



## Pressure vessels design methods using the codes, fracture mechanics and multiaxial fatigue

Fatima Majid, Jilali Nattaj, Mohamed Elghorba

*University of Hassan II, National Superior School of Electricity and Mechanics Casablanca (ENSEM) LCCMMS*

*majidfatima9@gmail.com, <http://orcid.org/0000-0001-8909-8232>*

**ABSTRACT.** This paper gives a highlight about pressure vessel (PV) methods of design to initiate new engineers and new researchers to understand the basics and to have a summary about the knowhow of PV design. This understanding will contribute to enhance their knowledge in the selection of the appropriate method.

There are several types of tanks distinguished by the operating pressure, temperature and the safety system to predict. The selection of one or the other of these tanks depends on environmental regulations, the geographic location and the used materials.

The design theory of PVs is very detailed in various codes and standards API, such as ASME, CODAP ... as well as the standards of material selection such as EN 10025 or EN 10028.

While designing a PV, we must design the fatigue of its material through the different methods and theories, we can find in the literature, and specific codes. In this work, a focus on the fatigue lifetime calculation through fracture mechanics theory and the different methods found in the ASME VIII DIV 2, the API 579-1 and EN 13445-3, Annex B, will be detailed by giving a comparison between these methods.

In many articles in the literature the uniaxial fatigue has been very detailed. Meanwhile, the multiaxial effect has not been considered as it must be. In this paper we will lead a discussion about the biaxial fatigue due to cyclic pressure in thick-walled PV. Besides, an overview of multiaxial fatigue in PVs is detailed.

**KEYWORDS.** Pressure vessel design; ASME VIII; Multiaxial fatigue; Fracture mechanics; Cumulative damage.



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### INTRODUCTION

The pressure vessels (PV) are among the most used storage means in many industries, particularly in the Ammonia, Gas, Petrochemical industries. They may be cylindrical, spherical depending on the nature of the stored product, its environment and its use. The PVs are more complex in design and safety component management. They are



interacting with the stored product and the external environment such as climate conditions and earthquakes. The higher number of PVs accidents [1] oblige us to be careful when using such equipments and to go beyond the codes and the standards for further detailed engineering design, develop new concepts in the performance framework and create a more dynamic vision and methodology as part of predictive and autonomous maintenance.

The designers do usually a routine design of the PVs, but they don't take into consideration the fatigue and the cumulative damage calculations for lifetime prediction. A lot of researchers are usually dealing with the uniaxial fatigue. But many other researchers tried to deal with the multiaxial fatigue to show the complexity of the phenomenon. A PV is always subjected to multiaxial loadings and multiaxial stresses. Meanwhile, the prediction of industrial equipments' reliability and availability still a difficult task for final clients and engineers. Thus five approaches dealing with multiaxial fatigue exist in the literature. The first approach is the stress or strain invariant approach led by many authors [2- 8]. The second one is the critical plan approach led by Brown and Miller [9-28]. The third one is the integral approach led by [29-32]. The fourth one is the energetic approach led by [33, 34]. The fifth and the last one are the empiric formulas led by [35-38] for high cycle fatigue et Mowbray [39], Manson and Halford Kalluri and Bonacuse for low cycle fatigue.

The metal's damage due to fatigue has a well-known cycle, going through micro crack initiation, then its propagation until the rupture at the end. The fatigue rupture causes 50% to 90% of all the mechanical failures. According to many researches as Fatemi, 2010 and NASA, 1994 for metal, micro cracks of about 10 to 100 micrometers uses 60 to 80% of the fatigue resistance, in other words the metal life time. That's why it is very interesting to study the small cracks in progress ie the first stage (Stage I) of crack. One of the major PV's failures is the fatigue's cracking. For that reason, we have to predict and analyze the cracks behavior, and specifically the crack propagation, in order to ensure the correct maintenance of PV. Many studies have been developed to face this kind of failures.

