}}{\partial\theta} + H_2\frac{d^{\lambda_2}}{\lambda_2}\frac{\partial f_{\theta\theta 2m}}{\partial\theta} + H_3\frac{d^{\lambda_3}}{\lambda_3}\frac{\partial f_{\theta\theta 3m}}{\partial\theta} = 0 \tag{4}$$

The first GSIF H_1 is factored out and the consequent ratios are denoted as $\Gamma_{k1} = H_k/H_1$. Equation has now the only unknown, the crack initiation direction $\theta_{0,m}$.

$$\Gamma_{11}\frac{d^{\lambda_1}}{\lambda_1}\frac{\partial f_{\theta\theta 1m}}{\partial\theta} + \Gamma_{21}\frac{d^{\lambda_2}}{\lambda_2}\frac{\partial f_{\theta\theta 2m}}{\partial\theta} + \Gamma_{31}\frac{d^{\lambda_3}}{\lambda_3}\frac{\partial f_{\theta\theta 3m}}{\partial\theta} = 0 \tag{5}$$

In general, global and local extremes can be found in bi-material problem. The crack will be initiated in the direction of maximum of $\overline{\sigma_{\theta\theta}}$ and corresponding minimum value of generalized fracture toughness [14]. Note that the crack initiation direction does not depend on the absolute values of GSIFs but rather on their ratios Γ_{k1}. The general form of the equation for n terms of the stress series is:

$$\sum_{k=1}^{n} \Gamma_{k1} \frac{d^{\lambda_k}}{\lambda_k} \frac{\partial f_{\theta\theta km}}{\partial \theta} = 0 \tag{6}$$

The crack initiation conditions of the SMI are described by means of critical stress quantity. In this way, Knésl in [13] assesses a stability of V-notch. He proposes comparison of such critical quantity $\overline{\sigma_{\theta\theta C}}$ of the crack and the V-notch. Since in both cases (crack tip and the V-notch tip) the stress is singular, he supposes that crack initiation mechanism in V-notch will be the same as the crack propagation mechanism. Then crack initiation conditions in case of GSSCs can be assessed by means of critical average stress ascertained for a crack. For a crack in homogeneous material and loaded in normal mode I, the critical mean tangential stress value is:

$$\overline{\sigma_{\theta\theta C}}\left(\theta_0 = 0\right) = \frac{2K_{IC}}{\sqrt{2\pi d}} \tag{7}$$

Whereas for the GSIF with consideration of first three singular and non-singular terms of the series (1), it is:

$$\overline{\sigma_{\theta\theta C,m}} = \frac{H_{1C,m}}{\sqrt{2\pi}} \left(\Gamma_{11} \frac{d^{\lambda_1 - 1}}{\lambda_1} f_{\theta\theta 1m} + \Gamma_{21} \frac{d^{\lambda_2 - 1}}{\lambda_2} f_{\theta\theta 2m} + \Gamma_{31} \frac{d^{\lambda_3 - 1}}{\lambda_3} f_{\theta\theta 3m} \right) \tag{8}$$

By comparison of the two relations above the generalized fracture toughness $H_{1C,m}$ is:

$$H_{1C,m} = \frac{2K_{IC,m}}{\Gamma_{11} \frac{d^{\lambda_1 - \frac{1}{2}}}{\lambda_1} f_{\theta\theta 1m}(\theta_{0,m}) + \Gamma_{21} \frac{d^{\lambda_2 - \frac{1}{2}}}{\lambda_2} f_{\theta\theta 2m}(\theta_{0,m}) + \Gamma_{31} \frac{d^{\lambda_3 - \frac{1}{2}}}{\lambda_3} f_{\theta\theta 3m}(\theta_{0,m})} \tag{9}$$

which in general form for n terms of the stress series is written:

$$H_{1C,m} = \frac{2K_{IC,m}}{\sum_{k=1}^{n} \Gamma_{k1} \frac{d^{\lambda_k - \frac{1}{2}}}{\lambda_k} f_{\theta\theta km}(\theta_{0,m})} \tag{10}$$