## PRESSURE VESSELS DESIGN

The tanks are classified into three groups according to the operating pressure. The atmospheric storage tanks for operating pressure of less than 18 kPa which are managed by the API 650 standard, The low-pressure storage tanks 18 kPa <P <100 kPa which are managed by the API 620 standard and PVs whose operating pressure P > 100 kPa which are managed by ASME Sec VIII [40].

In this part of work, we developed a standard methodology for PVs design. We start by defining the design assumptions through the PV's geometry, the site conditions, the service conditions, the test conditions and the design conditions. Then, the material choice is done through the clients recommendations and the international standards CODAP, ASME II, EN13345 or EN 10222-4 or standards for materials choice EN-10025, EN 10028, ISO 9327-4: 1999, JIS G 3202: 1988 and ASTM. In the next step, we define the codes for PV calculation, figure (a), such as ASME, CODAP or API. Next, we define earthquake, safety elements, metallic construction codes such as CM66, and the regulations for the stored product. After that, we start the PV element calculation through the shell's thickness calculation, figure (a), head's thickness calculation, figure (b), nozzles calculation, figure (c), saddles calculation, seismic through UBC 1997 ground supported code and wind through the building code ASCE 7-05 verifications, calculation of lifting lugs, figure (d), and finally the calculation of fire circuit tanks through NFPA or other recognized standard [50].

## PRESSURE VESSELS MULTIAXIAL FATIGUE DESIGN

PV is subjected to repeated loading that could cause failure by the development of progressive fracture, ASME Section VIII Division 2, API 579-1 and EN 13445-3 Annex B has detailed procedures for determining if a vessel in cyclic service requires a detailed fatigue analysis or not, and how to conduct the analysis. The ASME code is taking into consideration non conservative approaches, which are dealing combined load sources, rather than the other codes.

The exemption of fatigue calculation is given by 3 screening procedure. The first one is based on successful experience and the second one, method A, uses a simple six step procedure for material with tensile strength of 550 MPa maximum. The third one, method B, is the most important one and it is developed in the table below according to the Section VIII Division 2 Paragraph 5.5.2.4. We start by determining the history of the loading given by the specs (step1) and then we determine screening criteria factors, C1 and C2 (step2). Then, we proceed directly to fatigue analysis if any step inequation is false, else if we pass to the next step.

The fatigue life is predicted from the S-N curve, results of fatigue tests on smooth bar, based on fatigue strength reduction factors ( $K_f$ ).