The generalized fracture toughness $H_{1C,m}$ depends on fracture toughness $K_{IC,m}$ of the material m. It is obvious that in case of SMI two materials $m = 1$ and 2 should be considered and possibility of crack initiation into the matrix and into the inclusion should be evaluated. If the value $H_{1C,1}$ is lower than $H_{1C,2}$ crack initiation is expected into the inclusion, otherwise it occurs into matrix. In sake of completeness, the case of crack initiation into the interface should be also evaluated. In that case, the value of $H_{1C,interface}$ has to be calculated based on fracture toughness of the interface $K_{IC,interface}$. Note that for all the critical values $H_{1C,1}$, $H_{1C,2}$, and $H_{1C,interface}$ the shape functions $f_{\theta\theta,m}$ should contain corresponding angle of potential crack initiation $\theta_{0,m}$ (m = 1, 2, interface). The crack initiation occurs if the following stability criterion is violated (11):

$$H_1 < \min\left\{H_{1C,1}, H_{1C,2}, H_{1C,interface}\right\} \tag{11}$$

Crack is not initiated in SMI tip if the value of GSIF H_1 is lower than its critical value. Finally, the critical applied load σ_{crit} can be calculated as:

$$\sigma_{\text{crit}} = \sigma_{\text{appl}} \frac{\min\left(H_{1C,1}\left(\theta_{0,1}\right), H_{1C,2}\left(\theta_{0,2}\right), H_{1C,\text{interface}}\left(\theta_0 = \gamma_{1,2}\right)\right)}{H_1\left(\sigma_{\text{appl}}\right)} \quad (12)$$

NUMERICAL EXAMPLE

The SMI is modelled by FEM in 2D. The model geometry and boundary conditions are shown in Fig. 3. The rectangular inclusion with $\alpha = 90°$ in a plane strain conditions is studied. The modelled specimen size is 25 × 25 mm. The model is loaded with unit tension of $\sigma_{\text{appl}} = 1$ MPa. In this numerical example the inclusion more compliant than matrix is considered, which represents sandstone inclusion in cement paste of various stiffness. The modelled bi-material configurations are listed in Tab. 1. The Poisson's ratios are identical for both materials, $\nu_1 = \nu_2 = 0.2$. The eigenvalues λ_k are determined as a solution of an eigenvalue problem [2, 10]. The GSIFs H_k are calculated semi-analytically by means of overdeterministic method [2, 10], with resulting values as shown in Tab. 1. The diameter which provides inputs (nodal displacements) for GSIFs calculation is chosen as $r_1 = 1$ mm. The characteristic length d which corresponds to the fracture mechanism is for the case of cement paste composites chosen as $d = 1$ mm. Fig. 4 shows the tangential stress distribution for the case $E_1/E_2 = 0.5$, nevertheless distribution of this kind is characteristic to the cases of inclusion more compliant than matrix in general. The distribution disposes of two extremes, global which is found in the matrix $\theta_{0,2} = 180°$ and local found in the inclusion $\theta_{0,1} = 0°$. The values of crack initiation angles $\theta_{0,m}$ are given by the symmetry of the problem. For the same reason the odd term, the skew related GSIF $H_2 = 0$ MPa·m$^{1-\lambda_2}$. It is necessary to asses potential crack initiation in all possible ways, i.e. in the direction of the global maximum, local maximum and the interface. The crack initiation angle in the latter case is $\theta_{0,\text{interface}} = 45°$. In Tab. 1 the generalized fracture toughness is calculated for global function extremes, the local one and the interface. The fracture toughness value of matrix, inclusion and the interface is chosen equally, $K_{IC,m} = 1$ MPa·m$^{1/2}$ for $m = \{1, 2, \text{interface}\}$. Generalized fracture toughness for unit $K_{IC,m}$ can be understood as normalized value and in Tab. 1 is denoted as $H^*_{1C,m}$.