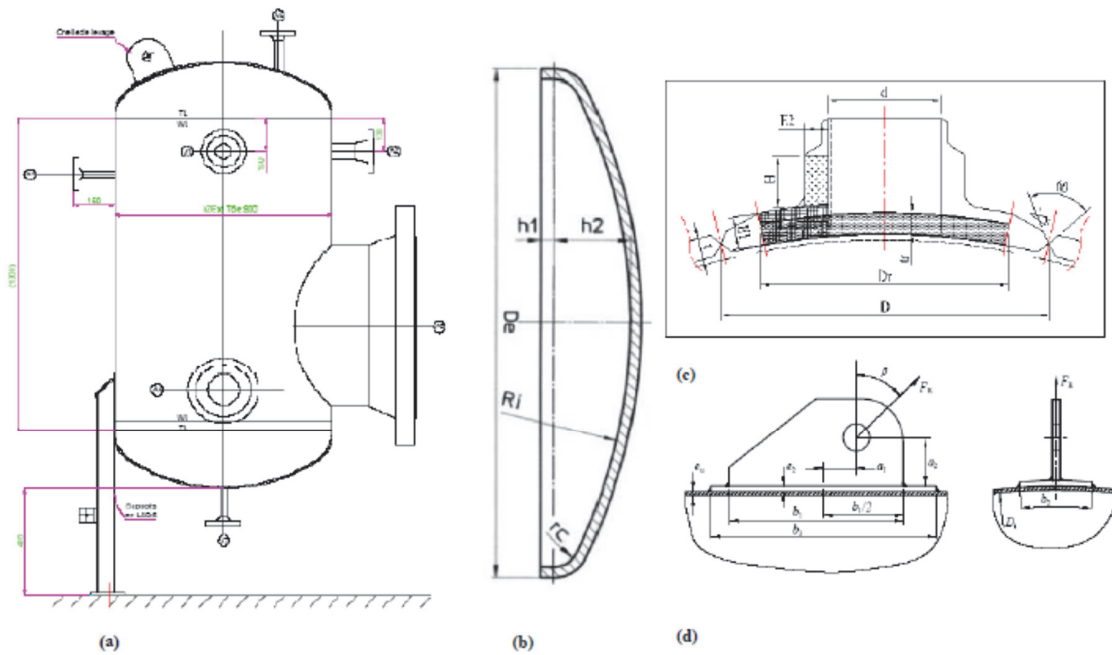


Figure 1: Illustration of pressure vessels elements.

Step	Step detail	Formulas
Exemption Method B	Step3- Full range pressure cycles	$N_{\Delta FP} \leq N(C_1 S)$ (1)
	Step4- Maximum range of pressure	$\Delta P_N \leq \frac{P}{C_1} \left( \frac{S_a(N_{\Delta P})}{S} \right)$ (2)
	Step5- Maximum temperature difference between two adjacent points	$\Delta T_N \leq \frac{P}{C_1} \left( \frac{S_a(N_{\Delta TN})}{C_1 E_{ym} \alpha} \right)$ (3)
	Step6- Maximum temperature difference fluctuation between two adjacent points	$\Delta T_R \leq \frac{P}{C_1} \left( \frac{S_a(N_{\Delta TN})}{C_1 E_{ym} \alpha} \right)$ (4)
	Step7- Maximum temperature difference fluctuation for different components materials	$\Delta T_R \leq \left( \frac{S_a(N_{\Delta TM})}{C_2 (E_{y1} \alpha_1 - E_{y2} \alpha_2)} \right)$ (5)
Fatigue Assessment	Step8- Equivalent stress range	$\Delta S_{MLR} \leq S_a(N_{\Delta S})$ (6)
	Elastic Stress Analysis Equivalent stress	$S = \frac{K_{e,k} (\Delta S_{p,k} - \Delta S_{LT,k}) + K_{v,k} \Delta S_{LT,k}}{2}$ (7)
		$S = \frac{E_{ya,k} \Delta \epsilon_{eff,k}}{2}$ (8)
	Elastic-Plastic Stress Analysis Equivalent strain	$\Delta \epsilon = \frac{\Delta S}{E_y} + \Delta \epsilon_{peq,k}$ (9)

Table 1: Pressure vessels fatigue design.



## MULTIAXIAL FRACTURE OF THICK WALL CYLINDER

The wall of the pressure cylinders generally undergoes triaxial loading Axial, circumferential and radial. In fact, many theories have been developed to predict the fracture of pressure cylinder by determining the limit charges. There are some theories which are dealing only with the internal pressure. Other theories are focusing on the applied axial stress. And the last category is dealing with both of them.

In the table below, we present a review of almost all the theories dealing with the limit internal pressure and the combined internal pressure and applied axial stress.

For the first category, they are predicting the rupture pressure. Meanwhile, the second category they are fixing either the internal pressure or the applied axial stress and predicting the other one.