E_1 [GPa]	E_2 [GPa]	E_1/E_2	H_1 [MPa·m$^{1-\lambda_1}$]	H_3 [MPa·m$^{1-\lambda_3}$]	K_{IC} [MPa·m$^{1/2}$]	$H^*_{1C,2}$ [MPa· m$^{1-\lambda_1}$]	$H^*_{1C,1}$ [MPa· m$^{1-\lambda_1}$]	$H^*_{1C,\text{interface}}$ [MPa· m$^{1-\lambda_1}$]
20	80	0.25	1.857	-0.014	1	0.618	0.822	1.078
20	60	0.33	1.592	-0.020	1	0.657	0.784	1.149
20	40	0.50	1.310	-0.022	1	0.715	0.765	1.238
20	26.6	0.75	1.114	-0.020	1	0.774	0.782	1.455

Table 1: Bi-material configurations with resulting GSIFs and values of generalized fracture toughness.

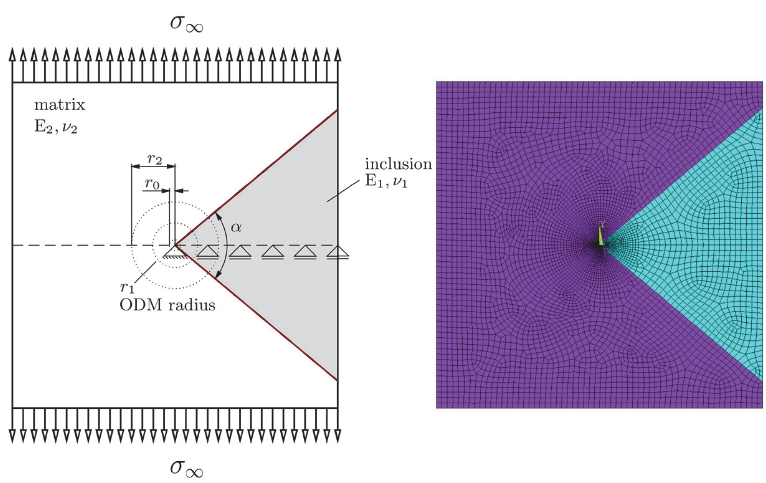

Figure 3: The numerical model.

It is evident that the stress description (Fig. 4) by means of the first and the third stress terms ($\sigma_{\theta\theta}^{\lambda_1,\lambda_3}$) is more precise than the description by the first stress term only ($\sigma_{\theta\theta}^{\lambda_1}$). It can be seen especially in directions $\theta = 90°$ and $270°$ corresponding to regions of lower tangential stresses. The stability criterion employing critical value H_{1C} following from all three stress terms (9), (10) is used. Fig. 5 shows results of the values H_1 and the critical values H^*_{1C} corresponding to crack initiation into matrix, into the inclusion and into the interface. The values H_1 follow from FEM numerical solution of model with unit loading 1 MPa. The critical values $H^*_{1C,m}$ are ascertained for unit fracture toughness $K_{IC,m}$ = 1 MPa·m$^{1/2}$ for m = {1, 2, interface}. Particular values $H_{1C,m}$ for given fracture toughness of the matrix, the inclusion and the interface are:

$$H_{1C,m} = H^*_{1C,m} K_{IC,m} \tag{13}$$

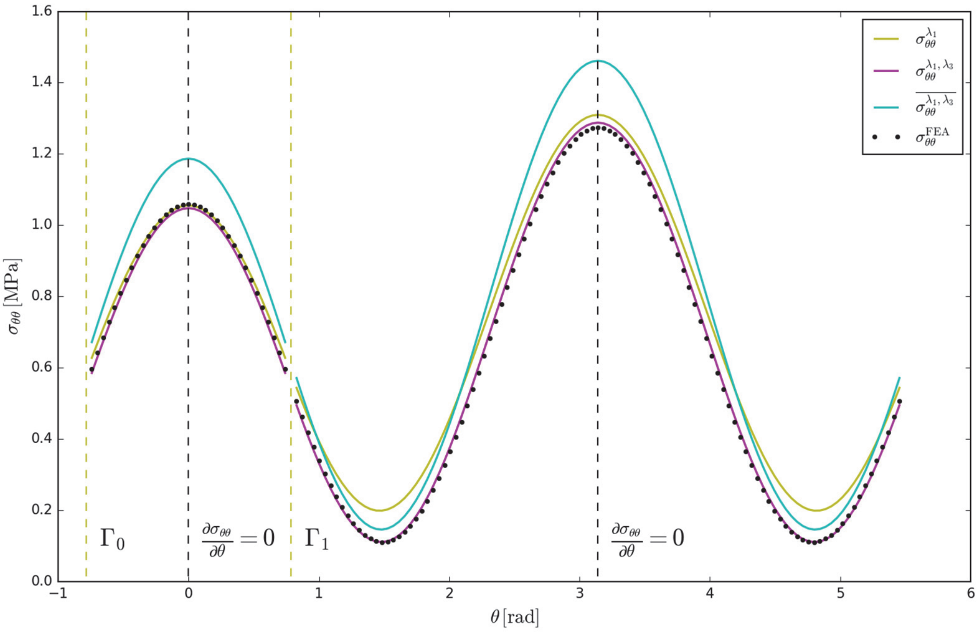

Figure 4: Tangential stress distribution for the case E_1/E_2 = 0.5. Yellow and magenta curve stay for the analytical tangential stress solution on r = 1 mm obtained by one term H_1 and by two terms H_1 and H_3 respectively. The cyan curve is the mean tangential stress obtained by two terms H_1 and H_3. Black dots represent the FEM solution on r = 1 mm. The yellow vertical lines denote the interfaces and black vertical lines the function local and global extreme.

The results show that crack initiation conditions depend on the ratio of materials Young's moduli, and on the fracture toughness of material components (matrix, inclusion) and the interface. In the numerical example E_1 corresponds to Young's modulus of the inclusion. The inclusion here is considered to be a sandstone aggregate with E_1 = 20 GPa. Corresponding fracture toughness of sandstone is between 0.28 and 0.52 MPa·m$^{1/2}$. On the other hand, E_2 corresponds to Young's modulus of matrix. In the numerical example it varies from 26.6 to 80 GPa. The fracture toughness of common hardened cement paste is between 0.1 and 0.8 MPa·m$^{1/2}$, where higher values of E_2 usually match to higher values of $K_{IC,2}$. The fracture toughness of interface can widely vary. In case of silicate based composites it is dependent on the manufacturing process and development of interfacial transition zone. The values $H_{1C,interface}$ are higher than $H_{1C,1}$ and $H_{1C,2}$ thus it seems that the crack kink to the interface is not probable, but mind that usually $K_{IC,interface}$ is lower than $K_{IC,matrix}$ and $K_{IC,inclusion}$. When considering particular values of $K_{IC,m}$ for particular ratios E_1/E_2, the curves of the critical values $H_{1C,m}$ for matrix, the inclusion and the interface will change by multiplying as indicated in (13). Thus every particular case will show if crack initiation occurs to matrix, to the inclusion or the interface, i.e. min{$H_{1C,1}$, $H_{1C,2}$, $H_{1C,interface}$}.

Let us note that studied material model is universal. Although it supposes sharp concave inclusion embedded in matrix, at the same time it can describe convex corner of stiffer inclusion filled with matrix. In this case E_1 would match to matrix and E_2 to the inclusion. Principally, the approaches will be the same and the results very similar.