Theories	Author,year	Equation
Internal pressure	Hill, 1950 [41].	$P = \frac{2}{\sqrt{3}} \sigma_y \ln \left( \frac{D_0}{D_i} \right) \quad (10)$
	Nadai, 1950 [42].	$P = \frac{2}{\sqrt{3}} \sigma_{UTS} \ln \left( \frac{D_0}{D_i} \right) \quad (11)$
	Faupel, 1956 [43].	$P = \frac{2}{\sqrt{3}} \sigma_y \left( 2 - \frac{\sigma_y}{\sigma_{UTS}} \right) \ln \left( \frac{D_0}{D_i} \right) \quad (12)$
	Asser Brabin, 2009 [44].	$P = \frac{2}{\sqrt{3}} \sigma_y \left\{ \left( 1 - \lambda \left( 1 - \frac{\sigma_y}{\sigma_{UTS}} \right) \right) \right\} \ln \left( \frac{D_0}{D_i} \right) \quad (13)$
	DNV,2010 [45].	$P = \frac{2}{\sqrt{3}} \frac{2t}{D_m} \sigma_y \quad (14)$
	KleverFJ, 2006 [46]. Stewart G, 1994 [47].	$P = \left( \frac{2}{(\sqrt{3})^{n+1}} + \frac{1}{2^{n+1}} \right) \frac{2t}{D_m} \sigma_{UTS} \quad (15)$
Combined internal pressure and axial applied stress	Klever FJ, 2006 [46]. Stewart G, 1994[47].	$\sigma_{eff} = \sigma_{UTS} \sqrt{1 - \left( 4^{1-n} - 3^{1-n} \right) \frac{3^{1+n}}{4} \left( \frac{P_i}{\frac{2t}{D_m} \sigma_{UTS}} \right)} \quad (16)$
		$P = \frac{2}{(\sqrt{3})^{n+1}} \sigma_{UTS} \sqrt{1 - \frac{4^{1-n} - 3^{1-n}}{3^{1-n}} \left( \frac{P_i D_m}{4t} \right)^2} \quad (17)$

Table 2: Overview of multiaxial fracture in the limit conditions.

## CUMULATIVE DAMAGE EVALUATION BY A THEORY COMBINATION

The prediction of intervention's time by the maintenance services is generally very difficult unless we figure out when the damage could occur. In fact, determining the damage, In the ASME code, is generally evaluated through linear methods like MINER, although the results obtained by this method are very approximate. However, the non-linear quantifications of the damage seem difficult due to the big number of parameter.



In this perspective, simplifying the testing procedures is required by opting for static tests instead of dynamic tests which are so expensive and difficult.

The unified theory developed by Bui Quoc in 1971, has the advantage of ensuring an assessment of the damage through dynamic and static tests.

In this paper we evaluated the damage through a combined theory using the unified theory [48,49] and burst pressure Eq. (16) and (17).

$$D_s = \frac{1 - \frac{P_{ur}}{P_u}}{1 - \frac{P_a}{P_u}} \quad (18)$$

where  $P_{ur}$  is the burst pressure for notched cylinder,  $P_u$  is the burst pressure for a unnotched cylinder and  $P_a$  is the pressure before rupture.

The approach presented in this part of the paper is based on artificial damage creation by creating a notch with a variable depth and then we evaluate the damage for each depth. The cylinder we are working on has a thickness of 5.8 mm, an external diameter of 63 mm and a length of 400 mm. The operating pressure for the case study is 0.6 MPa. The mechanical properties of the studied material are given in the Tab. 3 obtained from mechanical characterization we did through tensile tests.

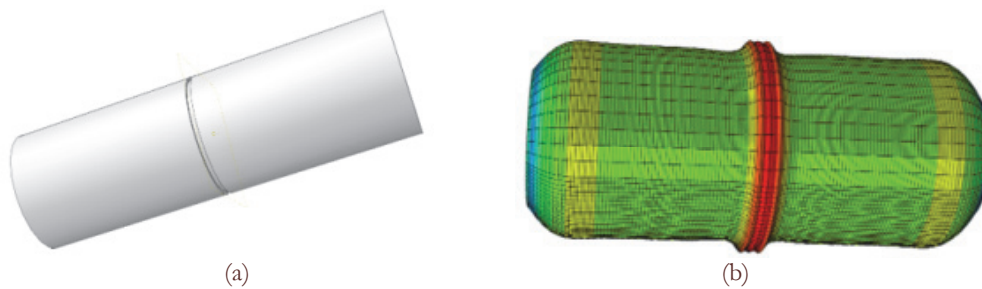


Figure 2: Notched cylinder (a) FEA of notched cylinder (b).