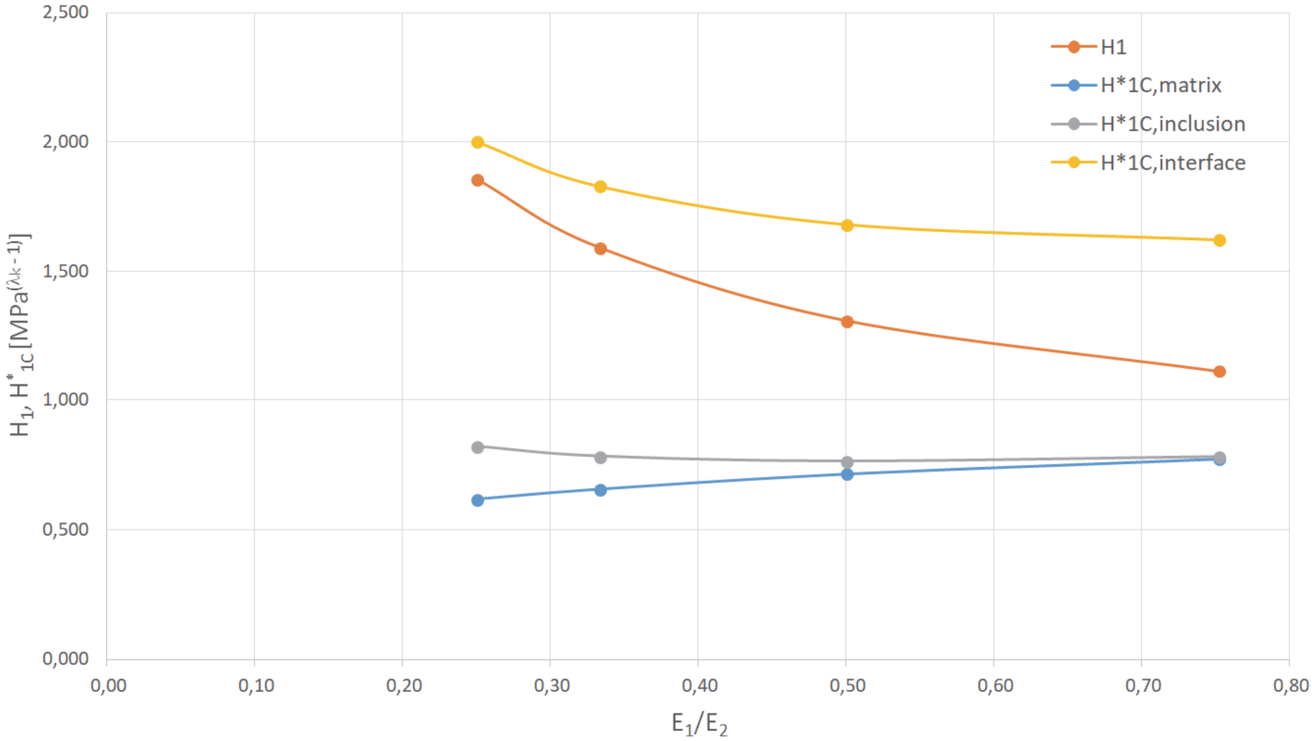

Figure 5: GSIF H_1 for unit applied load and the critical values H_{1C} needed for crack initiation to matrix and to the inclusion.

Conclusions

The article deals with generalized fracture mechanics of a crack initiation in a tip of polygonal material inclusion embedded in matrix. Stability criterion for determination of crack initiation conditions is proposed by means of average value of tangential stress. The stability criterion is written for stress described by singular and non-singular (higher order) terms. The criterion employing higher order terms is necessary for the assessment of sharp material inclusion, because singular stress terms only do not describe stress state satisfactorily in some material and geometrical configurations. The proposed criterion allows indicating whether crack initiation occurs into matrix, into the inclusion or into the interface. Its general form allows such assessment for particular known fracture toughness of the matrix, inclusion or the interface. Description of stability of stress concentrators of this kind contribute to better understanding of toughening mechanisms in particle composites. This kind of understanding leads to composite design optimisation or composite structure failure prevention.

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