Material	Yield stress $\sigma_y$ (MPa)	Ultimate stress $\sigma_u$ (MPa)
P265GH	320	470
A36	372	621

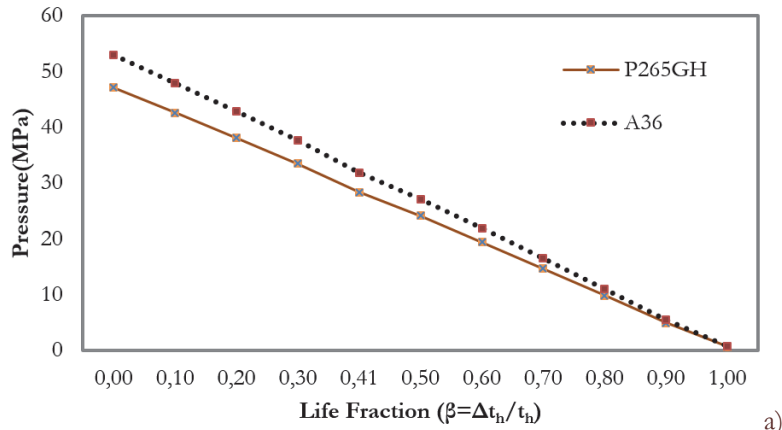
Table 3: Mechanical properties of materials.

In this part of the paper, we proved that a combination between the unified theory and the burst pressure formulas is possible. Then we showed that we can predict the fracture by theoretical calculations. We proved also that the unified theory can be used with burst pressure formulas based on combined applied axial stress and internal pressure. The burst pressure is decreasing while the notch depth increases. Meanwhile, the cumulative based on the burst pressure formulas is almost the same as the one obtained by experimental tests and the use of the unified theory.

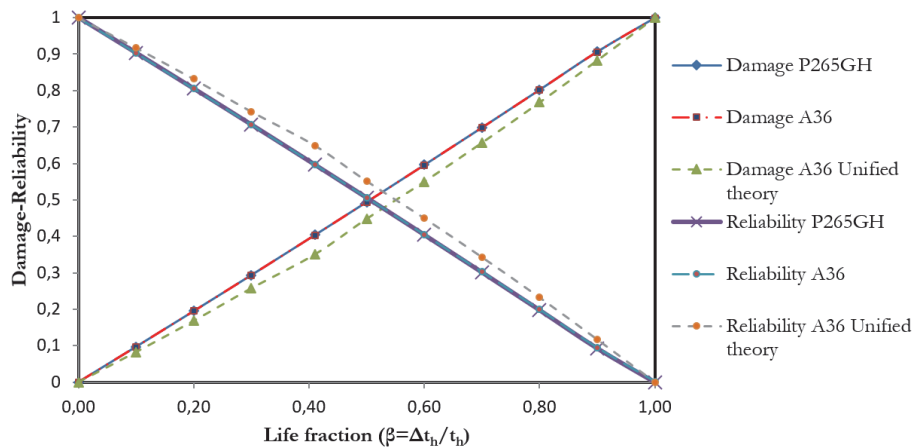
## CONCLUSION

Pressure vessel design pass through many steps as shown in this article. The minimum requirement according the ASME code has been resumed in the first part of the article. Then, a review and a discussion of pressure vessels fatigue design have been detailed. In the third part of the article, we discussed the multiaxial fracture by giving an overview of almost the methods and formulas of burst or rupture pressure. The limit pressure is determined through the

internal pressure, applied stress or the combination of both of them. In the last part, we want to make these formulas in proof by a combination of the unified theory and the burst pressure formulas for static damage evaluation. The obtained result was compared with the damage of A36 steel subjected to uniaxial fatigue tests and tensile tests. We noticed that the results are almost the same. The validation of this combination was done for P265GH and A36 steel.



a)



b)

Figure 3: Failure pressure (a) and Damage and reliability (b) of P265GH and A36 function of the ratio notch depth-thickness.

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